The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: December 8, 2025 Course: EE 445S Evans

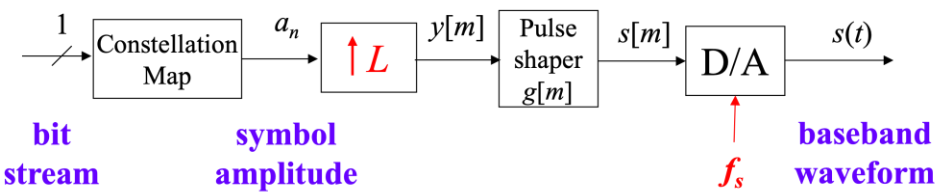
Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	24		Baseband PAM System
2	30		QAM Communication Performance
3	26		Automatic Gain Control
4	20		Communication System Tradeoffs
Total	100		

Problem 2.1. *Baseband PAM System. 24 points.*

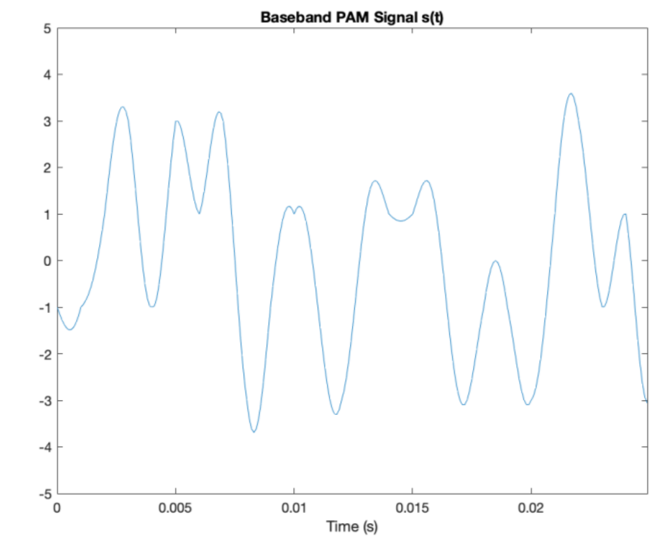
Consider the baseband pulse amplitude modulation (PAM) transmitter below whose parameters are described on the right:



PAM System Parameters

a_n	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

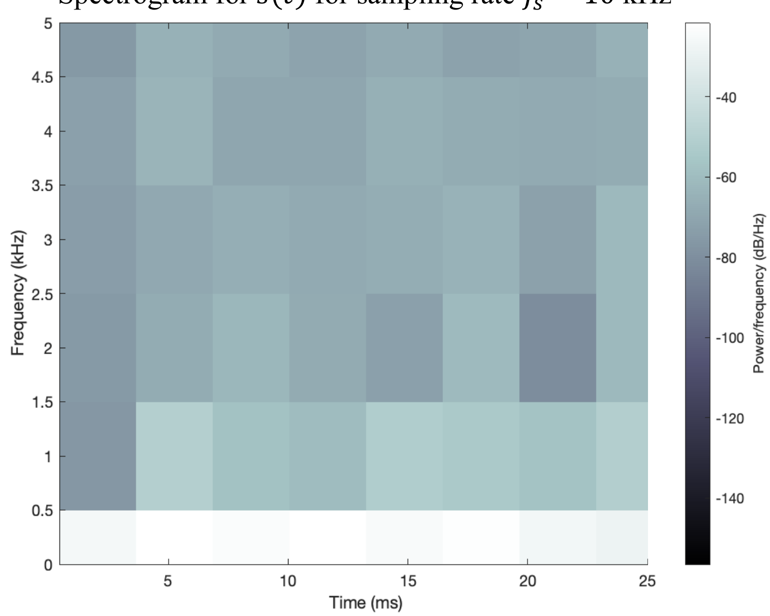
After 50 bits are input, the output $s(t)$ lasts from 0ms to 25ms and is plotted below. Its spectrogram is also computed below. The sampling rate in the baseband PAM transmitter is $f_s = 10$ kHz.



Determine numeric values for the following parameters and justify how you obtained them:

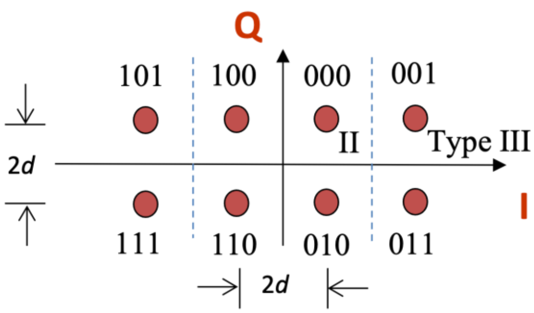
(a) d half the constellation spacing. *6 points*.
(b) J bits/symbol. *6 points*.
(c) L samples/symbol period. *6 points*.
(d) f_{sym} symbol rate. *6 points*.

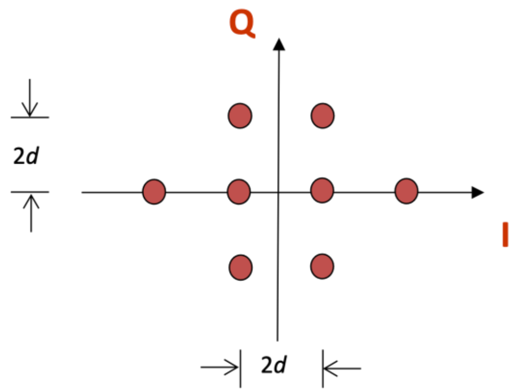
Spectrogram for $s(t)$ for sampling rate $f_s = 10$ kHz



Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

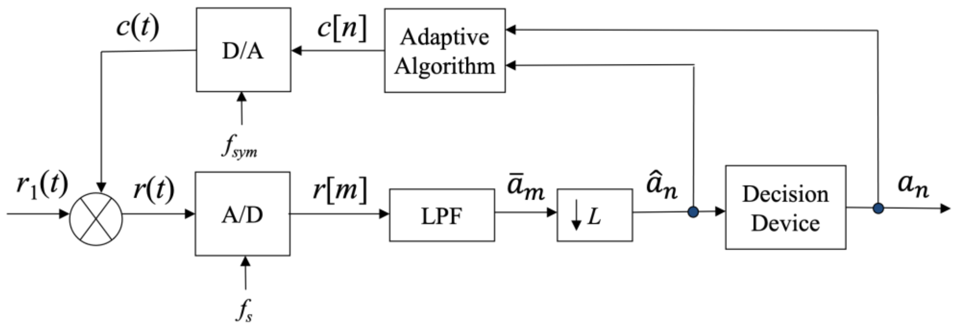
	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10\, d^2$	
(b) Average transmit power	$6\, d^2$	
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	4	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	

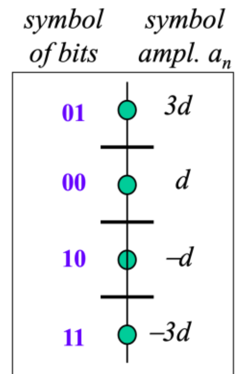
(j) Give a Gray coding for the right constellation or show that one does not exist. *3 points*.

Problem 2.3. *Automatic Gain Control. 26 points.*

Automatic gain control (AGC) is used to compensate for time-varying gain (e.g. fading).

In this problem, you'll design an adaptive AGC algorithm for a digital pulse amplitude modulation (PAM) receiver using decision-directed steepest descent algorithm:





4-level PAM Constellation Map

$r_1(t)$ is an analog continuous-time baseband PAM signal.

L is the number of samples in a symbol period.

Downsampling by L converts input $\bar{a}_m$ at the sampling rate to output $\hat{a}_n$ at the symbol rate.

The decision device finds a_n as the symbol amplitude in the PAM constellation map closest to $\hat{a}_n$.

(a) What are the two roles of the lowpass filter (LPF)? How would you design it? *6 points*.

(b) For the adaptive algorithm, what training signal would you recommend? Using a training signal would allow the receiver to know what the values of a_n are in the transmitter. *4 points*.

(c) For the decision-directed objective function $J(n) = (\hat{a}_n - a_n)^2$, give the update equation for the discrete-time gain $c[n]$. Assume $\hat{a}_n$ depends on $c[n]$, but a_n does not depend on $c[n]$. *12 points.*

(d) What value would you choose for the step size μ. Why? *4 points*.

Problem 2.4. *Communication System Tradeoffs. 20 points.*

Claude Shannon derived the following upper bound on the capacity, C, for a communication channel in units of bits/s for a QAM system:

$$C = B \log_2(1 + \text{SNR})$$

where

B is the transmission bandwidth in Hz

SNR is the Signal-to-Noise Ratio at the receiver in linear units (not in decibels) where

$$\text{SNR} = \frac{Signal\ Power}{Noise\ Power}$$

The upper bound on the number of bits/symbol, J, is $\log_2(1 + \text{SNR})$.

QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
N_g	number of symbol periods in a pulse shape
n	symbol index

We seek to increase the channel capacity in a QAM system:

Assuming the constellation spacing $2d$ stays the same, give formulas and an explanation as to how the following will increase or decrease or stay the same when increasing the transmission bandwidth, B:

(a) Bit rate. *4 points.*

(b) Probability of symbol error (also known as the symbol error rate). *4 points.*

(c) Baseband transmitter run-time implementation computational complexity. *4 points.*

(d) Power consumption in the D/A converter in the transmitter analog/RF front end. *4 points.*

(e) Transmitted power. *4 points.*