The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: December 4, 2023 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	22		Baseband PAM System
2	30		QAM Communication Performance
3	27		Decision-Directed Channel Equalization
4	21		Communication System Tradeoffs
Total	100		

Problem 2.1. *Baseband PAM System. 22 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

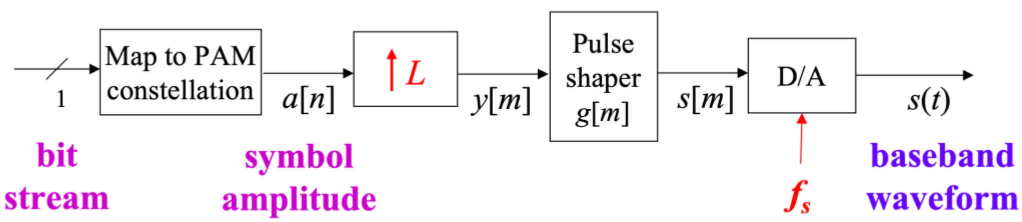
The system parameters are described on the right:

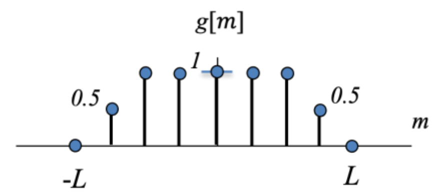
- $J = 1$ bit/symbol
- $L = 4$ samples per symbol period
- Pulse shape $g[m]$ is a trapezoidal pulse of 8 samples in duration from $m = -4$ to $m = 3$ inclusive of both and plotted below.
- A bit of value 0 is mapped to symbol amplitude -1, and a bit of value 1 is mapped to symbol amplitude 1.

PAM System Parameters

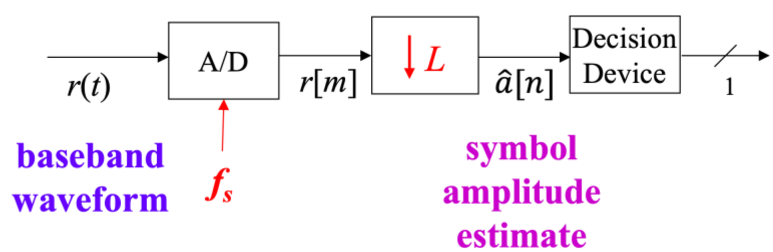
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

(a) For the **BPSK transmitter** below, the input bit stream is 01. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points.*



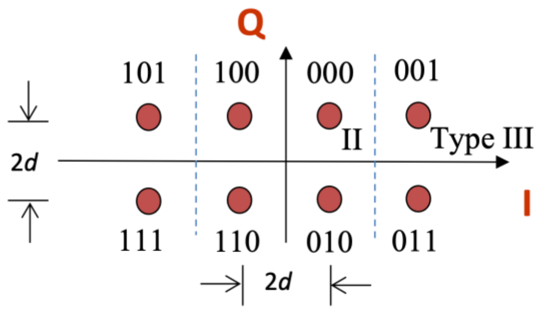


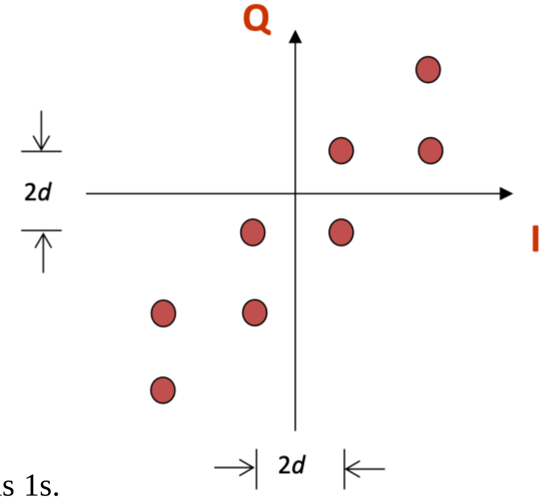
(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *10 points.*



Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

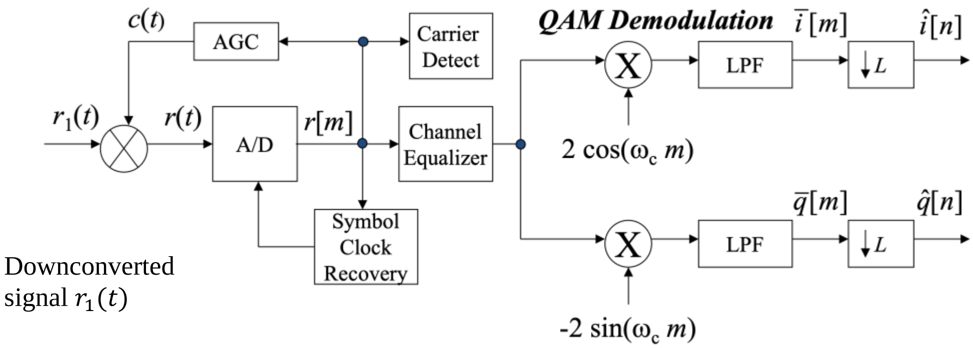
Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10d^2$	
(b) Average transmit power	$6d^2$	
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	4	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	

(j) Give a Gray coding for right constellation or show that one does not exist. *3 points*.

Problem 2.3. *Decision-Directed Channel Equalization. 27 points.*

For a quadrature amplitude modulation (QAM) receiver,



QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$i[n]$	Tx symbol amplitude in-phase component
$\hat{\imath}[n]$	Rx symbol amplitude in-phase component
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
$q[n]$	Tx symbol amplitude quadrature component
$\hat{q}[n]$	Rx symbol amplitude quadrature component

The adaptive FIR channel equalizer has N coefficients $\vec{w} = [\, w_0 \;\; w_1 \dots w_{N-1} \,]$.

The channel equalizer runs at the sampling rate but updates at the symbol rate.

Decision error $e[n]$ is the Euclidean distance between the transmitted symbol amplitude $i[n] + j\, q[n]$ and the received symbol amplitude $\hat{\imath}[n] + j\, \hat{q}[n]$ at symbol index n.

Consider adapting the decision-directed channel equalizer during training.

(a) What training sequence for the symbol amplitude values would you use? Why? *3 points.*

(b) Give an objective function $J(n)$. Why did you choose it? *6 points.*

(c) What is the initial value of the adaptive FIR equalizer coefficients you would use? Why? *3 points.*

(d) Derive an update equation for the adaptive FIR equalizer coefficients vector at iteration n

$$\vec{w}[n] = [\, w_0[n] \; w_1[n] \dots w_{N-1}[n] \,]$$

Compute all derivatives. Simplify the result. *12 points*

(e) What range of values would you recommend for the step size μ? Why? *3 points.*

Problem 2.4. *Communication System Tradeoffs. 21 points.*

Two-way communication systems have a data channel and a control channel in each direction.

The data channel conveys high speed data such as streaming audio or video whereas the control channel conveys configuration and feedback information. For example, a phone will send the received signal strength to the basestation on a control channel.

The control channel provides lower bit rates at lower symbol error rates with higher delay relative to the data channel.

(a) For an increase in each parameter below, indicate whether the pulse amplitude modulation (PAM) communication system performance measure in each column increases, decreases, or stays the same. As you consider increasing one of the parameters in the table, assume the other two parameters in the table are not changing in value. Assume the symbol rate, f_{sym}, does not change.

(b) Justify your answers.

Increasing Parameter	**Effect on the Bit Rate**	**Effect on Symbol Error Rate**	**Effect on Delay**
J bits/symbol			
L samples/symbol			
N_g symbol periods in pulse shape			