The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 17, 2014

Course: EE 445S Evans

Name: Code Lyoko

Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Discrete-Time Filter Analysis
2	24		Upconversion
3	30		Filter Design
4	18		Potpourri
Total	100		

Problem 1.1 *Discrete-Time Filter Analysis.* 28 points.

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following equation in the discrete-time domain:

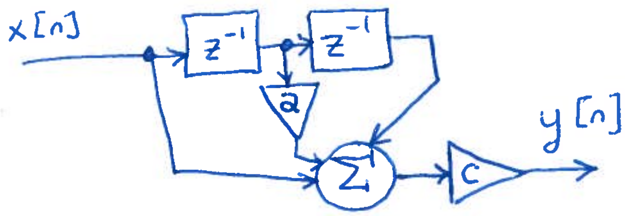
$$y[n] = C\,(\,x[n] + 2\,x[n-1] + x[n-2]\,)$$

Constant C is real-valued and is not equal to zero.

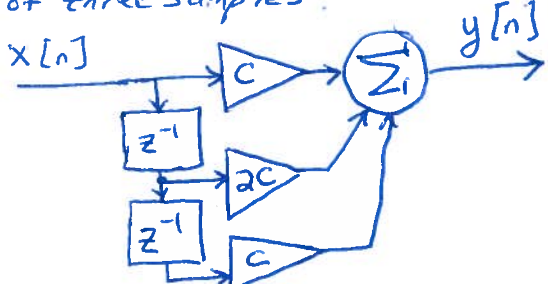
(a) Is this a finite impulse response filter or an infinite impulse response filter? Why? *4 points.*

FIR. Reason #1: Output value does not depend on previous output values.
Reason #2: Impulse response is $h[n] = C\,(\,\delta[n] + 2\,\delta[n-1] + \delta[n-2]\,)$ which has a non-zero extent of three samples.

(b) Give a block diagram for the filter. *4 points.*



or



(c) What are the initial condition(s)? What value(s) should they be assigned and why? *4 points.*

Let $n=0$: $y[0] = C\,(\,x[0] + 2\,x[-1] + x[-2]\,)$; Initial conditions $x[-1]$ and $x[-2]$. Set to zero to satisfy system properties causality, linearity, time-invariance.

Let $n=1$: $y[1] = C\,(\,x[1] + 2\,x[0] + x[-1]\,)$;

(d) Find the equation for the transfer function of the filter in the z-domain including the region of convergence. *4 points.*

Take z-transform of both sides of the difference equation,

$$Y(z) = C\,(\,X(z) + 2z^{-1}X(z) + z^{-2}X(z))$$

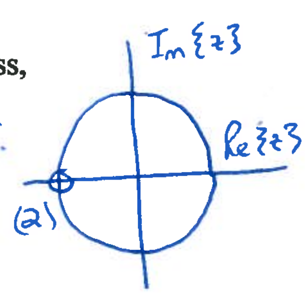
$$H(z) = \frac{Y(z)}{X(z)} = C\,(\,1 + 2z^{-1} + z^{-2}) \text{ for all } z \text{ except } z=0.$$

(e) Find the equation for the frequency response of the filter. *4 points.*

Because the region of convergence includes the unit circle,

$$H_{freq}(\omega) = H(z)\Big|_{z=e^{j\omega}} = C\left(1 + 2e^{-j\omega} + e^{-2j\omega}\right)$$

(f) What is the best description of the frequency selectivity of the filter: lowpass, highpass, bandstop, bandpass, allpass or notch? Why? *4 points*



Lowpass. Reason #1: Two zeros at $z=-1 \Rightarrow \omega = \pi, -\pi$.
Zeros indicate center of stop band.

$H(z) = C\,(1+z^{-1})^2$ Reason #2: Try values of ω. $H_{freq}(\omega)$ is $4C$ @ $\omega=0$, $2jC$ @ $\omega = \frac{\pi}{2}$, 0 @ $\omega=\pi$.

(g) Find a value for C that normalizes the magnitude response. *4 points.*

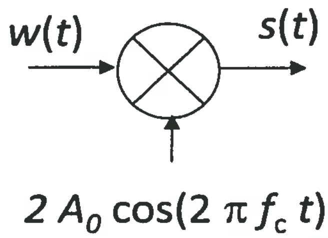
Peak value occurs at $\omega=0$ or $z=1$.

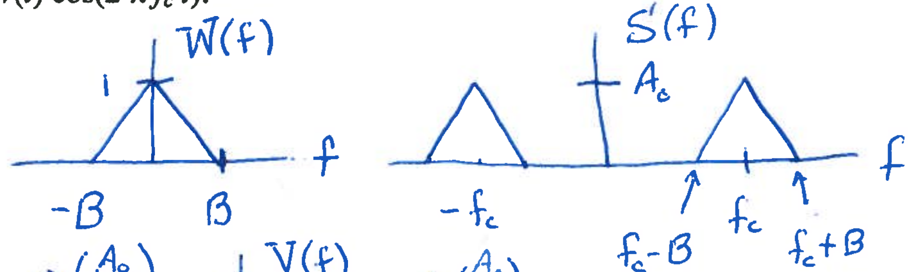
$$H_{freq}(0) = C\,(\,1 + 2 + 1\,) = 4C = 1 \Rightarrow C = \frac{1}{4}$$

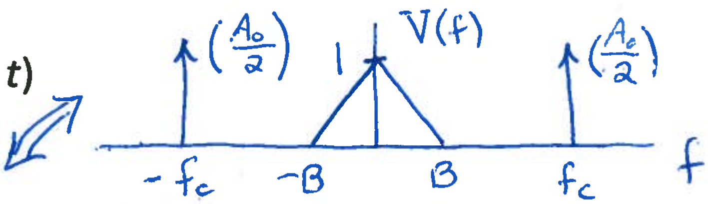
Problem 1.2 *Upconversion.* 24 points.

Here are two approaches to upconvert a baseband message signal $w(t)$ to a carrier frequency f_c to obtain an output signal $s(t) = 2\,A_0\,w(t)\cos(2\,\pi\,f_c\,t)$:

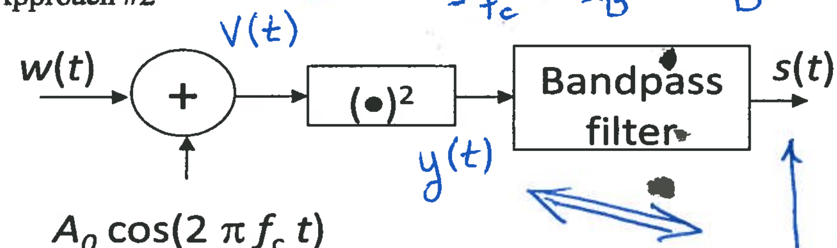
Approach #1

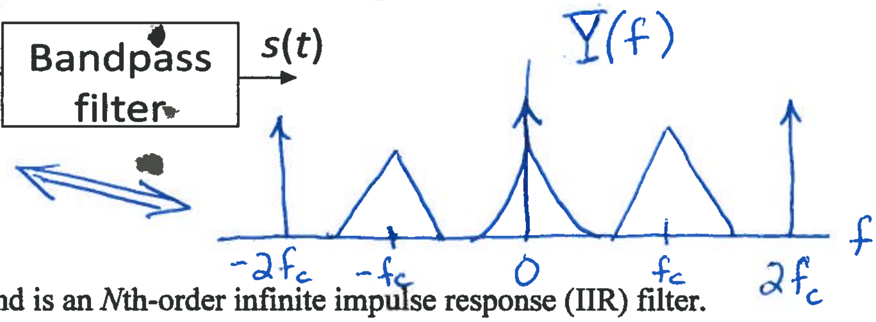






Approach #2





Bandpass filter has center frequency f_c and is an Nth-order infinite impulse response (IIR) filter.

Baseband message signal $w(t)$ has bandwidth B where $f_c > 2\,B$. $f_c > 3B$.

(a) For approach #1, please determine

- minimum sampling rate f_s needed for a discrete-time implementation. *3 points.*
- multiplication-addition operations/second for the discrete-time implementation. *9 points.*

$f_s > 2 f_{max}$ where $f_{max} = f_c + B$ as shown above.

Discrete-time implementation by sampling system at $t = nT_s = \frac{n}{f_s}$.

- Multiplication of $w[n]$ and $2A_0 \cos(\omega_c n)$ where $\omega_c = 2\pi \frac{f_c}{f_s} = 2\pi \frac{N}{L}$
- Generation of $\cos(\omega_c n)$ using lookup table with L entries. 0 mults.
- Pre-compute constant $2A_0$. $\boxed{2 f_s}$ multiplication-additions/s

(b) For approach #2, please determine

- minimum sampling rate f_s needed for a discrete-time implementation. *3 points.*
- multiplication-addition operations/second for the discrete-time implementation. *9 points.*

$f_s > 2 f_{max}$ where $f_{max} = 2 f_c$ as shown above.

Discrete-time implementation by sampling system at $t = nT_s = \frac{n}{f_s}$.

- Addition of $w[n]$ and $A_0 \cos(\omega_c n)$ plus multiplication for $A_0 \cos(\omega_c n)$.
- Generation of $\cos(\omega_c n)$ using lookup table with L entries. 0 mults.
- Squaring block takes 1 multiplication per sample
- Bandpass filter takes $(2N+1)$ multiplication-additions/sample

Total $\boxed{(2N+3)\,f_s}$ multiplication-additions/s.

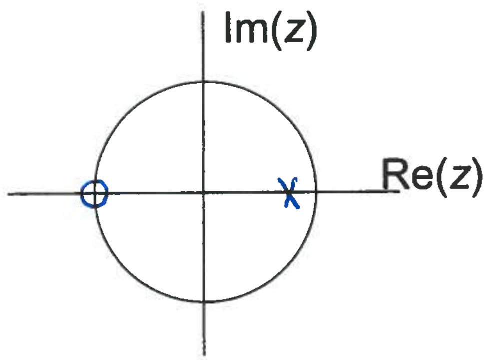
Problem 1.3 *Filter Design.* 30 points.

Consider design of discrete-time linear time-invariant filters by manually placing only real-valued poles and real-valued zeros.

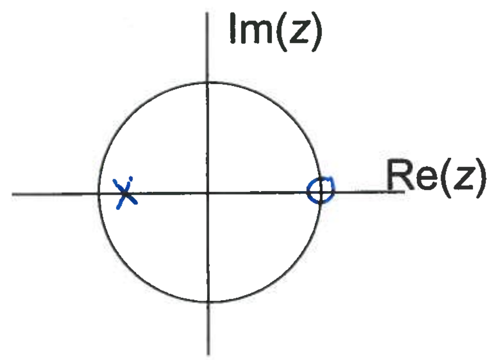
For each frequency selectivity below, indicate YES if at least one filter could be designed to give that selectivity, and NO if there isn't any filter that could be designed to give that selectivity.

If YES, please place real-valued pole(s) and zero(s) to achieve the frequency selectivity.

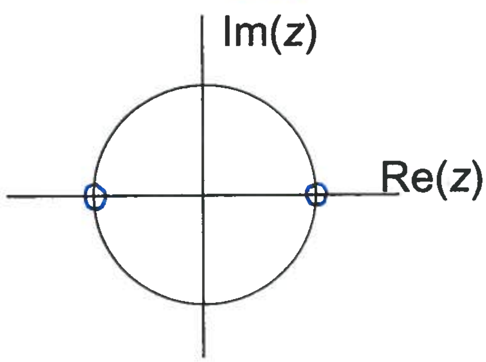
Lowpass: YES or NO



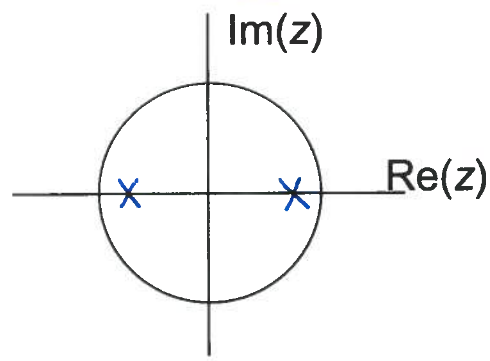
Highpass: YES or NO



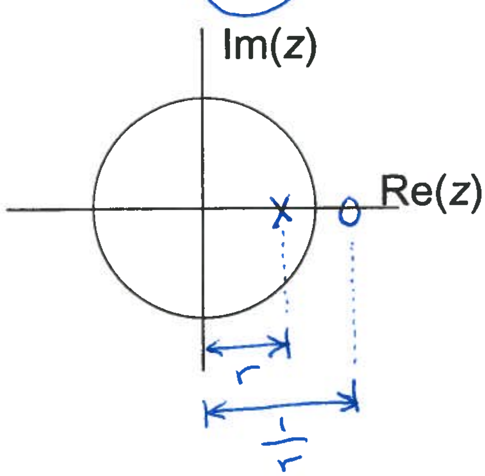
Bandpass: YES or NO



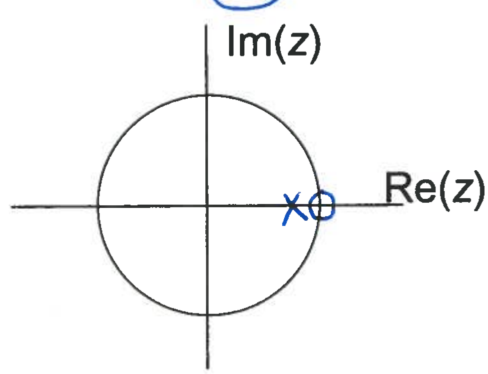
Bandstop: YES or NO



Allpass: YES or NO



Notch: YES or NO



Problem 1.4. *Potpourri.* 18 points.

Consider the design of a discrete-time linear time-invariant finite impulse response (FIR) filter by using the following steps: (1) design a discrete-time linear time-invariant infinite impulse response (IIR) filter to meet the design specification, and (2) truncate the impulse response of the IIR filter to a finite number of coefficients.

(a) How would you estimate the length of the FIR filter needed? *6 points.*

Possible answer #1: FIR length = IIR coefficients = $2N+1$ for an Nth-order IIR filter.

Possible answer #2: Estimate the FIR filter length by using a Kaiser window filter order estimator.

Possible answer #3: Keep enough of the IIR impulse response to contain 90% of total energy.

(b) If the FIR filter does not meet the design specification, how would you modify the design procedure to obtain an FIR filter of the same length that meets the design specification? *6 points.*

Possible answer #1: Increase the IIR filter order.

Possible answer #2: Find the amount of stopband attenuation specification missed by FIR filter and add this to the specification entered for the IIR design and repeat steps (1) and (2).

(c) Claim: The FIR filter would always have linear phase. Either prove the claim to be true for all possible designs, or give a counterexample to show the claim in false. *6 points.*

Counterexample: Let $h_{IIR}[n] = (0.9)^n u[n]$.

Keep first two samples:

$$h_{FIR}[n] = \delta[n] + 0.9\,\delta[n-1]$$

Impulse response is not symmetric or anti-symmetric with respect to its midpoint. Therefore, phase is not linear.

Claim is False.

Note: Truncating IIR impulse responses is a common method for modeling wireline/wired communication channels, which have IIR responses. We'll cover this in the Channel Impairments lecture.