The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 18, 2013

Course: EE 445S Evans

Name: Bollywood, Sally
Last, First

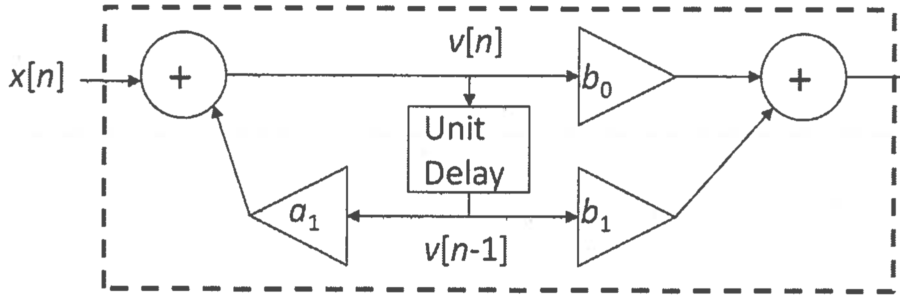
- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	~~27~~ 28		Discrete-Time Filter Analysis
2	24		Discrete-Time Filter Design
3	24		System Identification
4	24		Modulation and Demodulation
Total	100		

28

Problem 1.1 *Discrete-Time Filter Analysis.* ~~27~~ points.

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following block diagram:



First-Order IIR Section.

Derived from Slide 6-6 on Discrete-Time Biquad.

Constants a_1, b_0 and b_1 are real-valued, and $|a_1| < 1$.

(a) From the block diagram, derive the difference equation relating input $x[n]$ and output $y[n]$. Your final answer should not include $v[n]$. *6 points.*

Working backwards from transfer function in part (c) below,

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \Rightarrow (1 - a_1 z^{-1}) Y(z) = (b_0 + b_1 z^{-1}) X(z).$$

Applying the inverse z-transform to both sides,

$$y[n] - a_1 y[n-1] = b_0 x[n] + b_1 x[n-1]$$

(b) What are the initial condition(s)? What value(s) should they be assigned and why? *4 points.*

$v[-1] = 0$ for the system to be causal, linear and time-invariant.

Equivalently, $x[-1] = 0$ and $y[-1] = 0$.

(c) What is the transfer function in the z-domain? What is the region of convergence? *5 points.*

$$H(z) = \frac{Y(z)}{X(z)} = \frac{V(z)}{X(z)} \cdot \frac{Y(z)}{V(z)} = \frac{1}{1 - a_1 z^{-1}} \cdot (b_0 + b_1 z^{-1})$$

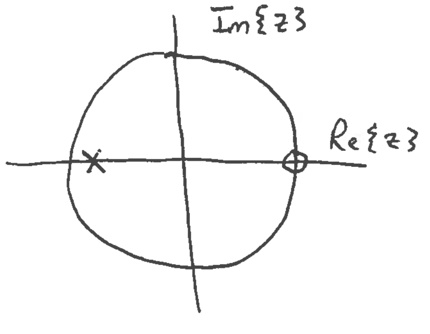
$$= \frac{b_0 + b_1 z^{-1}}{1 - a_1 z^{-1}} \quad \text{for } |z| > |a_1|$$

(d) Find the equation for the frequency response of the filter. Justify your approach. *6 points.*

Because $|a_1| < 1$, the region of convergence $|z| > |a_1|$ includes the unit circle.

$$H_{freq}(\omega) = H(z)\Big|_{z = e^{j\omega}} = \frac{b_0 + b_1 e^{-j\omega}}{1 - a_1 e^{-j\omega}}$$

(e) For $a_1 = -0.9$, $b_0 = 1$, and $b_1 = -1$, draw the pole-zero diagram. What is the best description of the frequency selectivity: lowpass, highpass, bandstop, bandpass, allpass or notch? *7 points.*



Passband is centered at $\omega = \pi$ due to pole at $z = -0.9$.

Stopband is centered at $\omega = 0$ due to zero at $z = 1$.

Highpass filter.

Problem 1.2 *Discrete-Time Filter Design.* 24 points. Configurable/programmable notch filte

Consider a causal second-order discrete-time infinite impulse response (IIR) filter with transfer function $H(z)$.

The filter is a bounded-input bounded-output stable, linear, and time-invariant system.

Input $x[n]$ and output $y[n]$ are real-valued.

The feedback and feedforward coefficients are real-valued. $\Rightarrow$ Poles are conjugate symmetric.
Zeros are conjugate symmetric.

You will be asked to design and implement a notch filter:

f_0 is the frequency in Hz to be eliminated, and

f_s is the sampling rate in Hz where $f_s > 2 f_0$

Assume that the gain of the biquad is 1. $\Rightarrow$ $C = 1$

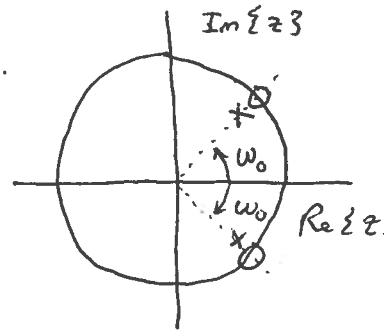
(a) Give a formula for the discrete-time frequency ω_0 in rad/sample to be eliminated. *3 points.*

$$\omega_0 = 2\pi \frac{f_0}{f_s}$$

(b) Give formulas for the two poles and the two zeros as functions of ω_0. *6 points.*

Poles: $p_0 = 0.9 e^{j\omega_0}$ and $p_1 = 0.9 e^{-j\omega_0}$

Zeros: $z_0 = e^{j\omega_0}$ and $z_1 = e^{-j\omega_0}$



(c) Give formulas for the three feedforward and two feedback coefficients. Simplify the formulas to show that all of these coefficients are real-valued. *9 points.*

$$H(z) = \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})} = \frac{1 - (z_0 + z_1) z^{-1} + z_0 z_1 z^{-2}}{1 - (p_0 + p_1) z^{-1} + p_0 p_1 z^{-2}}$$

Feed forward coefficients

$b_0 = 1$

$b_1 = -(z_0 + z_1) = -(e^{j\omega_0} + e^{-j\omega_0}) = -2\cos\omega_0$

$b_2 = z_0 z_1 = e^{j\omega_0} e^{-j\omega_0} = 1$

Feed back coefficients

$a_1 = p_0 + p_1 = 1.8 \cos\omega_0$

$a_2 = -p_0 p_1 = -0.81$

(d) How many multiplication-accumulation operations are needed to compute one output sample given one input sample? *3 points.*

$y[n] = a_1 y[n-1] + a_2 y[n-2] + x[n] + b_1 x[n-1] + x[n-2]$

3 multiplications and 4 additions $\Rightarrow$ 4 multiply-accumulates

(e) How many instruction cycles on the TI TMS3206748 digital signal processor used in lab will take to compute one output sample given one input sample? *3 points.*

$N = 5$ coefficients

$N + 28 = 33$ instruction cycles from Appendix N in reader.

Alternate Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{\frac{1}{1-z^{-1}}} = (1+z^{-1})(1-z^{-1}) = 1 - z^{-2} \Rightarrow h[n] = \delta[n] - \delta[n-2]$$

Problem 1.3 *System Identification.* 24 points. Solution uses de-convolution.

Consider a causal discrete-time finite impulse response (FIR) filter with impulse response $h[n]$.

The filter is a bounded-input bounded-output stable, linear, and time-invariant system.

For input $x[n] = u[n]$, the output is $y[n] = \delta[n] + \delta[n-1]$. Let $h[n]$ have $M+1$ coefficients.

(a) Determine the impulse response $h[n]$. *18 points.*

$$y[n] = x[n] * h[n] = \sum_{m=0}^{M} h[m]\, x[n-m]$$

n

0: $1 = y[0] = h[0]\, x[0] \Rightarrow 1 = h[0] \Rightarrow h[0] = 1$

1: $1 = y[1] = h[0]\, x[1] + h[1]\, x[0]$

$1 = h[0] + h[1] \Rightarrow h[1] = 0$

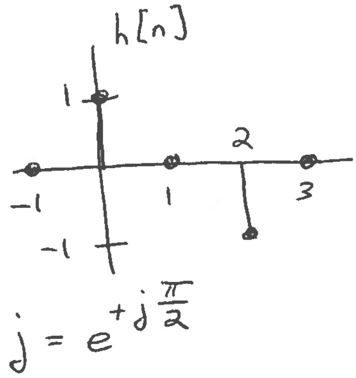
2: $y[2] = h[0]\, x[2] + h[1]\, x[1] + h[2]\, x[0]$

$0 = h[0] + h[1] + h[2] \Rightarrow h[2] = -h[0] \Rightarrow h[2] = -1$

3: $y[3] = h[0]\, x[3] + h[1]\, x[2] + h[2]\, x[1] + h[3]\, x[0]$

$0 = h[0] + h[1] + h[2] + h[3] \Rightarrow h[3] = 0$

Check: $h[n] * u[n] \stackrel{?}{=} \delta[n] + \delta[n-1]$ $\boxed{\text{YES}}$



(b) Compute the group delay through the filter as a function of frequency. *6 points.*

$$H_{freq}(\omega) = 1 - e^{-j2\omega}$$

$$= e^{-j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$$

$$= \underbrace{2\sin(\omega)}_{\text{amplitude term}}\ \underbrace{j\, e^{-j\omega}}_{\text{phase term}}$$

Note: $j = e^{+j\frac{\pi}{2}}$

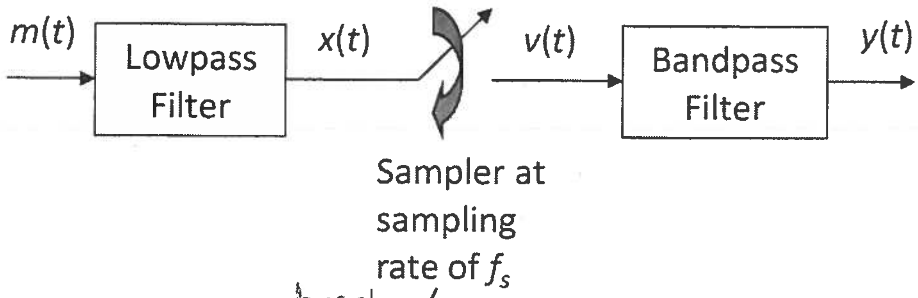
Group delay is 1 sample.

$$\text{Delay}(\omega) = -\frac{d}{d\omega} \angle H_{freq}(\omega) = 1$$

Except for two points of discontinuity, $\angle H_{freq}(\omega) = -\omega + \frac{\pi}{2}$

Problem 1.4. Modulation and Demodulation. *24 points*.

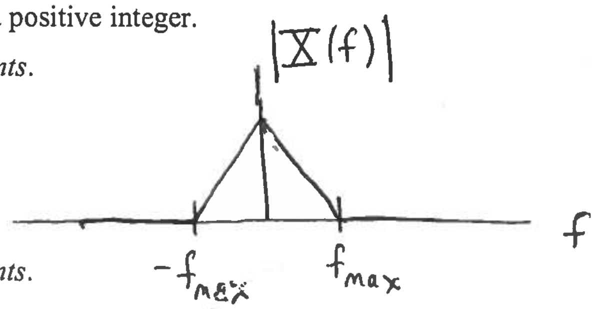
A mixer can be used to realize sinusoidal amplitude modulation $y(t) = x(t) \cos(2 \pi f_c t)$ for baseband signal $x(t)$:



Assume that $x(t)$ is a ideal ~~bandpass~~ baseband signal whose magnitude spectrum is zero for ~~$f > f_{max}$~~ $|f| > f_{max}$.

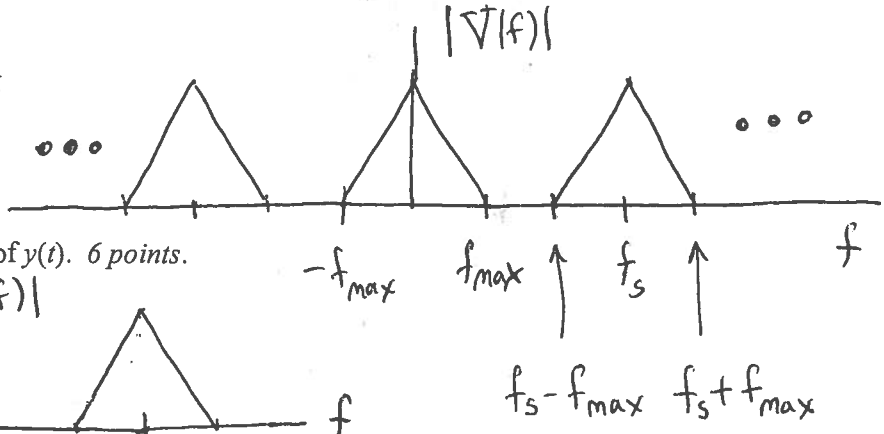
Assume that $f_s > 2 f_{max}$ and $f_c = m f_s$ where m is a positive integer.

(a) Draw the magnitude spectrum of $x(t)$. *6 points*.

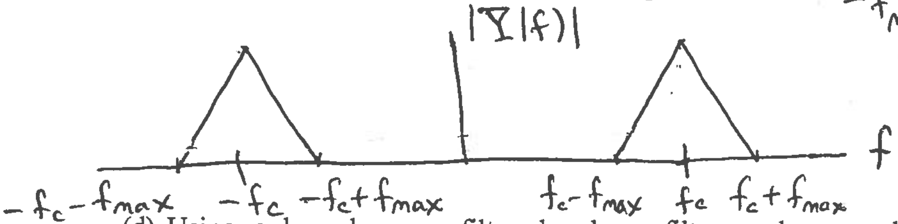


(b) Draw the magnitude spectrum of $v(t)$. *6 points*.

Spectrum of $X(f)$ is replicated at offsets in frequency equal to multiples of f_s

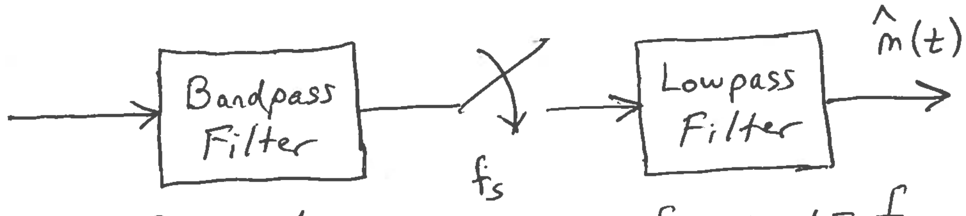


(c) Draw the magnitude spectrum of $y(t)$. *6 points*.



(d) Using only a lowpass filter, bandpass filter, and a sampler, give a block diagram for demodulation. *6 points*.

$f_c = m f_s$



Passband

$f_c - f_{max} < f < f_c + f_{max}$

$f_{passband} = f_{max}$

$f_{stopband} = f_s - f_{max}$