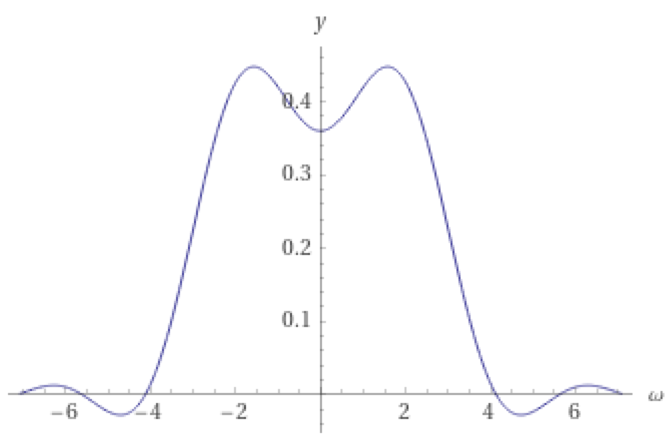
[10:30] Interpolation and pulse shaping

- Reconstruction can be expressed in terms of a mixed (continuous and discrete) signal convolution.

$$\tilde{y}(t) = \sum_{n=-\infty}^{\infty} y[n]\, p(t - T_s n)$$

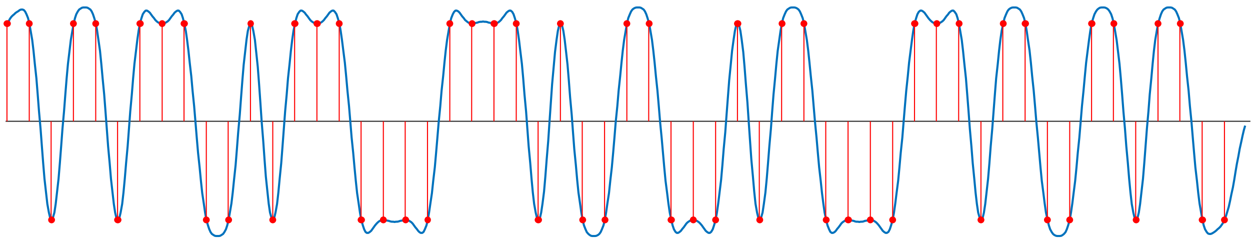
- Ideally, we would use an infinite, two-sided sinc for $p(t)$
  - This is a perfect lowpass filter to enforce the sampling theorem ($f_s > 2f_{\max}$)
  - Alternatives with finite duration:
    - Rectangular pulse with width T_s
    - Triangular pulse with width $2T_s$
    - Truncated sinc with zero crossings at multiples of T_s
    - Raised cosine with zero crossings at multiples of T_s
- Oversampling greatly improves the quality of the reconstruction
- Raised cosine
  - Still infinite in length, but has tails that decay much quicker than the sinc, so truncation has less effect
  - Parameterized by rolloff factor α (also sometimes called β)
    - $\alpha = 0$: identical to sinc, frequency response is a rectangle
    - $\alpha = 1$: Frequency response looks like $1 + \cos(f)$
    - $\alpha = 0$ is a better lowpass filter, but $\alpha = 1$ has faster decaying tails.
- What happens when we truncate the sinc in the time domain?
  - No longer a perfect rectangle in the frequency domain

$$\mathcal{F}\left\{\text{rect}\left(\frac{t}{4}\right)\frac{\sin(\pi t)}{\pi t}\right\} =$$

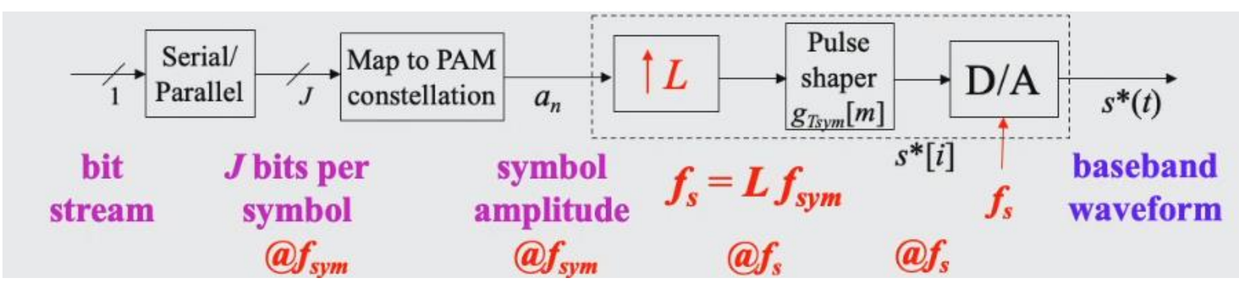


[11:20] Pulse amplitude modulation

- To transmit data, convert it to a sequence of bits, then convert to sequence of pulses.
  - Example: 2-PAM. Each bit becomes a pulse. A bit of 1 becomes a positive amplitude pulse and and bit of 0 becomes a negative amplitude pulse.



- It is possible to map multiple bits to each pulse to increase the data rate
  - 4-PAM example: Each pair of bits gets mapped to a pulse
    - 11 gets mapped to -3
    - 10 gets mapped to -1
    - 00 gets mapped to $+1$
    - 01 gets mapped to $+3$
  - Uniform spacing of symbol amplitudes (each spaced by distance of 2)
  - Mapping more bits to each symbol makes it harder to decode in the presence of noise.
- Peak in the baseband signal will be no greater than twice the maximum symbol amplitude.



- Upsampling by L lowers the data rate by a factor of L but also uses less bandwidth by a factor of L.
  - Transmission bandwidth when using raised cosine:

$$\frac{1}{2} f_{\text{sym}}(1+\alpha)$$