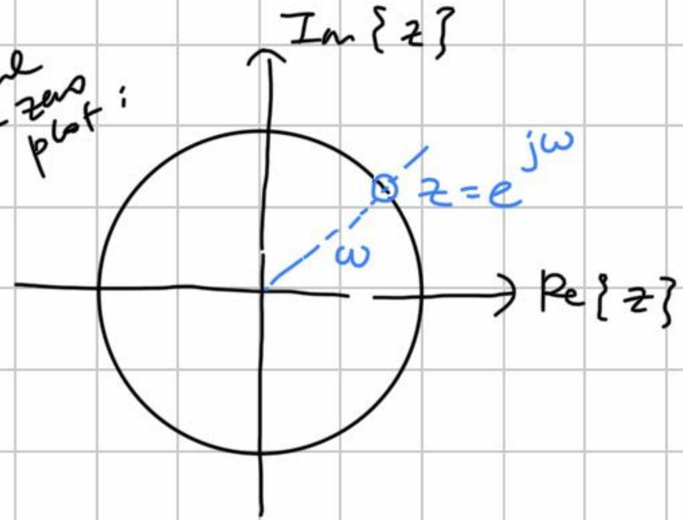
DSP Lecture 02/22/2023. Wednesday.

What was covered previously? IIR Filter Analysis / Design

$\exists$ poles & zeros tell us a lot about filter analysis/design!

general pole-zero plot:



First Order filter: (goal: build a Low pass filter!)

$\Rightarrow$ 1 pole, 1 zero

$$H(z) = C \cdot \frac{(z - z_0)}{(z - p_0)}$$

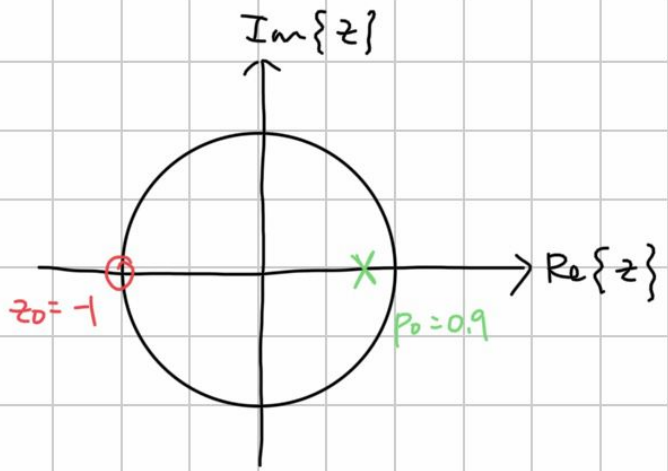
same transfer fxn.

$$H(z) = C \cdot \frac{1 - z_0 z^{-1}}{1 - p_0 z^{-1}} = \frac{Y(z)}{X(z)}$$

$$C(1 - z_0 z^{-1}) X(z) = Y(z) \cdot (1 - p_0 z^{-1})$$

$$Y(z) = C X(z) - C z_0 z^{-1} X(z) + p_0 z^{-1} Y(z)$$

$$y[n] = c\,x[n] - c z_0 x[n-1] + p_0 y[n-1]$$ $\Leftarrow$ difference eqtn.



where to put pole/zero for 1st order IIR to get Low pass filter?

$\Rightarrow$ ☆ because 1st order, we want to choose real-valued pole/zero.

pole: place near zero frequencies ($\omega = 0$)

let magnitude of pole be close to unit circle!

say, $p_0 = 0.9$

or $p_0 = 0.75, 0.8 \ldots$

zero: pretty much infinite options!

say, $z_0 = -1$ $\Rightarrow$ best choice!

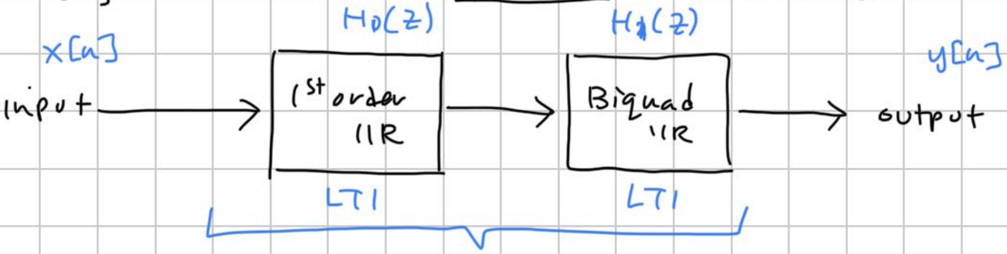
$z_0 = -1.1$

$z_0 = 0.9$ (etc.)

☆ $z_0 = -1$ is best choice for zero because it allows for greatest attenuation in stopband frequencies.

what if I want to build 3rd order IIR?

Say for example: Cascaded Implementation



$H(z) = H_0(z) \cdot H_1(z)$ ⇒ Cascaded filters

say, $H(z) = \dfrac{z^{-2}}{(1-p_0 z^{-1})(1-p_1 z^{-1})} = \dfrac{1}{(z-p_0)(z-p_1)}$ (@ 10:51AM)

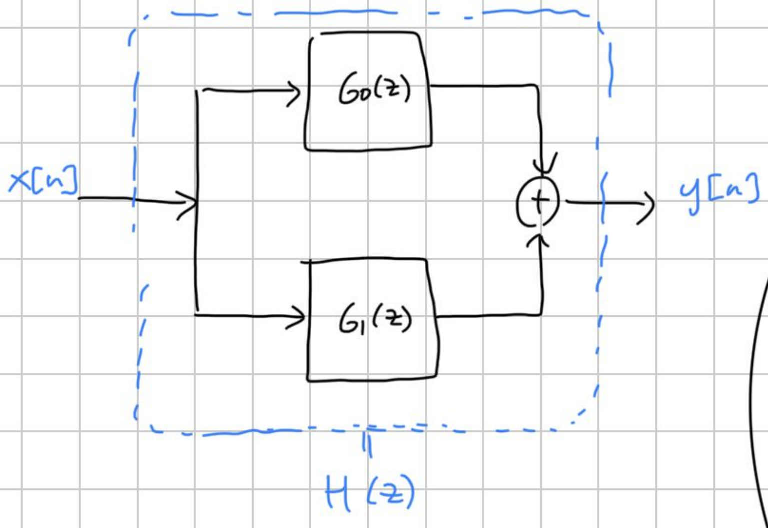
$H_0(z) = \dfrac{1}{z-p_0}$, $H_1 = \dfrac{1}{z-p_1}$

what's the trade off? ⇒ CON delay of filters!

your 2nd filter $H_1(z)$ must wait for 1st filter $H_0(z)$ to finish

PRO? flexibility in design (order of cascade does not matter. $H = H_1 \cdot H_0 = H_0 \cdot H_1$)

alternatively, can build Parallel Filters



partial fraction decomposition

$H(z) = \dfrac{A}{z-p_0} + \dfrac{B}{z-p_1}$

$A = \dfrac{1}{p_0 - p_1}$

$B = \dfrac{1}{p_1 - p_0}$

$G_0(z) = A/(z-p_0)$

$G_1(z) = B/(z-p_1)$

$H(z) = G_0(z) + G_1(z)$

trade offs? pro reduced delay!

G_1 doesn't depend on G_0!

much faster runtime

con there is only 1 configuration of $G(z)$

@ 10:55AM ⇒ move to slide deck. (slides 6–30)

Quality Factor of Digital Biquads (Cascade of Biquads)

⇒ measures stability of filter

(slide 6-31)

IIR Single-Sections : 1 big transfer fxn.

all feedforward coefficients together (Numerator, b)

all feedback coefficients together (Denominator, a)

"Direct Form" structure

→ a dot product between previous/current inputs/outputs and coefficients.

block diagram on Slide 6-32

what's the problem with this implementation? ⇒ loss of accuracy.

due to limited # of bits

⇒ introduces non-linearity

due to truncation of numerical calculations

⇒ Reduced Stability ☹

(slide 6-34)

so we don't like to implement

in Direct Form Structure/Single Section.

⇒ so instead we use Biquad format

(still have numerical error/inaccuracy,

but less severe!)

(break @ 11:08 AM)

(return @ 11:17 AM) ⇒ demo & in-lecture assignment.