The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Solutions Version 4.0***

Date: March 12, 2025 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	27		IIR Filter Analysis
2	24		Removing DC and Harmonics
3	25		Sinusoidal Amplitude Modulation
4	24		Mystery Systems
Total	100		

Lectures 3 & 6	Lab #3	HW 0.4, 1.1, 2.1, 3.1, 3.3
JSK Ch. 7	Midterm 1 Problem 1: Sp23, Sp18, Sp17, F16	

Problem 1.1 *IIR Filter Analysis*. 27 points.

Consider a causal linear time-invariant (LTI) discrete-time infinite impulse response (IIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$.

$$y[n] = x[n] + \alpha\, y[n-K]$$

where α is a real-valued positive constant and K is a positive integer.

(a) What are the initial condition(s) and their value(s)? Why? *6 points*.

Consider $n = 0$: $\boldsymbol{y[0] = x[0] + \alpha\, y[-K]}$ **where $y[-K]$ is an initial condition**

Consider $n = 1$: $\boldsymbol{y[1] = x[1] + \alpha\, y[1-K]}$ **where $y[1-K]$ is an initial condition**

⋮

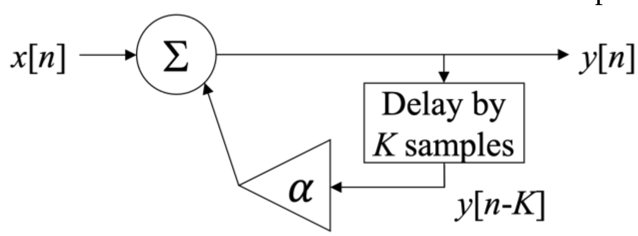
Consider $n = K$: $\boldsymbol{y[K] = x[K] + \alpha\, y[0]}$ **no more initial conditions are appearing**

The initial conditions must equal zero as a necessary condition for LTI to hold:

$$\boldsymbol{y[-K] = y[1-K] = \cdots = y[-2] = y[-1] = 0}$$

x[0] and y[0[are the initial input and output values, and not initial conditions of the system.

(b) Draw a block diagram. Be sure to use arrows to indicate the order of operations. *6 points*.



(c) Compute the transfer function in the z-domain including the region of convergence. 6 *points*.

Take the z-transform of both sides of the difference equation:

$$\boldsymbol{Y(z) = X(z) + \alpha\, z^{-K}\, Y(z)}$$

$$\boldsymbol{Y(z) - \alpha\, z^{-K}\, Y(z) = X(z)}$$

$$\boldsymbol{Y(z)(1 - \alpha\, z^{-K}) = X(z)}$$

$$\boldsymbol{H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \alpha\, z^{-K}}}$$

The poles are the roots of $1 - \alpha\, z^{-K} = 0$ which means $\alpha\, z^{-K} = 1$ or $z^{K} = \alpha$

All the poles will have magnitude $\sqrt[K]{\alpha}$.

The region of convergence is $|z| > \sqrt[K]{\alpha}$

(d) Give the range of values that α can take for the filter to be bounded-input bounded-output (BIBO) stable. *6 points*.

For a causal system, the poles must be inside the unit circle for BIBO stability.

Since $\alpha > 0$, the magnitude of the poles $\sqrt[K]{\alpha}$ will be inside the unit circle when $0 < \alpha < 1$.

(e) Derive a formula for the discrete-time frequency response of the filter. *3 points*.

Since the system is BIBO stable for $0 < \alpha < 1$, we can substitute $z = e^{j\,\omega}$ into $H(z)$ to find the discrete-time frequency response of the filter:

$$\boldsymbol{H_{freq}(\omega) = \frac{1}{1 - \alpha\, e^{-j\,K\,\omega}} \quad for\ 0 < \alpha < 1}$$

This is an IIR comb filter. You'll implement it in lab #7. See https://en.wikipedia.org/wiki/Comb_filter

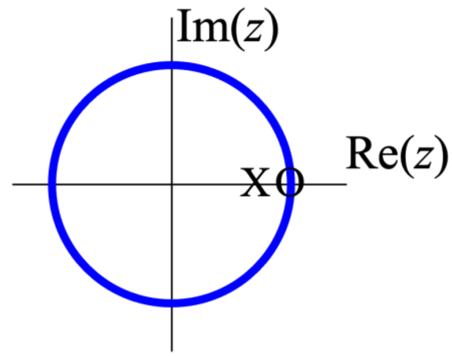
Designing Averaging Filters Handout | JSK Ch. 7
Lecture 6 | Lab #3 | HW 1.1, 2.1, 3.1
Midterm 1: F21 prob 2, Sp19 Prob 3(a)

Problem 1.2 *Removing DC and Harmonics.* 24 points.

This problem asks you to design linear time-invariant (LTI) invariant filters to remove specific frequencies from a signal.

(a) In many applications, we seek to remove DC (0 Hz). Since humans cannot hear below 20 Hz, so DC can be removed. Removing DC can reduce the number of bits needed to represent a signal.

i. Please design a first-order infinite impulse response (IIR) filter to remove DC by placing one pole and one zero on the pole-zero diagram on the right. *4 points.*



Placed a pole and zero on the right to create an IIR DC notch filter: pole at z = 0.9 and zero at z = 1.

ii. Give the transfer function of the first-order IIR filter designed in part (a)i. *4 points.*

$$\boldsymbol{H(z) = \frac{z-1}{z-0.9} = \frac{1-z^{-1}}{1-0.9\,z^{-1}}} \textbf{ with region of convergence } |z| > 0.9$$

iii. Give the discrete-time input-output relationship for the first-order IIR filter assuming the input signal is $x[n]$ and the output signal is $y[n]$. *4 points.*

$$\boldsymbol{\frac{Y(z)}{X(z)} = \frac{1-z^{-1}}{1-0.9\,z^{-1}}} \textbf{ which means } \boldsymbol{(1-0.9\,z^{-1})\,Y(z) = (1-z^{-1})\,X(z)}$$

$$\boldsymbol{\mathrm{Y}(\mathrm{z}) - 0.9\,z^{-1}\,Y(z) = X(z) - z^{-1}\,X(z)}$$

Taking the inverse z-transform of both sides gives $y[n] = 0.9\,y[n-1] + x[n] - x[n-1]$

(b) Harmonics can occur due to nonlinear distortion in a system. Design a discrete-time finite impulse response (FIR) filter to remove harmonics of continuous-time frequency, f_0. Harmonic frequencies of f_0 are $f_0, 2f_0, 3f_0, \ldots$ and $-f_0, -2f_0, -3f_0, \ldots$

i. For sampling rate f_s, how many harmonics in positive frequencies, N, would be captured by sampling? *4 points.*

From the sampling theorem, $f_s > 2\,f_{max}$ or equivalently $f_s > 2\,(N\,f_0)$, so $\boldsymbol{N < \frac{1}{2}\left\lfloor\frac{f_s}{f_0}\right\rfloor}$

where $\lfloor\cdot\rfloor$ is the floor operation because N is an integer.
In practice where we might use $f_s \geq 2\,f_{max}$ so $\boldsymbol{N = \frac{1}{2}\left\lfloor\frac{f_s}{f_0}\right\rfloor}$

ii. Give the discrete-time input-output relationship for the FIR filter assuming the input signal is $x[n]$ and the output signal is $y[n]$. *8 points.* **Many possible answers.**

$y[n] = h[0]\,x[n] + h[1]\,x[n-1] + \cdots + h[M-1]\,x[n-(M-1)]$ for M coefficients.

***Answer #1*. Averaging filter with N zeroes for positive harmonic frequencies and N zeros for the corresponding negative harmonic frequencies. Filter order is $2\,N$ which means $M = 2N+1$ coefficients. Impulse response: $h[n] = \frac{1}{M}$ for $n = 0, 1, \ldots, M-1$.**

***Answer #2*. FIR comb filter $y[n] = x[n] - x[n-2N]$ will remove zero frequency plus $2N-1$ harmonic frequencies. You'll implement this filter in lab #7.**

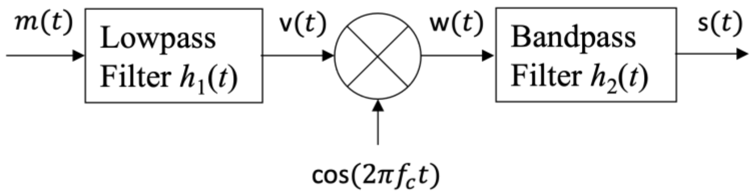
JSK Ch. 7 | Lectures 4 & 6

Midterm Problems 1.4 F20, 1.2 Sp19, 1.3 F18, 1.2 Sp18, 1.2 F17

Labs 2 & 3 | HW 0.1, 0.2, 0.3, 1.3, 3.1

Problem 1.3. *Sinusoidal Amplitude Modulation*. 25 points.

Sinusoidal amplitude modulation can be used to convey a baseband message signal $m(t)$ wirelessly as a bandpass RF signal $s(t)$ that can propagate further. Here's a block diagram representation:

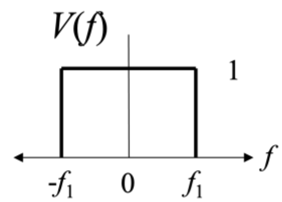


The Zigbee wireless standard is used in home automation. [1] In the unlicensed 2.4 GHz band, there are 16 Zigbee channels (numbered 11 to 26). Each Zigbee channel has 5 MHz transmission bandwidth. [2]

The carrier frequency for the kth channel is $(2.405 + (k - 11)\ 0.005)$ GHz where $k = 11, 12, \ldots, 26$.

Assume all signals can be observed for all time, i.e. $-\infty < t < \infty$.

Assume that there is no noise or interference.



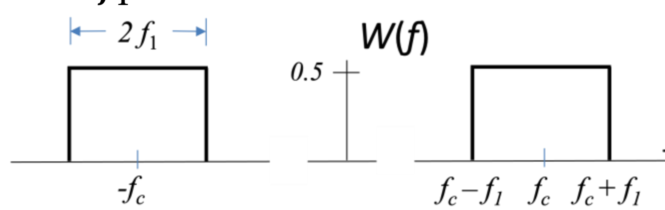
(a) For the Fourier transform for $v(t)$ on the right, indicate the value of f_1
This is the baseband bandwidth for one Zigbee channel. *3 points*.

f_1 is the baseband bandwidth. Transmission bandwidth is 2 f_1 for sinusoidal modulation. $f_1 = 2.5$ MHz.

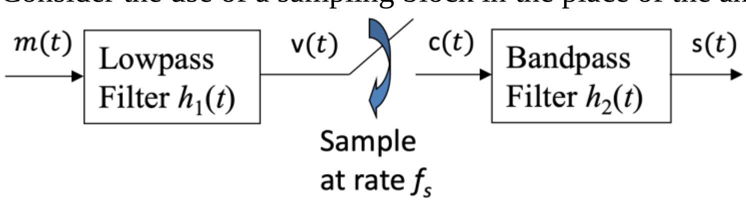
(b) Draw the Fourier transform for $w(t)$. *6 points*

(c) Draw the Fourier transform for $s(t)$. *6 points*.

Same as the Fourier transform for $\mathbf{w(t)}$.



(d) Consider the use of a sampling block in the place of the analog multiplication by $\cos(2\pi f_c t)$.



For channel k = 15, the carrier frequency f_c is 2.425 GHz.

What are the possible values of the sampling rate f_s you could use so that the output $s(t)$ would be the same as the output $s(t)$ in the sinusoidal amplitude modulator at the top of the page? *10 points*.

We want the mth replica of the baseband spectrum to be centered at the carrier frequency:

$$f_c = m\, f_s$$

And we want to avoid aliasing (overlap) among the replicas

$$f_s \geq 2\, f_1$$

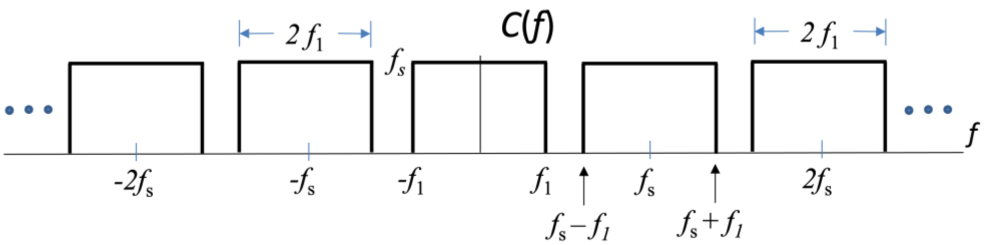
For $f_c = m\, f_s = m\,(2\, f_1)$,

$$m = \frac{1}{2}\frac{f_s}{f_1} = 485$$

Factors of 485 are 5 and 97.

Possible sampling rates are

$$\{\, 2f_1, 10f_1, 194f_1, f_c \}$$



[1] https://en.wikipedia.org/wiki/Zigbee
[2] C. Wong, T. Jiang, and Q. Zhang, *Zigbee Network Protocols and Applications*, 2014.

***Student answer*: f_s can be any divisor of f_c as along as $f_s \geq 5$ MHz. Factors of 2425 MHz are {5, 5, 97} MHz.**

Midterm 1: 1.4 Sp11, 1.3 Sp15, 1.5 Sp19, 1.4 F19, 1.4 Sp20, 1.4 Sp21, 1.4 Sp22, 1.4 F F23, 1.4 F24

Lecture 4	Handout Common Signals in Matlab
HW 1.2 1.3 & 2.2	In-Lecture #1 Assignment

Problem 1.4. *Mystery Systems*. 24 points.

You're trying to identify unknown discrete-time systems.

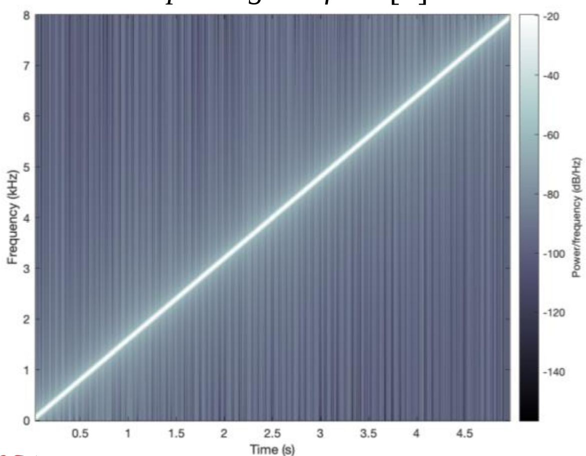
You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 8000 Hz over 0 to 5s

$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

where $f_1 = 0$ Hz, $f_2 = 8000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{8000 \text{ Hz}}{10\, s} = 800 \text{ Hz}^2$. Sampling rate f_s is 16000 Hz.

Spectrogram for $x[n]$

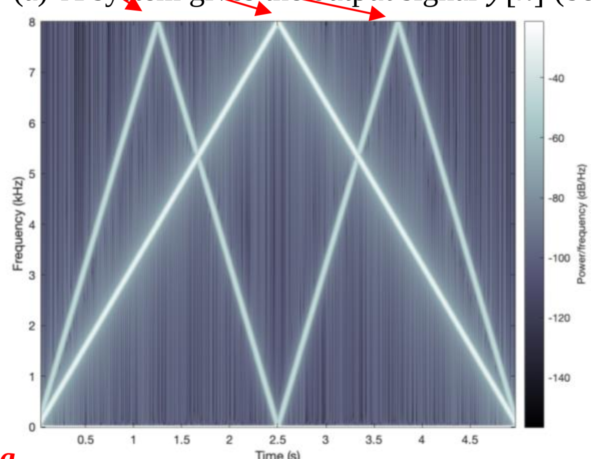


In each part below, identify the unknown system as one of the following **with justification**:

1. filter – give selectivity (lowpass, highpass, bandpass, bandstop) and passband/stopband frequencies
2. upsampler – give upsampling factor
3. downsampler – give downsampling factor
4. pointwise nonlinearity – give the integer exponent k to produce output $y[n] = x^k[n]$
5. amplitude modulation – give the amplitude modulation frequency f_0 to produce output $y[n] = \cos(\omega_0)\, x[n]$ where $\omega_0 = 2\pi\, f_0 \,/\, f_s$.

aliasing

(a) A system gives the output signal $y[n]$ (below) when the chirp signal $x[n]$ is input. *12 points*.



Please note that the output signal has a strong DC component (0 Hz) equally strong over all time.

Over a particular interval of time, output has frequencies not present in the input signal. Rules out filter.

Horizontal & vertical axes are same for output and input signals. No rate change. Rule out upsampler or downsampler.

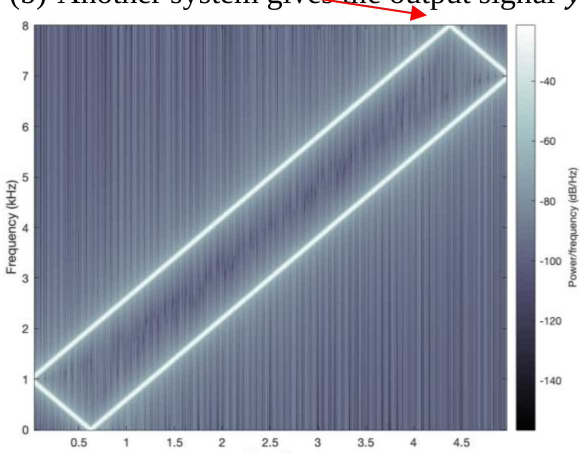
Output has 3 components: DC + frequency increasing at 2x input + frequency increasing at 4x input. $y[n] = x^4[n]$

For $x[n] = \cos(\omega_0\, n)$, $y[n] = \cos^4(\omega_0\, n) = (\cos^2(\omega_0\, n))^2$

$$\left(\frac{1}{2} + \frac{1}{2}\cos(2\,\omega_0\, n)\right)^2 = \frac{3}{8} + \frac{1}{2}\cos(2\,\omega_0\, n) + \frac{1}{8}\cos(4\,\omega_0\, n)$$

aliasing

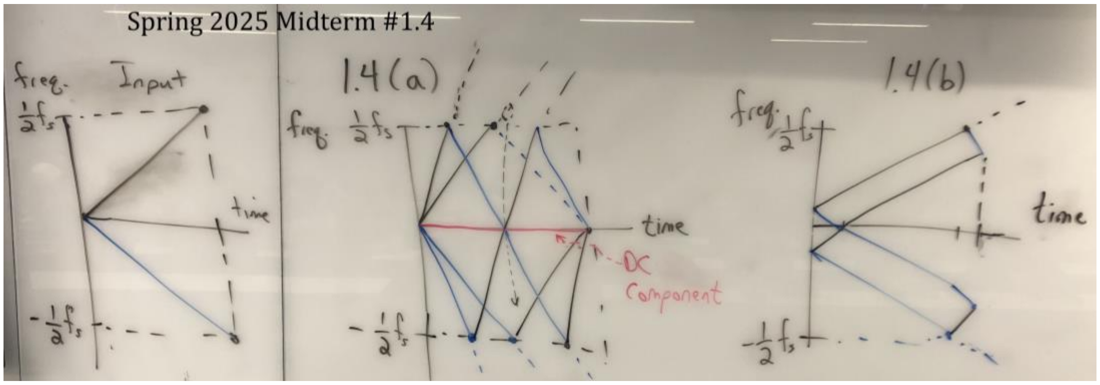
(b) Another system gives the output signal $y[n]$ below when the chirp signal $x[n]$ is input. *12 points*.



Over a particular interval of time, output has frequencies not present in the input signal. Rules out filter.

Horizontal & vertical axes are same for output and input signals. No rate change. Rule out upsampler or downsampler.

Amplitude modulation with modulation frequency of $f_0 = 1000$ Hz. The lines with positive slope are at the frequency of the input signal plus or minus 1000 Hz. The line with negative slope between 0 and 0.6s is due to the negative frequency component of $x[n]$. The line with negative slope that goes from 8 kHz to 7 kHz is due to aliasing of the frequency in the output that goes from -8 kHz to -9 kHz.




```matlab
%% Matlab code to generate the spectrograms for Problem 1.4
fs = 16000;
Ts = 1 / fs;
tmax = 5;
t = 0 : Ts : tmax;

%% Create chirp signal
f1 = 0;
f2 = fs/2;
mu = (f2 - f1) / (2*tmax);
x = cos(2*pi*f1*t + 2*pi*mu*(t.^2));

%% (a) Fourth-order nonlinearity
y = x .^ 4;

%%% Spectrogram parameters
blockSize = 1024;
overlap = 1023;

%%% Plot spectrogram of input signal
figure;
spectrogram(x, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;

%%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;

%% (b) Amplitude modulation
f0 = 1000;
y = cos(2*pi*f0*t) .* x;

%%% Spectrogram parameters
blockSize = 1024;
overlap = 1023;

%%% Plot spectrogram of input signal
figure;
spectrogram(x, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;

%%% Plot spectrogram of output signal
figure;
spectrogram(y, blockSize, overlap, blockSize, fs, 'yaxis');
colormap bone;
```