The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

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Course: EE 445S

Name: Asterix and Obelix
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your standalone computer system**.
- Please turn off all cell phones and other personal communication devices.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise**. When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

Problem	Point Value	Your score	Topic
1	21		Energy Detection
2	27		Communication Performance
3	30		Echo Cancellation
4	22		Transceiver Design
Total	100		

Problem 2.1. *Energy Detection. 21 points.* Fall 2014

Energy detection is useful in a wide variety of signal processing applications.

Definition of energy for a causal discrete-time signal $r[m]$ follows:

$$Energy = \sum_{m=0}^{\infty} |r[m]|^2$$

Several linear time-invariant energy detectors are given below. Assume all initial conditions are zero.

For each energy detector, input $x[m]$ is the instantaneous power $|r[m]|^2$. Output is $y[m]$.

Give the transfer function for each system and give the primary advantage and disadvantage of each.

(a) Running sum: $y[m] = y[m-1] + x[m]$. *7 points.*	Advantage	Disadvantage
$Y(z) = z^{-1}Y(z) + X(z)$ $(1 - z^{-1})Y(z) = X(z)$ $H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-z^{-1}}$	• Computationally simple (1 add/sample) • Little memory usage (1 word)	• BIBO Unstable • Same as (c)

(b) Sum of current input and previous M-1 inputs: $y[m] = x[m] + x[m-1] + \ldots + x[m-(M-1)]$. *7 points.*

$y[m] = \sum_{n=0}^{M-1} x[m-n]$ $Y(z) = \sum_{n=0}^{M-1} z^{-n} X(z)$ $H(z) = 1 + z^{-1} + \cdots + z^{-(M-1)}$	• BIBO Stable	• More Complex Implementation (M adds/sample and M words of memory) vs. (a)

(c) Weighted combination using constant c where $0 < c < 1$: $y[m] = c\,y[m-1] + (1-c)\,x[m]$.

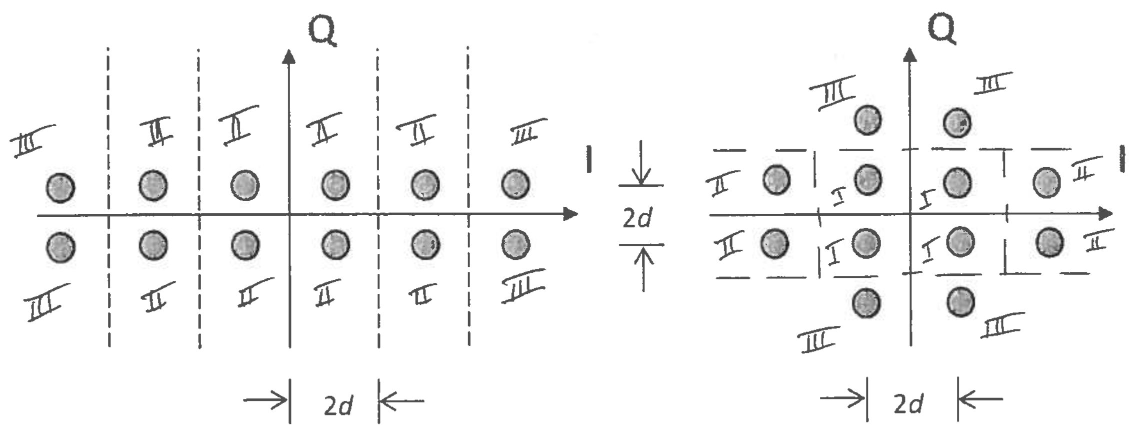
$Y(z) = c z^{-1} Y(z) + (1-c) X(z)$ $(1 - c z^{-1}) Y(z) = (1-c) X(z)$ $H(z) = \frac{Y(z)}{X(z)} = \frac{1-c}{1-c z^{-1}}$ Note: Constant c is real-valued.	• BIBO Stable • Good tradeoff in complexity (2 mults/sample, 1 adds/sample, 3 words memory)	• Number of bits needed for $y[m]$ grows without bound as $m \to \infty$ and no clear way to reset the system

All three LTI systems smooth the instantaneous power calculation of $|r[m]|^2$. LTI systems (b) and (c) are filters.

Fall 2014

Problem 2.2 *Communication Performance. 27 points.*

Consider the two 12-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the I axis, Q axis and dashed lines.

	Left Constellation	Right Constellation
(a) Peak power	$26\,d^2$	$10\,d^2$
(b) Average power	$12.67\,d^2$	$7.33\,d^2$
(c) Number of type I regions	0	4
(d) Number of type II regions	8	4
(e) Number of type III regions	4	4

Draw the decision regions for the right constellation on top of the right constellation. *3 points.*

The I axis and Q axis are also boundaries of decision regions.

Fill in each entry (a)-(e) in the above table for the right constellation. Each entry is worth *3 points.*

Due to symmetry, we can compute the average power in one quadrant:

$$\frac{2d^2 + 10d^2 + 10d^2}{3} = 7.33\,d^2$$

Which of the two constellations would you advocate using? Why? *9 points.*

The right constellation has

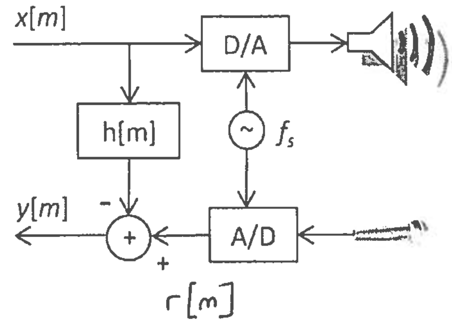
- Lower peak power
- Lower average power
- Lower peak power to average power ratio

The left constellation has a lower probability of symbol error as a function of d. Once we normalize in terms of SNR, the right constellation will have a lower probability of symbol error. Both constellations can be gray coded.

Correction: Only the left constellation can be gray coded.

Fall 2014

Problem 2.3. *Echo Cancellation. 30 points.*



A block diagram of a speakerphone is shown to the right.

During a phone call, the received speech plays out over the speaker. The sound from the speaker is captured by the microphone, and the caller will hear an echo of his/her voice.

$x[m]$ is a reference signal. It is either the digitized speech from the other person on the call or a training signal.

$h[m]$ is the impulse response of a finite impulse response filter.

The analog-to-digital (A/D) and digital-to-analog (D/A) converters are synchronized by use of a common sampling clock. The A/D and D/A converters quantize to B bits per sample.

Design the finite impulse response (FIR) filter to reduce echo by using an adaptive method.

(a) Give a reference signal for $x[m]$ to use for training $h[m]$. Why did you choose it? *6 points.*

Use a maximal length pseudo-noise sequence. This sequence is robust to frequency distortion and additive noise, and is easy to generate using a shift register and exclusive-OR operations.

(b) During training, the ideal value for $y[m]$ is 0, which would mean that all echo has been removed.

i. Give an objective function $J(y[m])$ to be minimized. *9 points.*

$$J(y[m]) = \frac{1}{2} y^2[m]$$

We seek to minimize $J(y[m])$ so that its minimum value corresponds to all echo being removed.

ii. Give the update equation for the vector $\vec{h}$ of FIR coefficients. *9 points.*

$$\vec{h} = [h_0 \quad h_1 \quad h_2 \quad \cdots \quad h_{M-1}]$$

$$\vec{h}[m] = [h_0[m] \quad h_1[m] \quad h_2[m] \quad \cdots \quad h_{M-1}[m]]$$

$$\vec{h}[m+1] = \vec{h}[m] - \mu \left. \frac{J(y[m])}{d\vec{h}} \right|_{\vec{h} = \vec{h}[m]}$$

iii. What values would you recommend for the step size μ? *6 points.*

- $\mu > 0$ and very small based on observations in homework problems (e.g. $\mu = 0.001$).
- Reusing the solution from midterm problem 2.1 in spring 2014:

$$0 < \mu < \frac{2}{x^2[m]}$$

$$y[m] = r[m] - x[m] * h[m]$$
$$= r[m] - h_0 x[m] - h_1 x[m-1] \cdots$$

$$\vec{h}[m+1] = \vec{h}[m] + \mu \, y[m] \, \vec{x}[m]$$

$$\vec{x}[m] = [x[m] \quad x[m-1] \quad \cdots]$$

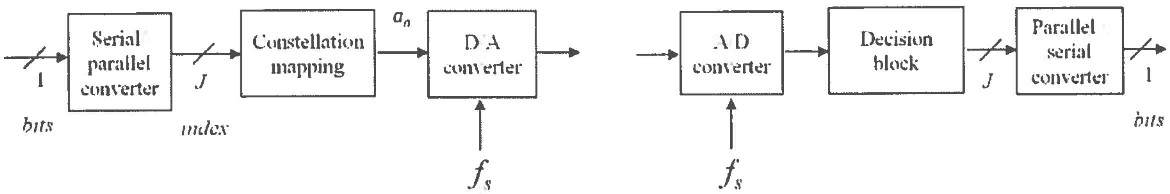
Spring 2014 objective function was y^2[m]; replace 2 u in spring 2014 solution with u

Problem 2.4. *Transceiver Design. 22 points.* Fall 2014

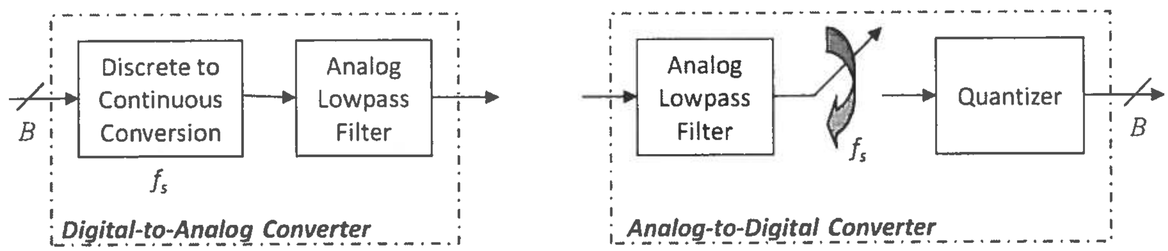
In certain discrete-time baseband transceivers, we remove the pulse shaping filter in the transmitter and the matched filter in the receiver to reduce complexity.

We also remove the upsampler in the transmitter and downsampler in the receiver.

Here is the resulting transmitter (left) and receiver (right) for baseband pulse amplitude modulation:



Here are block diagrams for the analog-to-digital (A/D) and the digital-to-analog (D/A) converters:



(a) What is the formula relating the symbol rate f_{sym} and the sampling rate f_s? *4 points*

$$\boxed{f_s = f_{sym}}$$

Alternate answer to (b)-i: The discrete-to-continuous conversion block modulates its input with an impulse train with impulses separated by T_s and then applies an interpolating (pulse-shaping) filter.

(b) Consider a channel model of only additive spectrally-flat Gaussian noise.

i. Which block in the block diagrams acts as a pulse shaping filter? In what way? *4 points.*

If the analog lowpass filter in the A/D converter is the matched filter, then the analog lowpass filter in the D/A converter is the pulse shaping filter.

ii. Which block in the block diagrams acts as a matched filter? In what way? *4 points.*

The matched filter increases the SNR at the input to the decision block, primarily by attenuating out-of-band noise. This role is played by the analog lowpass filter in the A/D converter in the receiver.

iii. How close does the matched filter you identified in part (b)-ii above come to the optimal matched filter? *4 points.*

The optimal matched filter has an impulse response of $h_{opt}(t) = k\, g_T^*(T_{sym} - t)$ where $g_T(t)$ is the pulse shape. $|H_{opt}(f)| = |G_T(f)|$. The analog lowpass filters in the A/D converter and the D/A converter have the same magnitude design specification.

(c) Consider a binary phase shift keying (BPSK) system, a.k.a. two-level Pulse Amplitude Modulation, with symbol amplitudes of $-d$ and $+d$. Give a formula for d. Why? *6 points.*

We would like to have as much transmit power as possible; hence, we would like d to be as large as possible.

$$d = 2^{B-1} - 1$$

B is the number of bits in the D/A converter in the transmitter.