The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Solutions 2.0***

Date: December 8, 2025 Course: EE 445S Evans

Name: ____
Last, First

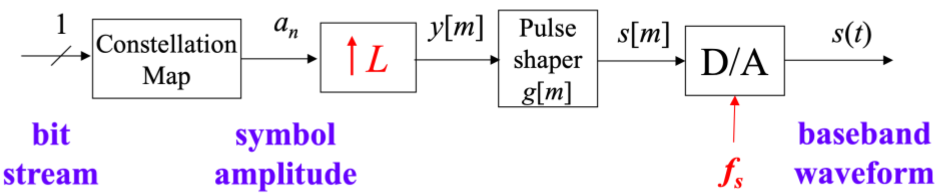
- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	24		Baseband PAM System
2	30		QAM Communication Performance
3	26		Automatic Gain Control
4	20		Communication System Tradeoffs
Total	100		

Lectures: 7 Pulse Shaping, 13 Digital PAM and 14 Matched Filtering; JSK Ch. 8 and 9; Labs 3, 5 & 6; HW 1.2, 2.1, 2.2, 4.2, 4.3, 5.2 & 6.2; Midterm 2: F21 Prob. 2.1, F24 Prob. 2.1, Sp25 Prob. 2.1

Problem 2.1. *Baseband PAM System. 24 points.*

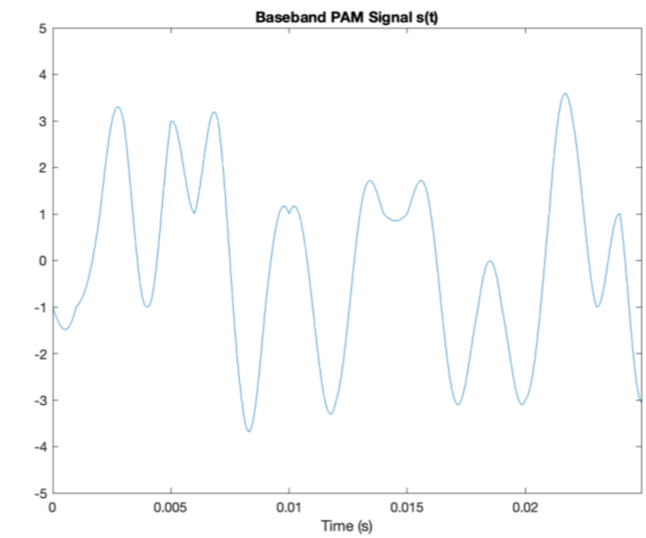
Consider the baseband pulse amplitude modulation (PAM) transmitter below whose parameters are described on the right:



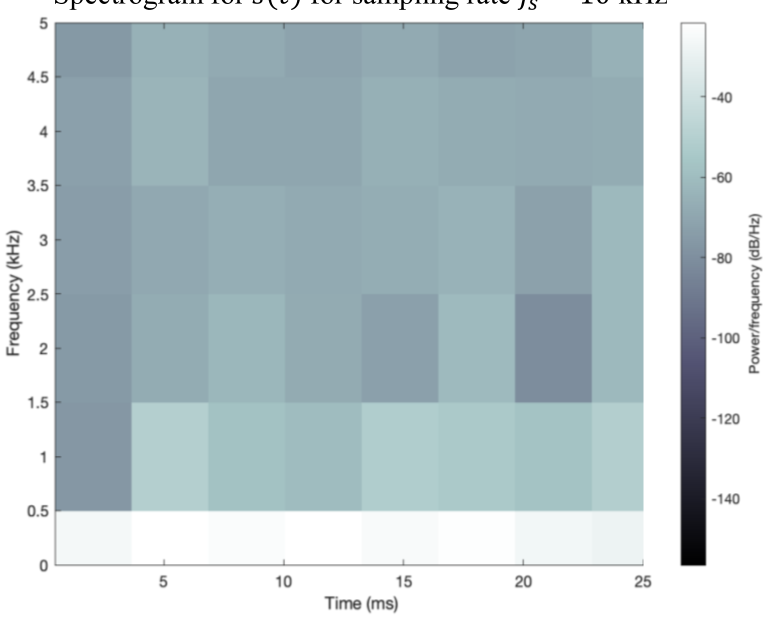
PAM System Parameters

a_n	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

After 50 bits are input, the output $s(t)$ lasts from 0ms to 25ms and is plotted below. Its spectrogram is also computed below. The sampling rate in the baseband PAM transmitter is $f_s = 10$ kHz.



Spectrogram for $s(t)$ for sampling rate $f_s = 10$ kHz



Determine numeric values for the following parameters and justify how you obtained them:

(a) d half the constellation spacing. *6 points*.

Analyzing the plot on the left for $s(t)$.
The value of at the origin is -1 which is a symbol amplitude.
The maximum symbol amplitude is less than the peak value in the plot which is around 3.5.
Symbol amplitudes are -3, -1, 1, 3. So $d = 1$.
***Comment:* Peaks and valleys are due to interpolation—they are close to, but not equal to, symbol amplitude values. See lecture slides 13-4 and 13-6 in Lecture 13 on Digital PAM.**

(b) J bits/symbol. *6 points*.

For 4-PAM, $J = 2$.

(c) L samples/symbol period. *6 points*.

$f_s = L\, f_{sym}$ and $f_s = 10000$ Hz. Find f_{sym}.

Approach #1: $Bit\ Rate = J\, f_{sym}$.

$$Bit\ Rate = \frac{50 \text{ bits}}{0.025 \text{ s}} = 2000 \text{ bits/s}$$

With $J = 2$, $f_{sym} = 1000$ Hz and $L = 10$.

Approach #2: The spectrogram shows a maximum baseband frequency of 0.5 kHz which is $\frac{1}{2} f_{sym}$. This gives $f_{sym} = 1000\ Hz$ and hence $L = 10$.

(d) f_{sym} symbol rate. *6 points*.

See the answer in part (c).

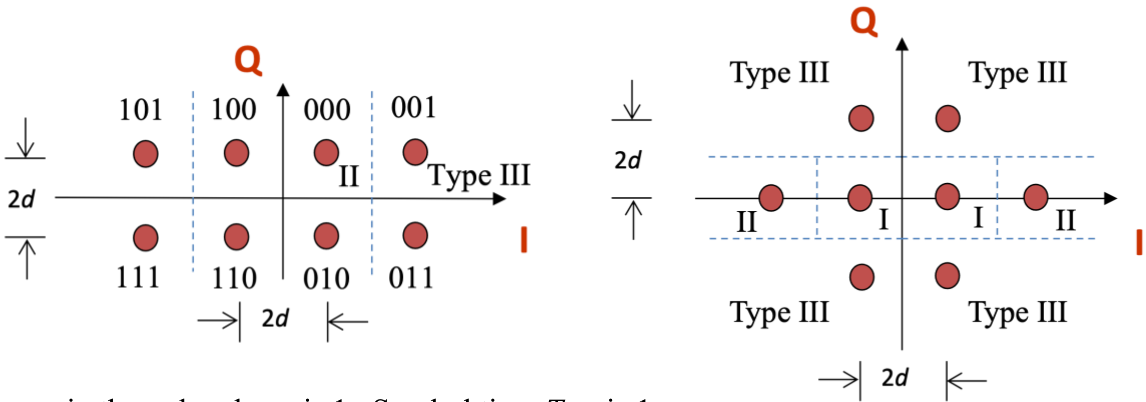
***Epilogue*: Given the symbol rate of 1000 Hz computed two different ways, the symbol time is 1 ms. With 25 symbol periods in 25 ms, each symbol has 2 bits to match 50 bits transmitted.**

```
% Baseband PAM Signal Generation
% by Prof. Brian L. Evans, UT Austin
% Modified from ECE 445S Lecture Slide 13-5

% m is sample index; n is symbol index

% Simulation parameters
N = 25;      % Number symbol periods

% Pulse shape g[m]
Ng = 4;      % Number symbol periods
L = 10;      % Samples/symbol period
f0 = 1/L;
midpt = Ng*L/2;
m = (-midpt) : (midpt-1);
g = sinc(f0*m);

% Adjust for group delay
N = N + (Ng/2);

% M-level PAM symbol amplitudes
d = 1;
M = 4;
ioffset = M + 1;
symAmp = (2*randi(M,[1,N]) - ioffset)*d;

% Discrete-time baseband PAM signal
mmax = N*L;
v = zeros(1,mmax);
v(1:L:end) = symAmp;  % interpolation
s = conv(v, g);       % pulse shaping
slength = length(s);  % trim result
s = s(midpt+1:slength-midpt+1);

% Interpretation in continuous time
Tsym = 10^(-3); % Symbol period in sec
fsym = 1/Tsym;    % Symbol rate in Hz
fs = L*fsym;      % Sampling rate in Hz
Ts = 1/fs;        % Sampling time in sec

% Plots
Mmax = length(s);
m = 0 : (Mmax-1);
t = m*Ts;
Nmax = Mmax / L;
n = 0 : (Nmax-1);
figure;
plot(t,s);
% hold on;
% stem(n*Tsym,symAmp);
% hold off;
xlim( [0 (Nmax-(Ng/2))*Tsym-Ts] );
ymax = 5;
ylim( [-ymax ymax] );
xlabel('Time (s)');
title('Baseband PAM Signal s(t)');

figure;
Nfft = L;
Noverlap = Nfft-1;
spectrogram(s, [], Noverlap, Nfft, fs, 'yaxis');
colormap bone;
ylim( [0 fs/2] / 1000 );  % put units in kHz
```

Lectures 13-16; JSK Ch. 16; Labs 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3; Handout P PAM vs. QAM; Midterm 2.2: F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20, F20, Sp21, F21, Sp22, Sp25

Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**	
(a) Peak transmit power	$10\,d^2$	$\mathbf{9\,d^2}$	
(b) Average transmit power	$6\,d^2$	$\mathbf{5\,d^2}$	**See below**
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	$\frac{9d^2}{5d^2} = \frac{9}{5} \approx 1.8$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***			
(e) Number of type I QAM regions	0	**2**	
(f) Number of type II QAM regions	4	**2**	
(g) Number of type III QAM regions	4	**4**	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ².	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	$P_e = \frac{11}{4} Q\left(\frac{d}{\sigma}\right) - 2Q^2\left(\frac{d}{\sigma}\right)$	**See below**
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	$\text{SNR} = \frac{5d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{5}}$	

(j) Give a Gray coding for the right constellation or show that one does not exist. *3 points*.

Gray coding means that every pair of adjacent symbols (based on their constellation regions) differ only by one bit. For 8-QAM, we have three bits for each symbol. Yet, the symbol at $(0, d)$ has four neighbors, and it's not possible to Gray code 4 neighbors with 3 bits.

(b) The average transmit power is proportional to $\frac{1}{8}(4(d)^2 + 2(2d^2 + d^2) + (3d)^2) = 5d^2$.

(h) Let $q = Q\left(\frac{d}{\sigma}\right)$. $P_c = \frac{2}{8}P_c^{I} + \frac{2}{8}P_c^{II} + \frac{4}{8}P_c^{III} = \frac{2}{8}(1-2q)^2 + \frac{2}{8}(1-q)(1-2q) + \frac{4}{8}(1-q)^2$

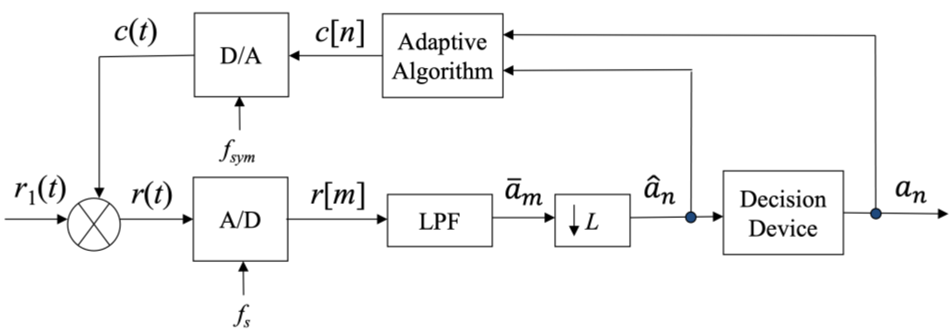
and $P_e = 1 - P_c$

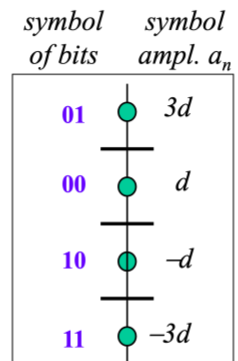
JSK Sec. 3.7, 6.5-6.8, 9.3; HW 5.1, 6.1, 6.2, 7.2; HW 7.3 from Sp14; In-Lecture Assignments 3 & 4; PAM Lectures 12-14; AGC Lecture Slide 16-4; AGC Midterm #2 Prob. 2.3 in Sp15 and Sp19; Decision Directed Midterm #2 Prob. 2.3 in F17, F21, Sp 23 and F23

Problem 2.3. *Automatic Gain Control. 26 points.*

Automatic gain control (AGC) is used to compensate for time-varying gain (e.g. fading).

In this problem, you'll design an adaptive AGC algorithm for a digital pulse amplitude modulation (PAM) receiver using decision-directed steepest descent algorithm:





4-level PAM Constellation Map

$r_1(t)$ is an analog continuous-time baseband PAM signal.

L is the number of samples in a symbol period.

Downsampling by L converts input $\bar{a}_m$ at the sampling rate to output $\hat{a}_n$ at the symbol rate.

The decision device finds a_n as the symbol amplitude in the PAM constellation map closest to $\hat{a}_n$.

(a) What are the two roles of the lowpass filter (LPF)? How would you design it? *6 points.*

- **Anti-aliasing filter to reduce the amount of aliasing caused by downsampling by *L*.**
- **Matched filter to maximize the SNR at downsampler output assuming the only impairment is additive noise. The LPF impulse response would be $h[m] = k\, g^*[L - m]$ where $g[m]$ is the pulse shape used by the transmitter and k is a non-zero gain. Hence, the AGC does not affect the optimality of the matched filter.**

(b) For the adaptive algorithm, what training signal would you recommend? Using a training signal would allow the receiver to know what the values of a_n are in the transmitter. *4 points.*

Training sequence should be easy to generate, have good correlation properties, and contain all discrete-time frequencies because the channel will attenuate/reject some frequencies.
Option #1: **Long maximal-length pseudo-noise sequence. Length is $2^r - 1$ bits where *r* is the number of states in the PN generator. Map '1' bit to $3d$ and '0' bit to $-3d$ in amplitude.**
Option #2: **Chirp signal that sweeps from 0 Hz to $\frac{1}{2} f_s$. Acceptable as a test answer but how to map the chirp signal to symbol amplitudes? What pulse shape to use?**

(c) For the decision-directed objective function $J(n) = (\hat{a}_n - a_n)^2$, give the update equation for the discrete-time gain $c[n]$. Assume $\hat{a}_n$ depends on $c[n]$, but a_n does not depend on $c[n]$. *12 points.*

$$c[n+1] = c[n] - \mu \frac{d}{dc[n]} J(n) = c[n] - 2\,\mu\,(\hat{a}_n - a_n) \frac{d}{dc[n]} \hat{a}_n = c[n] - \bar{\mu}\,(\hat{a}_n - a_n)\, a_n$$

See next page for additional work

(d) What value would you choose for the step size μ. Why? *4 points.*
Small positive values for μ such as 0.001 to ensure convergence of the iterative algorithm. Using $\mu = 0$ would not allow the iterative algorithm to update. Using a negative μ would convert the iterative algorithm into a steepest ascent algorithm to maximize the objective function. A large positive value would cause the steepest descent algorithm to diverge.

In part (c), steepest descent is used to minimize the objective function $J(n) = (\hat{a}_n - a_n)^2$ and the answer given on the previous page would be enough for a test:

$$c[n+1] = c[n] - \mu \frac{d}{dc[n]} J(n) = c[n] - 2\,\mu\,(\hat{a}_n - a_n)\,\frac{d}{dc[n]}\,\hat{a}_n$$

The automatic gain $c[n]$ is updated at the symbol rate and not the sampling rate.

To compute the derivative of the received symbol amplitude $\hat{a}_n$ with respect to the automatic gain $c[n]$, we work backwards on the block diagram (lower branch)

$$\hat{a}_n = \bar{a}_{nL}$$

and $\bar{a}_{nL}$ is the output of the LPF filter with impulse response $h[m]$ with $N_g\,L$ coefficients

$$\bar{a}_{nL} = h[0]\,r[nL] + h[1]\,r[nL-1] + \cdots + h\left[N_g L\right] r\left[nL - N_g L - 1\right]$$

Expanding this to express what happens every L terms (one symbol period of samples)

$$\bar{a}_{nL} = h[0]\,r[nL] + h[1]\,r[nL-1] + \cdots + h[L-1]\,r[nL-(L-1)] +$$

$$h[L]r[(n-1)L] + h[L+1]\,r[(n-1)L-1] + \cdots + h[2L-1]\,r[(n-1)L-(L-1)] + \cdots$$

$$h\left[N_g L\right] r\left[nL - N_g L - 1\right]$$

Moreover, over the *n*th symbol period, for $m = nL, nL+1, \dots, nL+(L-1)$,

$$r[m] = r_1[m]\,c[n]$$

Combining the two previous equations,

$$\bar{a}_{nL} = h[0]\,r_1[nL]\,c[n] + h[1]\,r_1[nL-1]\,c[n] + \cdots + h[L-1]\,r_1[nL-(L-1)]\,c[n] + \cdots$$

The omitted terms after $h[L-1]\,r_1[nL-(L-1)]\,c[n]$ do not depend on $c[n]$ but instead on $c[n-1], c[n-2], \dots$.

$$\frac{d}{dc[n]}\,\hat{a}_n = \frac{d}{dc[n]}\bar{a}_{nL} = h[0]\,r_1[nL] + h[1]\,r_1[nL-1] + \cdots + h[L-1]\,r_1[nL-(L-1)]$$

At this point, we're stuck. We don't know the received baseband signal $r_1(t)$ and hence we don't know $r_1[m]$. The transmitted baseband signal, which is proportional to a_n, experiences linear and nonlinear distortion and additive impairments by time it becomes $r_1(t)$. The objective function $J(n)$ attempts to capture all these impairments. For the purpose of computing the derivative of $\hat{a}_n$ w/r to $c[n]$, we simplify $r_1[m]$ to be a_n.

$$\frac{d}{dc[n]}\,\hat{a}_n = \frac{d}{dc[n]}\bar{a}_{nL} = a_n\,(h[0] + h[1] + \cdots + h[L-1])$$

The term $h[0] + h[1] + \cdots + h[L-1]$ is a constant and can be rolled into the step size:

$$\frac{d}{dc[n]}\,\hat{a}_n = \frac{d}{dc[n]}\bar{a}_{nL} = a_n$$

where $\bar{\mu} = 2\,\mu\,(h[0] + h[1] + \cdots + h[L-1])$. The update equation becomes

$$c[n+1] = c[n] - \bar{\mu}\,(\hat{a}_n - a_n)\,a_n$$

Lectures 13-16; JSK Sec. 14.4; JSK Ch. 2, 3, 16; Labs 3, 5 & 6; HW 3.3, 5.3, 6.3, 7.1, 7.2; Reading from Ch. 4 of Simon Haykin's Communication Systems book; Midterm Prob. 2.4 in F20, 2.3 in Sp21, 2.4 in F23, 2.4 in Sp24, 2.4 in F24, 2.4 in Sp25

Problem 2.4. *Communication System Tradeoffs. 20 points.*

Claude Shannon derived the following upper bound on the capacity, C, for a communication channel in units of bits/s for a QAM system:

$$C = B \log_2(1 + \text{SNR})$$

where

B is the transmission bandwidth in Hz

SNR is the Signal-to-Noise Ratio at the receiver in linear units (not in decibels) where

$$\text{SNR} = \frac{\mathit{Signal\ Power}}{\mathit{Noise\ Power}}$$

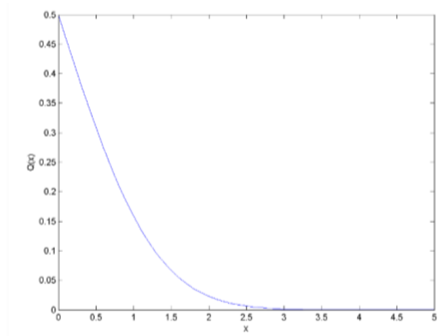
The upper bound on the number of bits/symbol, J, is $\log_2(1 + \text{SNR})$.

We seek to increase the channel capacity in a QAM system:

Assuming the constellation spacing $2d$ stays the same, give formulas and an explanation as to how the following will increase or decrease or stay the same when increasing the transmission bandwidth, B:

QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
N_g	number of symbol periods in a pulse shape
n	symbol index



Baseband bandwidth: $\frac{1}{2} f_{sym}$

Transmission bandwidth: $B = f_{sym}$

(a) Bit rate. *4 points.*

Bit Rate $= J\, f_{sym} = J\, B$ **in units of bits/s.**

Increasing B linearly increases the Bit Rate.

(b) Probability of symbol error (also known as the symbol error rate). *4 points.*

$P_e = C_0\, Q\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right) + C_1\, Q^2\left(\frac{d}{\sigma}\sqrt{T_{sym}}\right)$ **where** $T_{sym} = \frac{1}{f_{sym}} = \frac{1}{B}$, C_0 **and** C_1 **are positive constants, the Q function (plotted above) decreases as its argument increases.**

As B increases and d and σ remain the same, the Probability of Symbol Error increases.

(c) Baseband transmitter run-time implementation computational complexity. *4 points.*

The pulse shaping filter is a finite impulse response filter with $N_g\, L$ coefficients running at sampling rate f_s and requires $(L\, N_g)(L\, f_{sym}) = L^2\, N_g\, f_{sym} = L^2\, N_g\, B$ multiplications/s. A polyphase filter bank implementation would need $L\, N_g\, B$ multiplications/s. Either way, the run-time implementation computational complexity would increase linearly with B.

(d) Power consumption in the D/A converter in the transmitter analog/RF front end. *4 points.*

D/A power consumption is proportional to $f_s\, 2^{Bits}$ where $Bits$ is the number of bits on the input to the D/A converter and f_s is the sampling rate for the transmitter. For a transmitter, $Bits \geq M$ and $f_s = L\, f_{sym} = L\, B$. D/A power consumption will increase linearly with B.
High-resolution, high-speed D/A converters consume as much as power as $f_s^2\, 4^{Bits}$.

(e) Transmitted power. *4 points.* ***#1.*** **Transmit power is proportional to $d^2$and d is not changing.**

#2. Maximum transmit power is not dependent on B. The government or communication standard will set the maximum transmit power allowed for a particular system. This is to reduce interference for transmitters operating in the same or nearby frequency bands.