Marker Board Notes for September 30, 2020

Takeaways from Lecture 3 Part 2 on FIR Filters for Linear Time-Invariant (LTI) Systems

- Linear Phase
  - Important in some applications
  - Same delay from input to output for all frequencies
  - Filter must be FIR and have even or odd symmetry about midpoint of impulse response
- Fundamental Theorem
  - LTI systems do not generate new frequencies
  - Any frequencies on the output had to be present in the input signal

i**mageRampsCosines.m** image processing demonstration

- 512 x 512 images with each pixel being an eight-bit unsigned value [0, 255]
- Horizontal frequency increased from 0 to π by decreasing the period L
  - For a cosine signal $\cos(\omega_0 n)$, discrete-time frequency is $\omega_0 = 2\pi / L$ in rad/sample
  - Low-frequency regions of the image have constant or slowly-varying amplitude values
  - High-frequency regions correspond to fast alternation between min and max values
  - Edges/texture have significant high-frequency content and some low-frequency content
- Consider an edge in a row of the image: row[n] = 255 $u[n-n_0]$. It has a sudden jump in value from 0 to 255 at $n = n_0$. This has high-frequency and some low-frequency content

Lecture Slide 5-18 Averaging Filter (LTI System)

Normalized: $y_1[n] = ½\, x[n] + ½\, x[n-1]$. 2 multiplications and 1 additions per output sample.
Unnormalized: $y_2[n] = x[n] + x[n-1]$. 1 addition per output sample. Lower comp. complexity.
Impulse responses $h_1[n] = ½\, \delta[n] + ½\, \delta[n-1]$ and $h_2[n] = \delta[n] + \delta[n-1] = 2\, h_1[n]$:

$$H_2(\omega) = 2\, H_1(\omega) = 2\, |\, H_1(\omega)\, |\, e^{j \prec H_1(\omega)} = (2\, |\, H_1(\omega)\, |)\ e^{j \prec H_1(\omega)}$$

- No difference in the phase responses: $\prec H_2(\omega) = \prec H_1(\omega)$
- Magnitude response (in linear units) scaled by constant; no change in shape—still lowpass

Lecture Slide 5-19

Rewrite j term in the frequency response as a phase of $\pi/2$: $e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = 0 + j = j$

Lecture Slide 5-20

Rewrite the frequency response for the first-order difference filter (from lecture slide 5-19)

$$H(\omega) = \sin\left(\frac{\omega}{2}\right) e^{j\left(\frac{\pi}{2} - \frac{\omega}{2}\right)}$$

When ω is in $[0, \pi]$, $H(\omega)$ is already in magnitude-phase form because $\sin(\omega/2)$ is non-negative.
When ω is in $(-\pi, 0)$, $\sin(\omega/2)$ is negative. We can negate both terms and replace -1 with $e^{-j\pi}$:

$$\left(-\sin\left(\frac{\omega}{2}\right)\right)\left(-e^{j\left(\frac{\pi}{2} - \frac{\omega}{2}\right)}\right) = \left(-\sin\left(\frac{\omega}{2}\right)\right)\left(e^{-j\pi}\, e^{j\left(\frac{\pi}{2} - \frac{\omega}{2}\right)}\right) = \left(-\sin\left(\frac{\omega}{2}\right)\right)\, e^{j\left(-\frac{\pi}{2} - \frac{\omega}{2}\right)}$$

Here's the resulting magnitude-phase form, which is plotted on lecture slide 5-20:

$$H(\omega) = \begin{cases} \left(-\sin\left(\frac{\omega}{2}\right)\right)\, e^{j\left(-\frac{\pi}{2} - \frac{\omega}{2}\right)} & \text{for } \omega \in (-\pi, 0) \\ \sin\left(\frac{\omega}{2}\right)\, e^{j\left(\frac{\pi}{2} - \frac{\omega}{2}\right)} & \text{for } \omega \in [0, \pi] \end{cases}$$