The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1 ***Solutions 3.0***

Date: March 11, 2026 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	24		IIR Filter Analysis
2	24		Real-Time Audio
3	28		Sinusoidal Amplitude Modulation
4	24		Potpourri
Total	100		

Lectures 3 & 6 | Lab #3 | HW 0.4, 1.1, 2.1, 3.1, 3.3 | JSK Ch. 7

Midterm 1 Problem 1: Sp25, Sp23, Sp18, Sp17, F16

Problem 1.1 *IIR Filter Analysis*. 24 points.

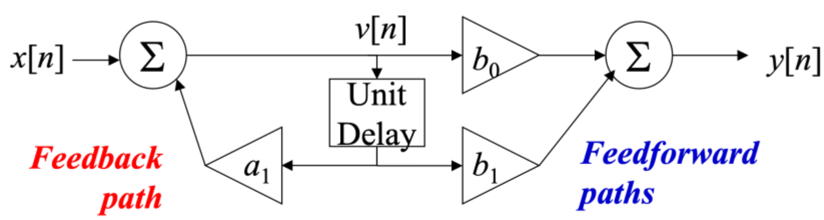
Consider a causal linear time-invariant (LTI) bounded-input bounded-output (BIBO) discrete-time infinite impulse response (IIR) filter with input $x[n]$ and output $y[n]$ observed for $n \geq 0$ described by

$$y[n] = a_1\, y[n-1] + b_0\, x[n] + b_1\, x[n-1]$$

where the coefficents a_1, b_0 and b_1 are real-valued and non-zero.

(a) What are the initial condition(s) and their value(s)? Why? *3 points*.
At $n = 0$: $y[0] = a_1\, y[-1] + b_0\, x[0] + b_1\, x[-1]$ where $x[-1]$ and $y[-1]$ are initial cond.
At $n = 1$: $y[1] = a_1\, y[0] + b_0\, x[1] + b_1\, x[0]$ which do not contain any initial cond.
Initial conditions must be zero as a necessary condition for LTI: $x[-1] = 0$ and $y[-1] = 0$.
x[0] and y[0] are the initial input and output values, and not initial conditions of the system.

(b) Draw a block diagram. Be sure to use arrows to indicate the order of operations. *3 points*.
The following is the biquad block diagram from lecture slide 6-5 with $a_2 = 0$ and $b_2 = 0$:



Several possible answers.

(c) Compute the transfer function in the z-domain including the region of convergence. 6 *points*.
Take the z-transform of both sides of difference equation with initial conditions being zero:

$$Y(z) = a_1\, z^{-1}\, Y(z) + b_0\, X(z) + b_1\, z^{-1}\, X(z)$$

$$Y(z) - a_1\, z^{-1}\, Y(z) = b_0\, X(z) + b_1\, z^{-1}\, X(z)$$

$$(1 - a_1\, z^{-1})\, Y(z) = (b_0 + b_1\, z^{-1})\, X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1\, z^{-1}}{1 - a_1\, z^{-1}} \quad \text{where } |z| > |a_1|$$

(d) Derive a formula for the discrete-time frequency response of the filter. *3 points*.
Since the filter is BIBO stable, the pole at $z = a_1$ is inside the unit circle and hence the region of convergence $|z| > |a_1|$ includes the unit circle. So, we can substitute $z = e^{j\,\omega}$:

$$H_{freq}(\omega) = H(z)|_{z=e^{j\,\omega}} = \frac{b_0 + b_1\, e^{-j\,\omega}}{1 - a_1 e^{-j\,\omega}}$$

(e) Give the numeric values for the coefficents a_1, b_0 and b_1 that would allow the discrete-time IIR filter to meet the following frequency domain specification: *9 points*. **Notch filter at $\omega = \pi$.**

- Completely removes discrete-time frequency of π rad/sample. **Zero at radius 1 & angle π**
- Passes as many other frequencies as possible with a constant gain. **Pole radius 0.9 & angle π**
- Has frequency response of 1 at discrete-time frequency of 0 rad/sample. **Normalize @ $\omega = 0$.**

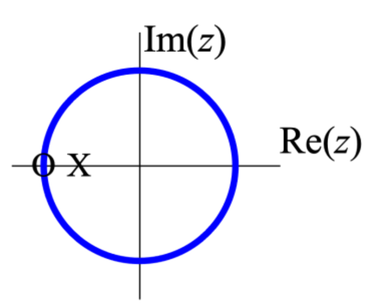
$$H(z) = \frac{b_0 + b_1\, z^{-1}}{1 - a_1\, z^{-1}} = b_0 \frac{1 + \frac{b_1}{b_0}\, z^{-1}}{1 - a_1\, z^{-1}}$$

Zero at $-\frac{b_1}{b_0} = e^{j\,\pi} = -1$ which means $b_0 = b_1$

Pole at $a_1 = 0.9\, e^{j\,\pi} = -0.9$

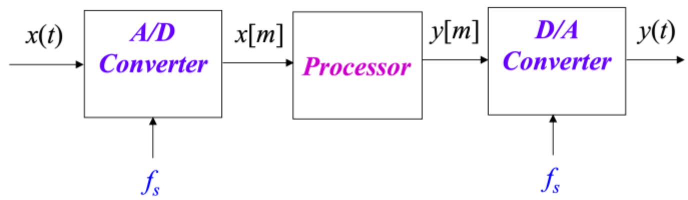
Normalize at $z = e^{j\,0} = 1$: $H(1) = b_0 \frac{1+1}{1-(-0.9)} = 1$

which means $b_0 = \frac{1.9}{2} = 0.95$



JSK Ch. 2, 3, 5 & 7	Lectures 4, 5 & 6	HW 0.3, 2.3, 3.3	Labs 2 & 3

Problem 1.2 *Real-Time Audio.* 24 points.



Consider a real-time single-channel audio system.

Sampling rate is $f_s = 48$ kHz.

The processor implements a finite impulse response (FIR) filter to extract 200-2000 Hz woofer frequencies.

The linear phase bandpass FIR filter has 1000 coefficients.

(a) Computational requirements. *6 points.*

1. How many multiplications per second are required to process the signal in real time?

   For each input sample, the FIR filter multiplies the 1000 FIR coefficients and the current and previous 999 input samples and adds the results to produce an output sample:

$$\mathbf{1000\, f_s = 48 \times 10^6 \text{ multiplications/s}}$$

2. If each multiply-add operation takes 1 CPU cycle, what is the minimum processor clock frequency required to sustain the number of multiplications required? **48 MHz**

(b) Latency for sample-by-sample filtering. *6 points.*

1. What is the group delay (in samples) of the FIR filter? Why? **Since the FIR filter has linear phase, the group delay is $\frac{1000-1}{2} = 499.5$ samples.** **Each frequency component in input signal $x[n]$ is delayed by 499.5 samples before it appears in the output signal.**

2. Convert this group delay into milliseconds for the given sampling rate.

The sampling rate f_s is in units of samples/s and sampling time T_s is in units of s/sample.

$$(499.5 \text{ samples})\, T_s = \frac{499.5 \text{ samples}}{48000 \text{ samples/s}} = 10.4 \text{ ms}$$

(c) Block-based processing. A/D samples are put in a buffer of 64 samples before filtering any of them, and the filter output samples are placed in a buffer of 64 samples before sending any of them through the D/A Converter. *6 points*

1. Does the latency increase, decrease, or stay the same? Why?

2. If you said increase or decrease, please say by how many samples. **(Below, both are true.)**

   Increase **by 128 samples. A/D output samples are delayed by 64 samples before being filtered, and the filter output is delayed by 64 samples before being input into the D/A.** ***Decrease*.** **If we consider the delays to service the interrupt service routine for each sample in (b) vs. for each buffer in this part, we reduce the ISR latency by a factor of 64.**

(d) Tradeoffs. If the processor becomes overloaded due to other tasks, propose two ways to reduce the computational load while still extracting 200-2000 Hz woofer frequencies. *6 points.*

Design changes to algorithms and system settings

- **Design discrete-time IIR filter to extract woofer frequencies.** **20x savings. See next page. Some phase distortion (approx. linear phase over passband). High group delay at 200 Hz.**
- **Reduce sampling rate f_s in both data converters. $f_s > 2\, f_{max}$ where $f_{max} = 2200$ Hz due to 10% rolloff in analog filters. Can reduce f_s to 4400 Hz which gives 10.9x savings.**
- **Decimate by 6 and change D/A sampling rate to 8000 Hz. 7.7x savings. See next page.**
- **Decimate by factor of 6, bandpass filter, and interpolate by factor of 6. 4x savings.**
- **Redesign FIR filter using Parks-McClellan algorithm to be shorter. Up to 2x savings.**

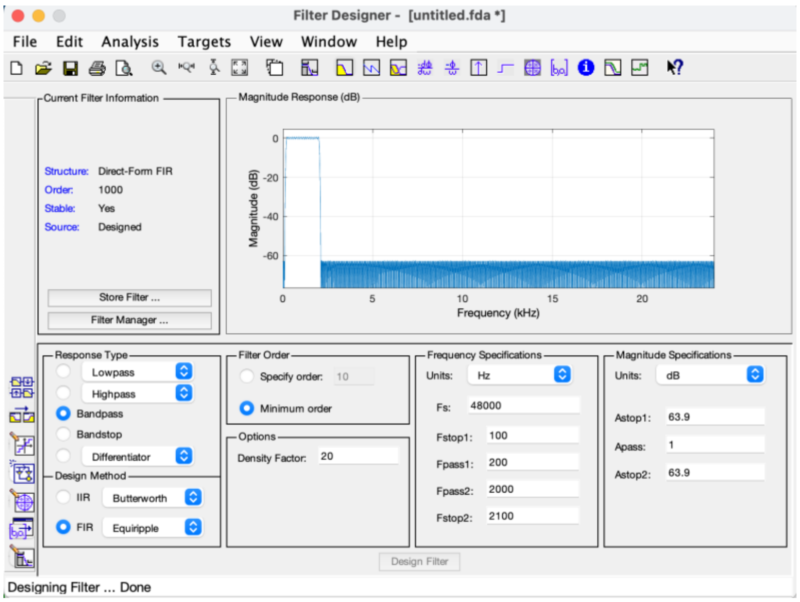
ChatGPT was used to create the initial version of this question but was not used in formulating answers.

Implementation efficiency (additional gains over design changes above)

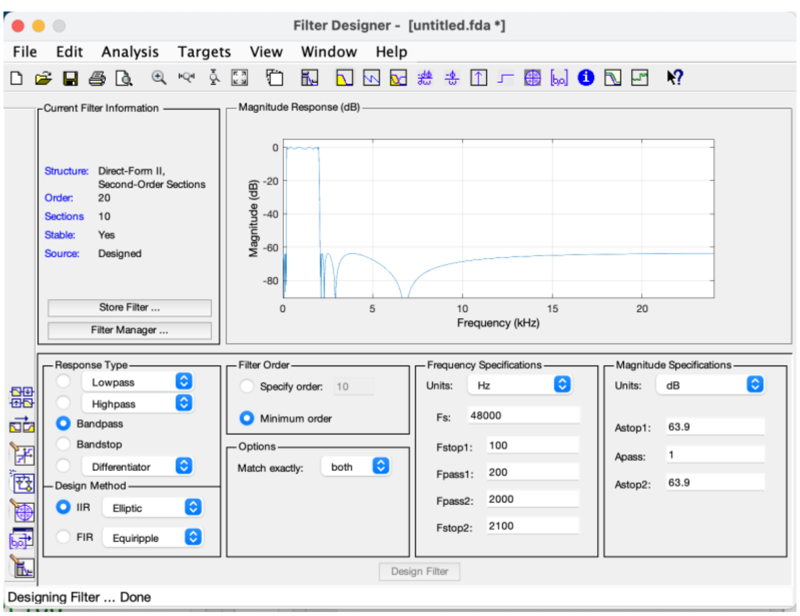
- **FIR filter: for the internal buffering of the previous input values, use a circular buffer instead of a linear buffer. For an FIR with N coefficients, savings by a factor of (N/2).**
- **Use DMA controllers for block-based external buffer I/O to reduce CPU load.**

More information for the first answer in part (d)…

Here's a bandpass FIR filter to pass the woofer frequencies using a sampling rate of 48000 Hz, designed using an Equiripple design method (a.k.a. Parks-McClellan, Remez Exchange, and Chebyshev FIR) with 1001 coefficients which needs 1001 multiplications/sample:



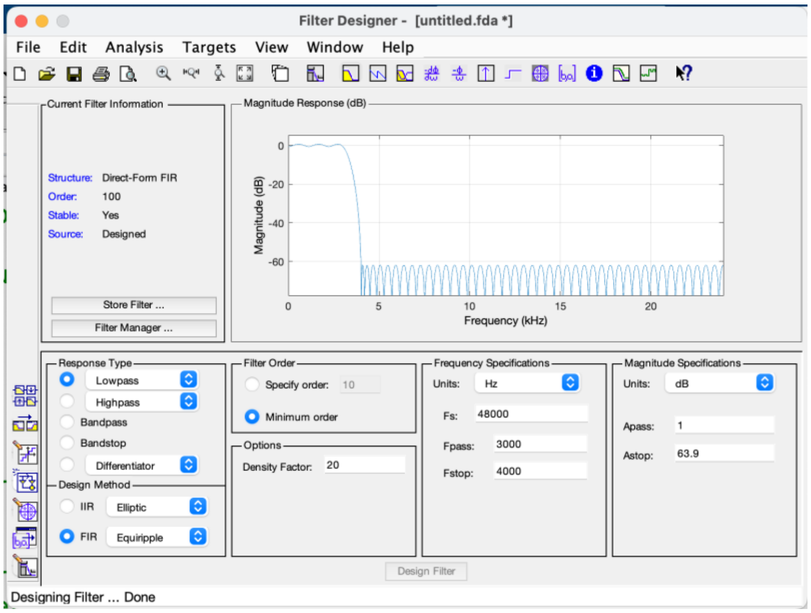
Here's a bandpass IIR filter to pass the woofer frequencies using a sampling rate of 48000 Hz, designed by the Elliptic design method. An IIR filter of 20th order takes 50 multiplications per sample when implemented as a cascade of 10 biquads, <u>which is a 20x savings</u>:



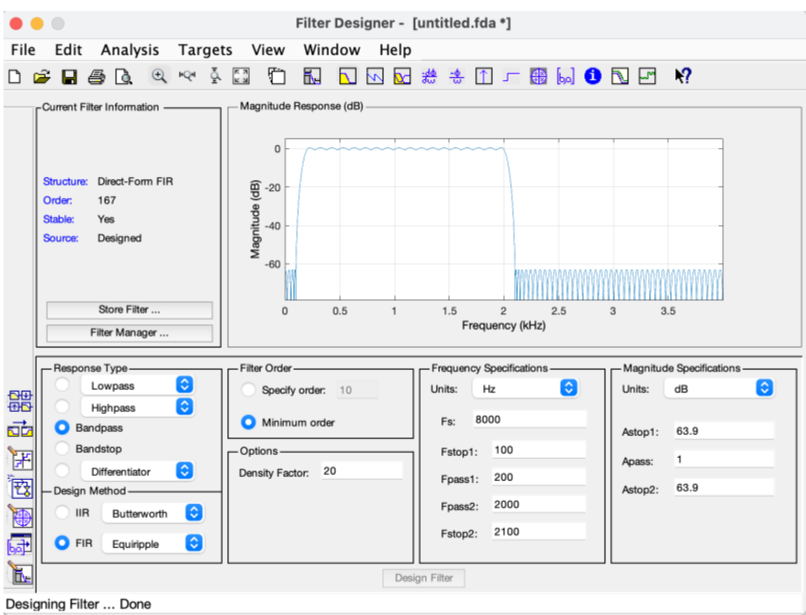
More information for the third answer in part (d)….

Here's a design to decimate by 6 and then bandpass filter. Needs 101 multiplications/sample for the anti-aliasing filter prior to downsampling by 6 and 168 coefficients for the bandpass filter running at the lower sampling rate of 8000 Hz.

Lowpass linear phase FIR filter using a sampling rate of 48000 Hz with cutoff frequency of 4000 Hz prior to downsampling by 6. Needs 101 FIR coefficients.



Bandpass linear phase FIR filter to extract woofer frequencies using a sampling rate of 8000 Hz. Needs 168 coefficients.



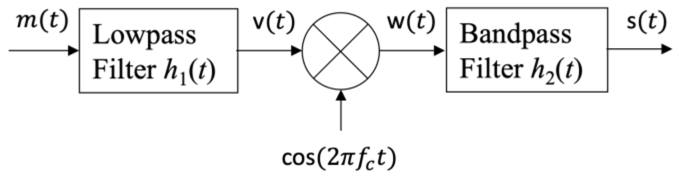
JSK Ch. 2, 3, 5 & 7	Lectures 4 & 6	HW 0.1, 0.2, 0.3, 1.1(d), 1.3, 2.1(d), 3.1, 3.2

Midterm Problems 1.3 Sp25, 1.4 F20, 1.2 Sp20, 1.2 Sp19, 1.3 F18, 1.2 Sp18, 1.2 F17

Labs 2 & 3

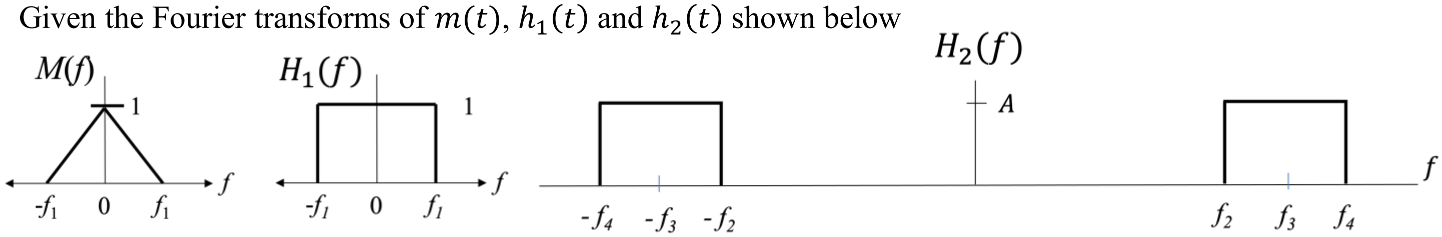
Problem 1.3. *Sinusoidal Amplitude Modulation*. 28 points.

Sinusoidal amplitude modulation can be used to convey a baseband message signal $m(t)$ wirelessly as a bandpass RF signal $s(t)$ that can propagate further. Here's a block diagram representation:

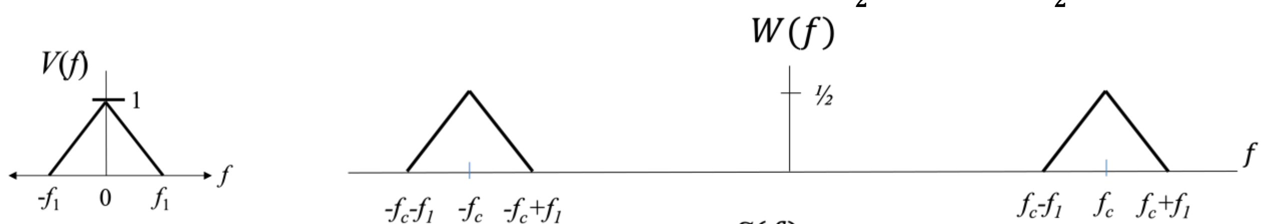


Assume that we can observe the system for $-\infty < t < \infty$ and f_c is much greater than f_1.

Given the Fourier transforms of $m(t)$, $h_1(t)$ and $h_2(t)$ shown below

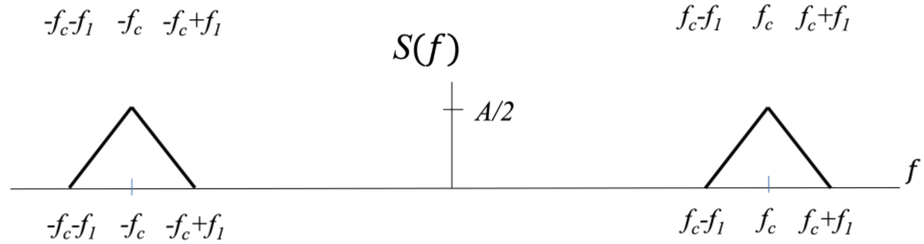


(a) Draw $V(f)$ and $W(f)$. *6 points*. $\boldsymbol{V(f) = H_1(f)\, M(f)}$ **and** $\boldsymbol{W(f) = \frac{1}{2}\, V(f - f_c) + \frac{1}{2}\, V(f + f_c)}$



(b) Draw $S(f)$. What is the value of A so that the maximum value of $S(f)$ is 1? *3 points*.

$\boldsymbol{A = 2}$



(c) Give equations for f_2, f_3 & f_4 in terms of f_1 & f_c. *6 points*. $\boldsymbol{f_2 = f_c - f_1; f_3 = f_c; f_4 = f_c + f_1}$

(d) To implement the modulation system in discrete time, give a formula in terms of f_1 and f_c for the choice of sampling rate f_s for the entire system that would avoid aliasing. All continuous-time signals will be sampled by the same sampling rate f_s. *6 points*.

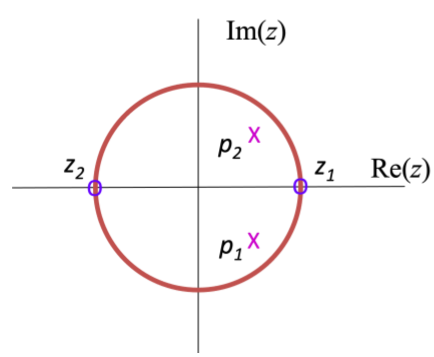
The maximum continuous-time frequency anywhere in the system is $\boldsymbol{f_4 = f_c + f_1}$.
The sampling rate $\boldsymbol{f_s > 2\, f_4}$ **or equivalently** $\boldsymbol{f_s > 2\,(f_c + f_1)}$.

(e) Give two poles and two zeros for the discrete-time bandpass filter. The zeros should be on the unit circle. *7 points*. **The discrete-time bandpass filter is the discrete-time version of** $\boldsymbol{H_2(f)}$**.**

When poles and zeros are separated in angle, angles of the zeros on/near the unit circle indicate stopband frequencies and angles of the poles near (but inside) the unit circle indicate passband frequencies. Passbands are centered at continuous-time frequencies $\boldsymbol{-f_c}$ **and** $\boldsymbol{+f_c}$**.**

Poles: $\boldsymbol{p_1 = 0.9\, e^{-j\omega_c}}$ **and** $\boldsymbol{p_2 = 0.9\, e^{j\omega_c}}$ **where** $\boldsymbol{\omega_c = 2\pi\, \frac{f_c}{f_s}}$.

Zeros: $\boldsymbol{z_1 = e^{j0} = 1}$ **and** $\boldsymbol{z_2 = e^{j\pi} = -1}$.



JSK Ch. 2, 3, 5 & 7 | Lectures 1 & 4 | HW 0.3, 1.3, 2.2 | Lab 2

Problem 1.4. *Potpourri.* 24 points.

(a) Consider a continuous-time signal that is a sum of two cosine signals observed for $-\infty < t < \infty$

$$x(t) = \cos(2\,\pi\, f_0\, t) + \cos(2\,\pi\, f_1\, t)$$

where $f_1 = 10\, f_0$. After sampling at a sampling rate of $f_s > 2\, f_0$, the discrete-time signal is

$$x[n] = \cos(\omega_0\, n) + 1$$

where $\omega_0 = 2\,\pi\, \frac{f_0}{f_s}$ and frequency f_0 does not alias; however, the frequency f_1 aliases.

What are all the possible values of f_s ? Give expressions for f_s in terms of f_0 . *12 points*.

After sampling at sampling rate of f_s, continuous-time frequency f_0 does not alias because $f_s > 2\, f_0$. However, the continuous-time frequency $f_1 = 10\, f_0$ aliases to 0 Hz.

After sampling at sampling rate of f_s, the continuous-time frequency f_1 has replicas $f_1 \pm k f_s$ where k is an integer and its negative counterpart has replicas $f_1 \pm m f_s$ where m is an integer.

The possible values of f_s for $10 f_0 \pm k f_s = 0$ for $f_s > 2\, f_0$ are $\left\{ 2.5\, f_0, \frac{10}{3} f_0, 5\, f_0, 10\, f_0 \right\}$.

Problem is analogous to the in-class demo of a videocamera recording a scene using a fixed frame rate (f_s) where the motion of a boat (f_0) is captured correctly but the rotating helicopter blades appear to stand still (f_1): https://www.youtube.com/watch?v=yr3ngmRuGUc

(b) Consider a continuous-time cosine signal of a single frequency observed for $-\infty < t < \infty$

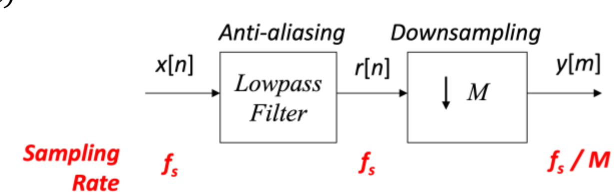
$$x(t) = \cos(2\,\pi\, f_2\, t)$$

After sampling at a sampling rate of $f_s = 10\, f_2$, the discrete-time signal is

$$x[n] = \cos(\omega_2\, n)$$

where $\omega_2 = 2\,\pi\, \frac{f_2}{f_s}$ and frequency f_2 does not alias.

The signal $x[n]$ is decimated as shown on the right.



Assume that the maximum passband frequency of the lowpass filter is π / *M*.

1. Assuming an ideal lowpass filter, what is the maximum integer value of *M* that can be used and still allow us to reconstruct $x(t)$ from $y[m]$? *6 points*.

   When sampling $x(t)$ at $f_s = 10\, f_2$, the discrete-time frequency $\omega_2 = 2\pi \frac{f_2}{f_s} = 2\pi \frac{f_2}{10\, f_2} = \frac{\pi}{5}$.

   The ideal LPF has a frequency response of a rectangular pulse from -π/*M* to π/*M*. $M = 5$.
   We can use $M = 5$ because $x[n]$ is a cosine. It wouldn't have worked for sine.

2. Assuming a practical lowpass filter, what is the maximum integer value of *M* that can be used and still allow us to reconstruct $x(t)$ from $y[m]$? *6 points.*

   The magnitude response of a practical lowpass filter would have a 10% rolloff from passband to stopband. With maximum passband frequency of π/*M*, the stopband frequency would be 1.1 π/M. Any transition and stopband frequencies in the input signal will alias. Because we're observing $x(t)$ for $-\infty < t < \infty$, the only frequencies present are at $-f_2$ and f_2. So, we can use $M = 5$.