The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Prof. Brian L. Evans

Date: May 4, 2012 Course: EE 445S

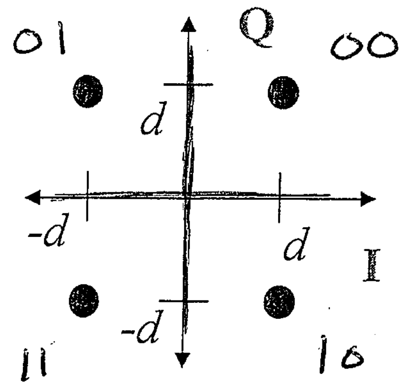
Name: Set, Solution
Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets. You may not share materials with other students.
- Calculators are allowed.
- You may use any stand-alone computer system, i.e. one that is not connected to a network. **Disable all wireless access from your stand-alone computer system.**
- Please turn off all cell phones, pagers, and personal digital assistants (PDAs).
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.** When justifying your answers, you may refer to the Johnson, Sethares & Klein textbook, the Welch, Wright and Morrow lab book, course reader, and course handouts. Please be sure to reference the page/slide number and quote the particular content you are using in your justification.

Problem	Point Value	Your score	Topic
1	27		Quadrature Amplitude Modulation
2	27		Channel Estimation
3	27		Pulse Amplitude Modulation Receiver
4	19		Noise Shaping
Total	100		

Problem 2.1 *Quadrature Amplitude Modulation (QAM). 27 points.*

A 4-level QAM constellation is shown on the right.
Assume that the symbol time is 1s. $T_{sym} = 1$
Assume that the energy in the pulse shape is 1.



(a) On the 4-QAM constellation on the right, please specify an encoding for each level that minimizes the number of bit errors when a symbol error occurs. *6 points.* Gray coding →

(b) Compute the average and peak transmitted power. *3 points.*

Total power : $2d^2 + 2d^2 + 2d^2 + 2d^2 = 8d^2$

Average power : $\frac{8d^2}{4} = 2d^2$

Peak power : $2d^2$

(c) Draw decision regions at the receiver on the above constellation. *6 points.* I axis and Q axis.

(d) Based on your decision regions in (c), give the fastest algorithm possible to decode/quantize the estimated symbol amplitude in the receiver into a symbol of bits. *6 points.*

Symbol of bits has 2 bits $S_0 S_1$. Symbol amplitude (estimated): $\hat{a}_n + j\hat{b}_n$

If $(\hat{a}_n > 0)$ $S_1 = 0$ else $S_1 = 1$

If $(\hat{b}_n > 0)$ $S_0 = 1$ else $S_0 = 0$

Two comparisons using divide-and-conquer strategy.

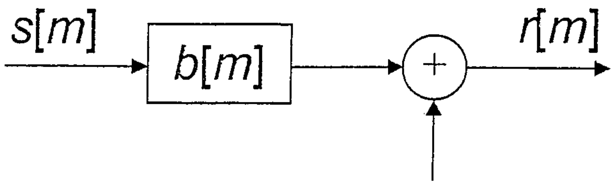
(e) Based on the decision regions in (c), give a formula for the probability of symbol error. *6 points.*

Based on QAM transmitter lecture slides 15-13 and 15-14, 4-QAM decision regions are type-3 QAM regions (i.e. corner regions that are not edges).

$$P_3(c) = \left(1 - Q\left(\frac{d}{\sigma}\right)\right)^2$$

$$P(e) = 1 - P_3(c) = 1 - \left(1 - Q\left(\frac{d}{\sigma}\right)\right)^2 = 2Q\left(\frac{d}{\sigma}\right) - Q^2\left(\frac{d}{\sigma}\right)$$

Problem 2.2. *Channel Estimation.* 27 points.



A sparse communication channel is modeled as a linear time-invariant (LTI) finite impulse response (FIR) comb filter $b[m]$ plus additive white Gaussian noise $w[m]$.

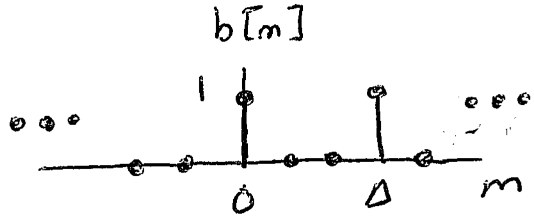
~~$b[m] = 1 + \delta[m-\Delta]$~~ where $\Delta > 1$ $\quad b[m] = \delta[m] + \delta[m-\Delta]$

$w[m]$ has zero mean and variance σ^2.

(a) Give an equation in the discrete-time domain for the received signal $r[m]$ for the transmitted signal $s[m]$ and the channel model. *6 points.*

$$r[m] = s[m] * b[m] + w[m]$$

$$r[m] = s[m] + s[m-\Delta] + w[m]$$



(b) Give a training signal $s[m]$ that would enable accurate estimation of Δ for $\Delta > 1$. How would you determine the length of the training signal? *6 points.*

Use a long maximal-length pseudo-noise sequence for the training signal $s[m]$. PN sequences are robust to frequency selective channels and to additive noise. The PN sequence would have values of +1 or −1. Longer sequences will better results in (c). PN length can be less than, equal to, or greater than Δ.

(c) Using your answer in (b), give an algorithm in the receiver to estimate Δ. *6 points*

In the receiver, we would correlate the received signal $r[m]$ against the training sequence. Two peaks should result at $m = 0$ and at $m = \Delta$.

(d) Give a formula for the impulse response or transfer function of a channel equalizer in the receiver to compensate for the frequency selectivity of the channel. You may ignore the noise. *9 points.*

$$b[m] = \delta[m] + \delta[m-\Delta]$$

$$B(z) = 1 + z^{-\Delta} \quad \leftarrow \Delta \text{ zeros on the unit circle}$$

Equalizer filter $G(z)$

We can't use $G(z) = \frac{1}{B(z)}$ because $G(z)$ would have Δ poles on unit circle.

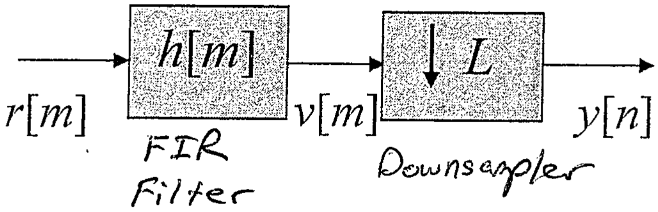
$$\text{Let } G(z) = \frac{1}{1 + 0.95\, z^{-\Delta}} \quad \leftarrow \Delta \text{ poles with radius } (0.95)^{\Delta}.$$

$\boxed{\text{IIR Comb Filter}}$

Cascade of $B(z)$ and $G(z)$ gives LTI system with Δ notches.

Problem 2.3. *Pulse Amplitude Modulation (PAM) Receiver.* 27 points.

For a discrete-time baseband PAM receiver when the channel is modeled as additive white Gaussian noise, the first two blocks are:



$r[m]$ is the discrete-time received signal.

$g[m]$ is the pulse shape used in the transmitter.

N_g is the number of symbol periods in the pulse shape.

f_{sym} is the symbol rate.

Note that $v[m] = h[m] * r[m]$ and $y[n] = v[L\, n]$

$h_{causal}[m] = k\, g^*[L N_g - m]$

(a) Give a formula for the causal impulse response $h[m]$ that maximizes a measure of signal-to-noise ratio at $y[n]$. *6 points.*

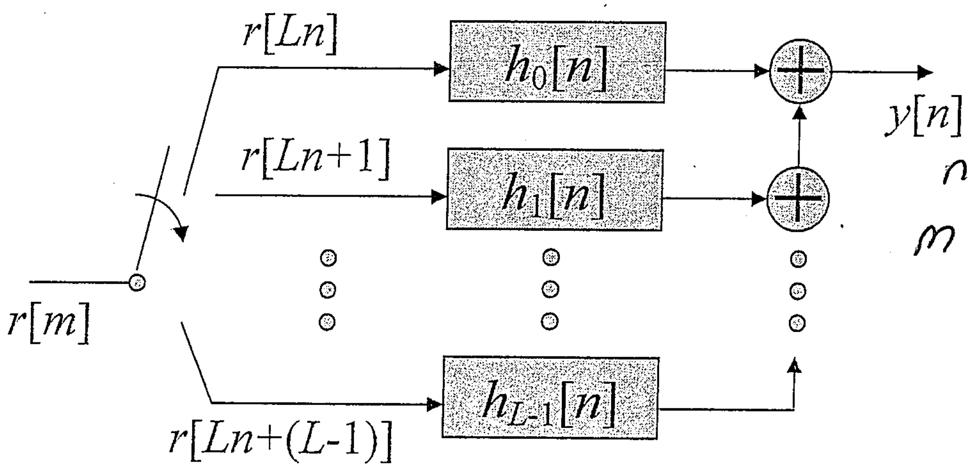
Matched filter $h[m] = k\, g^*[L - m]$ where $k \in \mathbb{R}$. Delay to make causal.

(b) How many multiplication-accumulation operations per second are needed for the two blocks above? *6 points.*

For an FIR filter of $L N_g$ coefficients, $L N_g$ multiplication-accumulation (MAC) operations are needed for each output sample. MACs/s $(L N_g)(L f_{sym})$

sampling rate

The above cascade can be efficiently implemented as a polyphase filter bank as follows:



n is symbol index.

m is sample index.

(c) Give a formula of $h_0[n]$ in terms of $h[n]$. Hint: Compare $y[N_g]$ for the direct form with $y[N_g]$ of the polyphase filter bank. *6 points.*

$\boxed{h_0[n] = h[Ln] \text{ for } n = 0, 1, \ldots, N_g - 1}$

From midterm #2 review slide 10:

$y[1] = v[L] = h[0]\,r[L] + h[1]\,r[L-1] + \cdots + h[L-1]\,r[1] + h[L]\,r[0]$

(d) How many multiplication-accumulation operations per second are needed to implement the above polyphase filter bank? *9 points.*

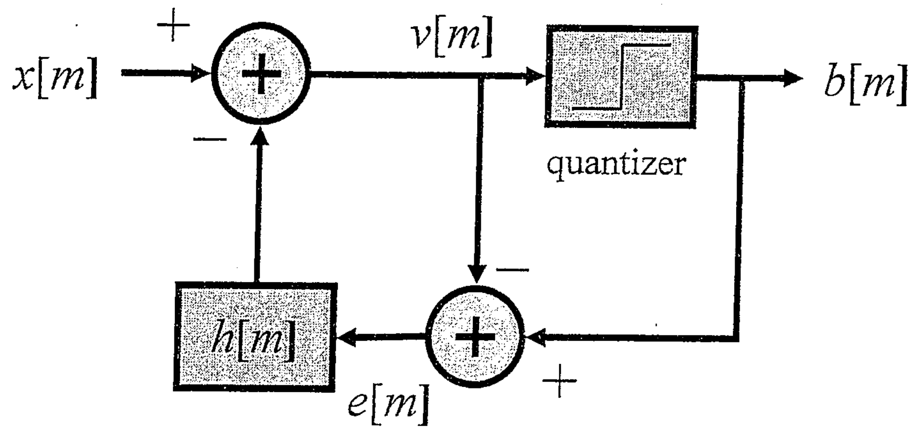
L filters with N_g coefficients each. Executes at symbol rate.

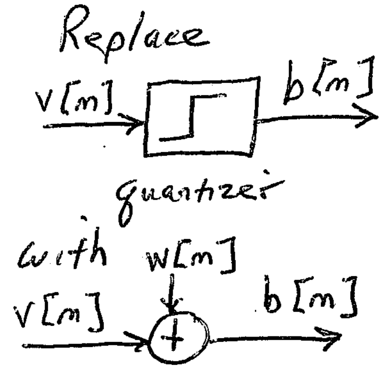
MACs/s : $L N_g f_{sym}$

Computational savings over direct implementation by a factor of L.

Problem 2.4. *Noise Shaping.* 19 points.

Here is a block diagram of a noise-shaping feedback coder used in data conversion.





$h[m]$ is the impulse response of a linear time-invariant (LTI) finite impulse response (FIR) filter.

This problem asks you to analyze the noise shaping.

(a) Replace the quantizer with an additive noise source $w[m]$, i.e. $b[m] = v[m] + w[m]$, and derive the transfer function in the frequency domain from the noise source $w[m]$ to the output $b[m]$. Assume that the input $x[m]$ is zero. *10 points.* Set $x[m] = 0$.

$b[m] = v[m] + w[m]$

$v[m] = \cancel{x[m]}^{0} * h[m] * e[m]$

$e[m] = w[m]$

$B(\omega) = V(\omega) - W(\omega)$

$V(\omega) = -H(\omega)W(\omega)$

$\Downarrow$

$B(\omega) = -H(\omega)W(\omega) + W(\omega)$

$B(\omega) = (1 - H(\omega))W(\omega)$

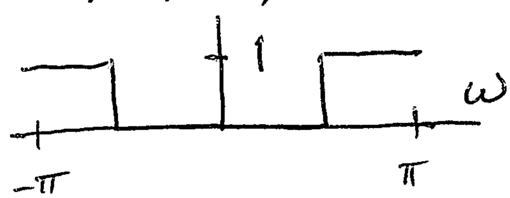
$\frac{B(\omega)}{W(\omega)} = 1 - H(\omega) \Leftarrow$

(b) If the frequency selectivity of $h[m]$ were lowpass, what is the frequency selectivity of the noise transfer function? Sketch example frequency responses for both to help justify your answers. *9 points.*

$H(\omega)$

Lowpass

$1 - H(\omega)$



Highpass

$\frac{B(\omega)}{W(\omega)} = 1 - H(\omega)$ is highpass.