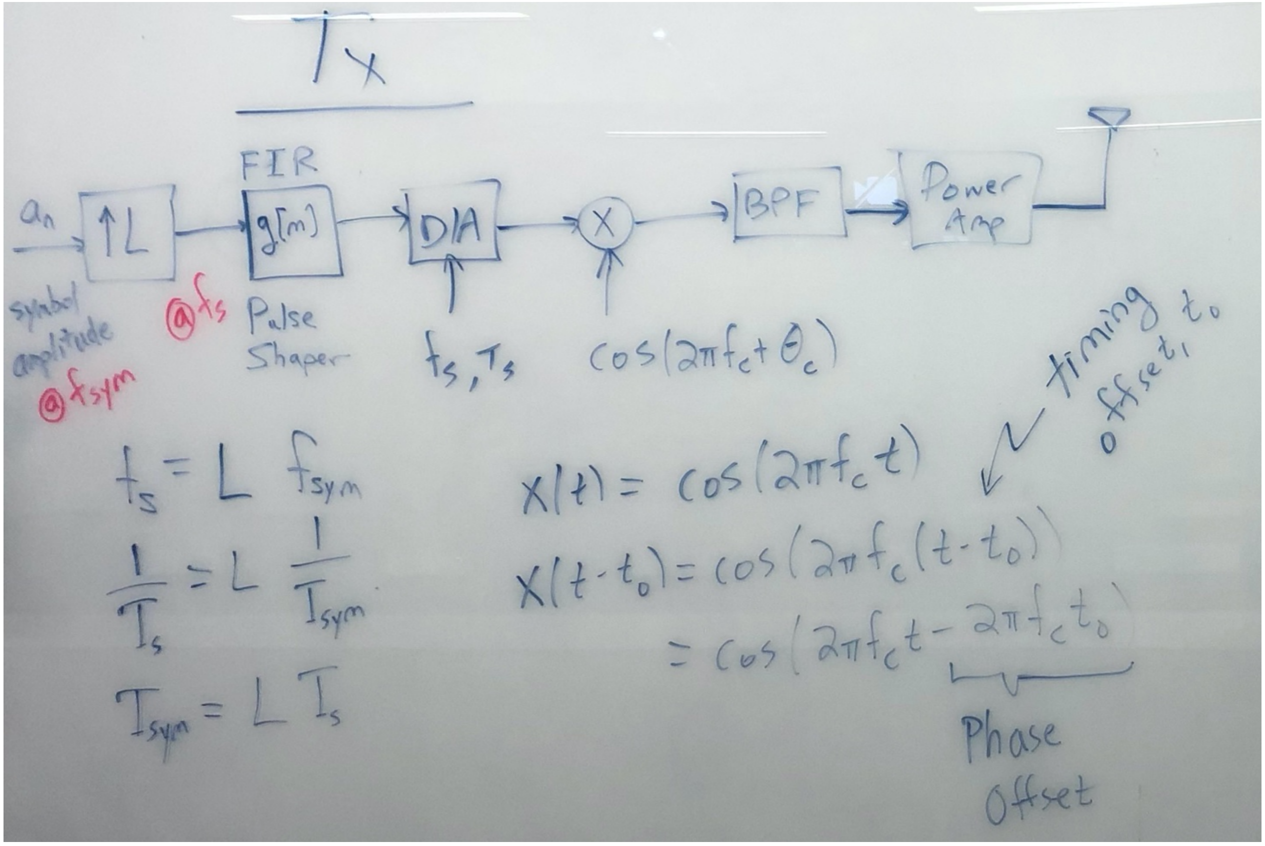
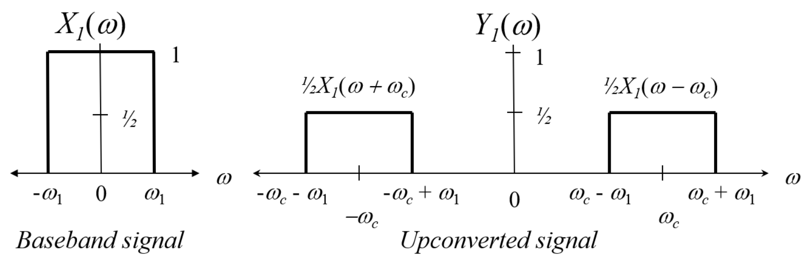


[10:30am] Implementation details for receiver to go with the transmitter above

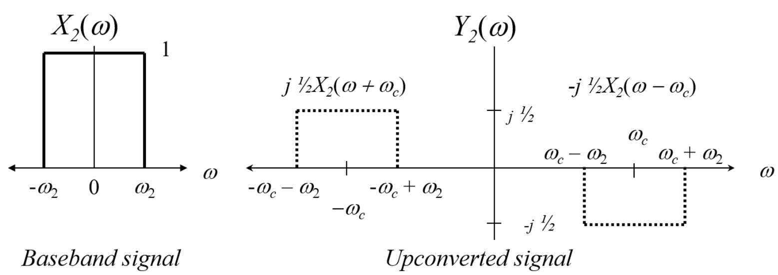
- Receiver must track f_c, θ_c
  - Assume f_c approximately correct
  - Use phase locked loop to adapt and correct for θ_c (HW 6.1)
- Receiver must track f_{sym}, τ_{sym}
  - Assume f_{sym} is approximately correct
  - Perform symbol timing recovery to estimate τ_{sym} and compensate (HW 6.2)
- Receiver must compensate for spreading in time and distortion in frequency due to the channel impulse response by using an equalizer (HW 7.1 and 7.2)
- Receiver must compensate for thermal noise – matched filter
  - Impulse response to maximize SNR is $h_{opt}[m] = k\ g^*[L - m]$ for real constant k
  - When $g[m]$ is real and even symmetric, given that the delay L can be any multiple of L and still maximize SNR, $h_{opt}[m] = g[m]$
  - Correlating filter (HW 4.2, 4.3, 5.2)
- Receiver must track fading and compensate using automatic gain control (HW 7.3, HW 5.1 prolog)

[10:50] Quadrature Amplitude Modulation (QAM)

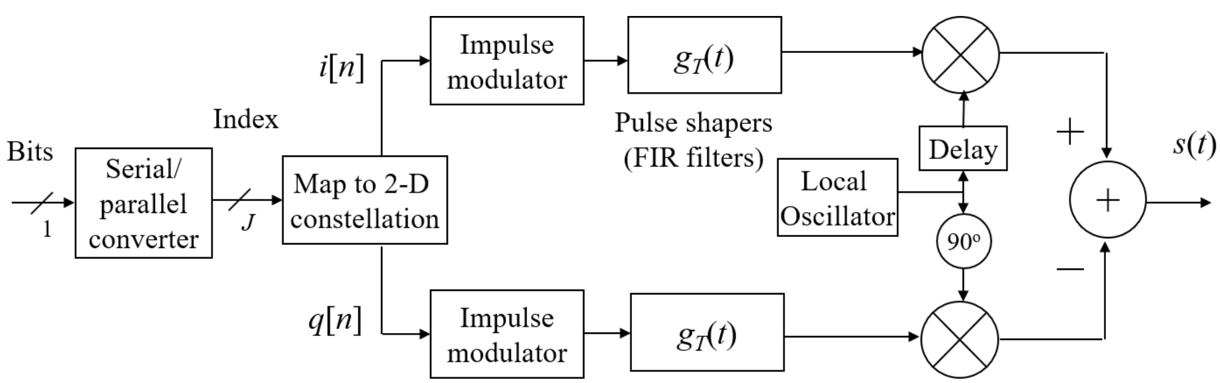
- Recall that PAM modulates information into amplitude of pulse
- Quadrature amplitude modulation is a two-dimensional extension of PAM
- Combine two PAM signals in a way that they do not interfere with each other
    - Amplitudes of sine and cosine
    - Equivalently: amplitude and phase of sinusoid
    - Doubles spectral efficiency
    - Increases sensitivity to phase error
- Amplitude modulation by cosine
    - $y_1(t) = x_1(t)\cos(\omega_c t) \overset{\mathcal{F}}{\Longleftrightarrow} Y_1(\omega) = \frac{1}{2}X_1(\omega+\omega_c) + \frac{1}{2}X_1(\omega-\omega_c)$
    - Results in doubling of bandwidth



- Amplitude modulation by sine
    - $y_2(t) = x_2(t)\sin(\omega_c t) \overset{\mathcal{F}}{\Longleftrightarrow} Y_2(\omega) = \frac{j}{2}X_2(\omega+\omega_c) - \frac{j}{2}X_2(\omega-\omega_c)$
    - Still results in doubling of bandwidth



Continuous Time QAM Transmitter



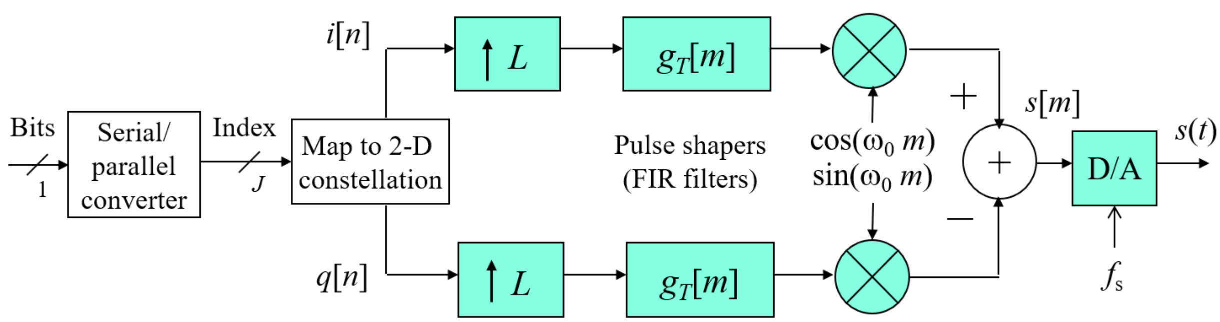
- Use same oscillator to generate sine and cosine
  - 90° phase shift can be performed using Hilbert transformer
  - $\cos(2\pi f_0 t) \Rightarrow \frac{1}{2}\delta(f+f_0) + \frac{1}{2}\delta(f-f_0)$
  - $\sin(2\pi f_0 t) \Rightarrow \frac{j}{2}\delta(f+f_0) - \frac{j}{2}\delta(f-f_0)$
  - Need a response of j for negative frequencies and $-j$ for positive frequencies:

$$H(f) = -j\,\mathrm{sgn}(f) = \begin{cases} j & \text{if} \quad f<0 \\ 0 & \text{if} \quad f=0 \\ -j & \text{if} \quad f>0 \end{cases}, \qquad j = \underbrace{\overbrace{e^{j\pi/2}}^{90^\circ}}_{\text{mag. of 1}}$$

$$\cos(2\pi f_0 t) \rightarrow \boxed{\text{Hilbert Transformer}} \rightarrow \sin(2\pi f_0 t)$$
$$\sin(2\pi f_0 t) \rightarrow \boxed{\text{Hilbert Transformer}} \rightarrow -\cos(2\pi f_0 t)$$

  - Discrete-time Hilbert transformer: approximate as FIR filter

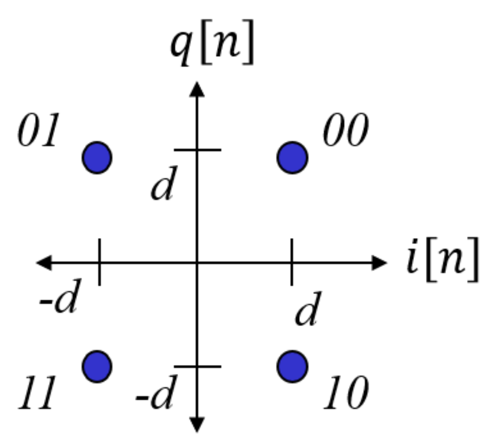
Discrete Time QAM Transmitter



[11:30] QAM Constellation

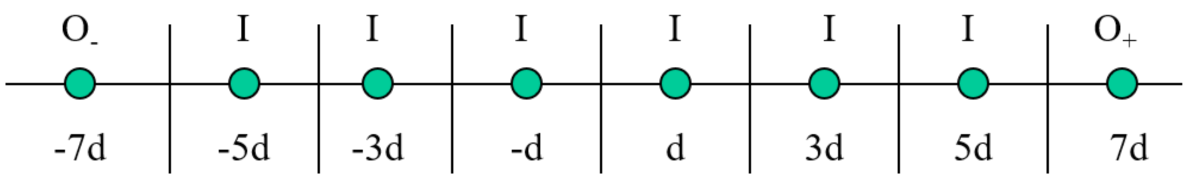
- QAM constellation :
- Quadrature $q[n]$ (PAM symbol amplitude)
- In-Phase $i[n]$ (PAM symbol amplitude)

Symbol of bits	$i[n]$	$q[n]$
00	d	d
01	$-d$	d
10	d	$-d$
11	$-d$	$-d$



[10:35] Performance analysis for M-level PAM

- Sampling matched filter output without ISI:
  $x(nT_{sym}) = s(nT_{sym}) + v(nT_{sym})$ is received signal
  $s(nT_{sym}) = a_n = (2i - 1)d$ for $i = -M/2 + 1, \dots, M/2$ is transmitted signal
  $v(nT) \sim N(0; \sigma^2 / T_{sym})$ is output of matched filter for input of gaussian noise
- 8-PAM example



  - Symbol error probability will be different for outer and interior points
  - Outer points only have decision errors on one side of distribution
  - Inner points can have decision errors on both sides of distribution

$$P_{\text{Inner}}(\text{error}) = P\left(\left|v(nT_{sym})\right| > d\right) = 2Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

$$P_{\text{outer}}(\text{error}) = P\left(v(nT_{sym}) > d\right) = Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

$$P(\text{error}) = \frac{2(M-1)}{M} Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$$

  - Factor $\frac{2(M-1)}{M}$ has small variation: $\frac{2(2-1)}{2} = 1,\ \lim_{M\to\infty} \frac{2(M-1)}{M} = \lim_{M\to\infty} \left(2 - \frac{2}{M}\right) = 2$
  - The Q function decays faster than exponential for large values of its argument
  - What can a system designer do to decrease the symbol error probability?
    - Increase d, which is in Volts. Also increases transmit power.
    - Increase T_{sym}. Decreases bit rate $J\ f_{sym}$ where J is number of bits/symbol.