

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
Midterm #2

Date: December 9, 2024

Course: EE 445S Evans

Name: \_\_\_\_\_  
Last, First

- **Exam duration.** The exam is scheduled to last 75 minutes.
- **Materials allowed.** You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks.** Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed.** As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers.** When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab.** No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test.** All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

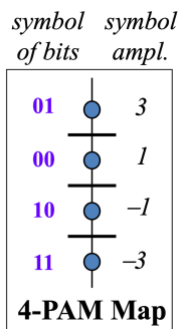
Problem	Point Value	Your score	Topic
1	26		Baseband PAM System
2	30		QAM Communication Performance
3	20		Steepest Descent Algorithms
4	24		Communication System Tradeoffs
Total	100		

**Problem 2.1. Baseband PAM System. 26 points.**

Consider a pulse amplitude modulation (4-PAM) system.

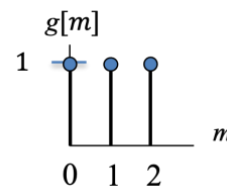
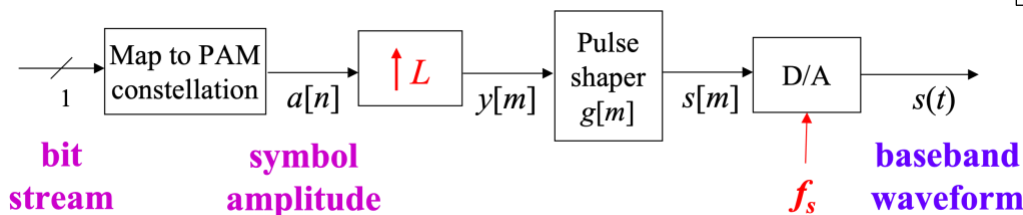
The system parameters are described on the right:

- $J = 2$  bits/symbol; i.e.,  $M = 4$  levels/symbol
- $L = 3$  samples per symbol period
- Pulse shape  $g[m]$  is a rectangular pulse of 3 samples in duration plotted below.
- Constellation map is shown to the right ( $d = 1$ ).

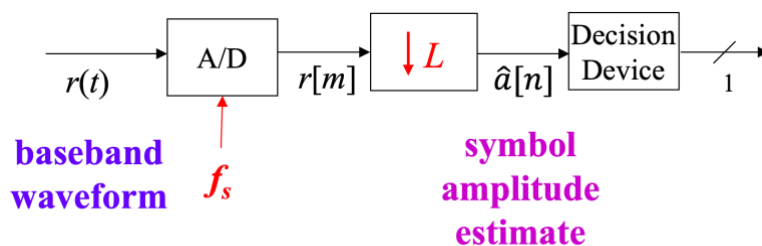


<b>PAM System Parameters</b>	
$a[n]$	symbol amplitude
$2d$	constellation spacing
$f_s$	sampling rate
$f_{sym}$	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse response
$J$	bits/symbol
$L$	samples/symbol period
$M$	levels, i.e. $M = 2^J$
$m$	sample index

(a) For the **4-PAM transmitter** below, input bit stream is 01001011. Plot the discrete-time signals  $a[n]$ ,  $y[m]$  and  $s[m]$ . 12 points.



(b) For the **4-PAM receiver** to the right, assume there is no channel distortion or additive noise and assume that  $r[m] = s[m]$ . **There is no matched filter.** Plot the discrete-time signal  $\hat{a}[n]$  and give the received bit stream based on the 4-PAM transmitter in (a). 7 points.

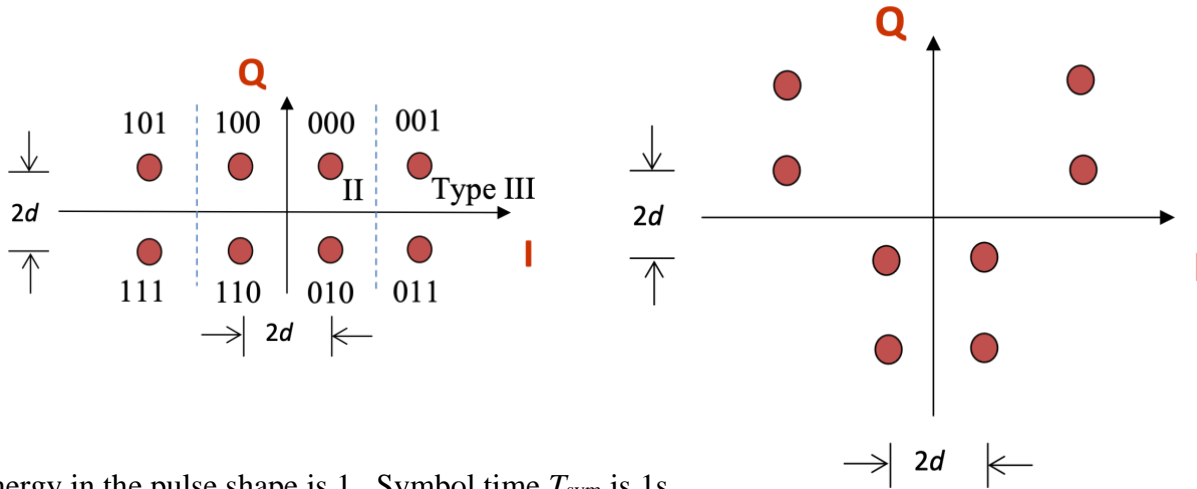


(c) Concerning the impulse response  $h[m]$  for optimal matched filter: 7 points.

- Give the general formula for  $h[m]$  in terms of the pulse shape  $g[m]$  used in a transmitter?
- Plot  $h[m]$  given the pulse shape  $g[m]$  in part (a).
- What is the optimal matched filter impulse response optimizing?

**Problem 2.2 QAM Communication Performance.** 30 points.

Consider the two 8-QAM constellations below. Constellation spacing is  $2d$ .



Energy in the pulse shape is 1. Symbol time  $T_{\text{sym}}$  is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	Right Constellation
(a) Peak transmit power	$10d^2$	
(b) Average transmit power	$6d^2$	
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation <b>that will minimize the probability of symbol error using such decision regions.</b>		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	4	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance $\sigma^2$ .	$P_e = \frac{5}{2}Q\left(\frac{d}{\sigma}\right) - \frac{3}{2}Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the $Q$ function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	

(j) Give a Gray coding for the right constellation or show that one does not exist. 3 points.

**Problem 2.3. Steepest Descent Algorithms. 20 points.**

Consider using the steepest descent algorithm to maximize the objective function

$$J(x) = 8 - x^2 + 6 \cos(6x)$$

The objective function is plotted on the right.

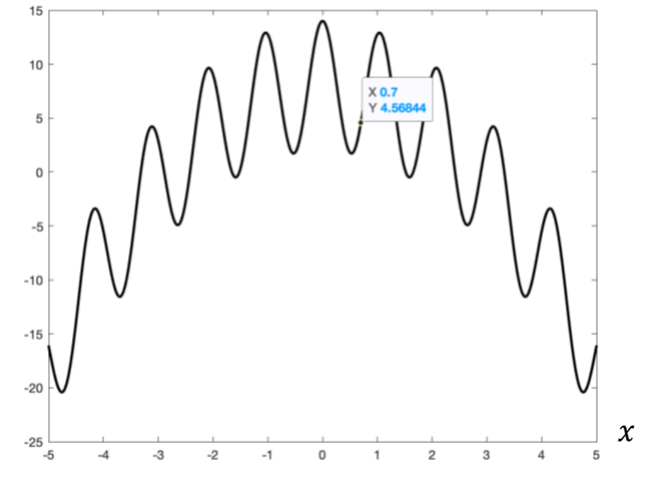
The steepest descent update equation at iteration  $k$  is

$$x[k + 1] = x[k] + \mu \left. \frac{dJ(x)}{dx} \right]_{x=x[k]}$$

For the problem, assume the initial guess for  $x$  is 0.7, which is shown on the plot of the objective function on the right.

Someone else has implemented the steepest descent algorithm in MATLAB and you're running their code.

Their algorithm runs the update equation for 50 iterations.



- (a) If their algorithm were to give an answer of  $x = 0.7$ , what's a possible cause? 4 points.
  
  
  
  
  
  
  
  
  
  
- (b) If their algorithm were to give an answer of  $x = 6.9818 \times 10^{63}$ , what's a possible cause? 4 points
  
  
  
  
  
  
  
  
  
  
- (c) If their algorithm were to give an answer of  $x = 0.53$ , which gives a value of  $J(x) = 1.72$ , what's a possible cause? 4 points
  
  
  
  
  
  
  
  
  
  
- (d) If their algorithm were to give an answer of  $x = 0.7782$ , what are **two possible causes**? 8 points.

**Problem 2.4. Communication System Tradeoffs. 24 points.**

Two-way communication systems have a data channel and a control channel in each direction.

The data channel supports high bit rates such as for streaming audio or video whereas the control channel has low bit rates for configuration and feedback information (e.g. received SNR).

In this problem, compare a

- Control channel transmitter with 4-level QAM ( $J = 2$  bits/symbol), a bit rate of 100 kbps,  $d = 1$ ,  $L = 10$ , and  $N_g = 16$ .
- Data channel transmitter with 256-level QAM ( $J = 8$  bits/symbol), a bit rate of 10 Mbps,  $d = 1$ ,  $L = 10$ , and  $N_g = 2$ .

Both quadrature amplitude modulation (QAM) constellations are square; e.g., the 256-level QAM constellation has 16 points in the in-phase dimension and 16 points in the quadrature dimension.

The pulse shape is a raised cosine with a rolloff factor of 1.

In the table, give a numeric value for each quantity and explain your reasoning with a formula to compute the numeric value.

**QAM System Parameters**

$2d$	constellation spacing
$f_s$	sampling rate
$f_{sym}$	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
$J$	bits/symbol
$L$	samples/symbol period
$M$	levels, i.e. $M = 2^J$
$m$	sample index
$N_g$	number of symbol periods in a pulse shape
$n$	symbol index

Quantity	Control Channel Transmitter	Data Channel Transmitter
Symbol rate, $f_{sym}$ , in Hz		
Transmission bandwidth, in Hz		
Peak transmit power consumption in Watts		
Baseband run-time complexity in multiplications/s		