The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Solution 3.0***

Date: April 24, 2023 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	**Point Value**	**Your score**	**Topic**
1	22		Baseband PAM System
2	30		QAM Communication Performance
3	24		Symbol Timing Recovery
4	24		Potpourri
Total	100		

Problem 2.1. *Baseband PAM System. 22 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

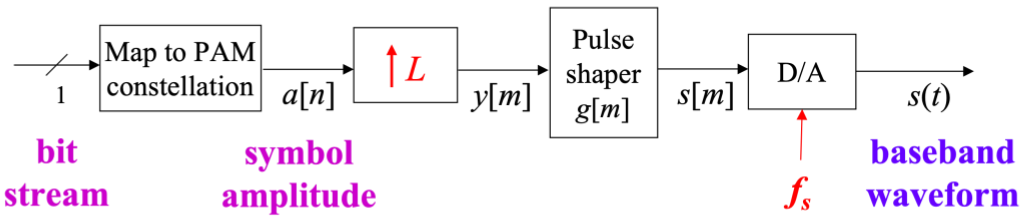
The system parameters are described on the right:

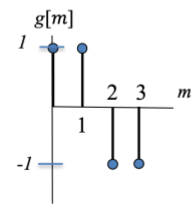
- $J = 1$ bit/symbol
- $L = 4$ samples per symbol period
- Pulse shape $g[m]$ is a **Manchester non-return-to-zero** pulse of 4 samples in duration plotted below on the right.
- A bit of value 0 is mapped to symbol amplitude *-1*, and a bit of value 1 is mapped to symbol amplitude *1*.

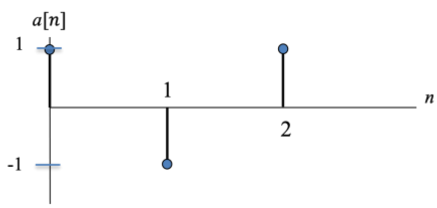
PAM System Parameters

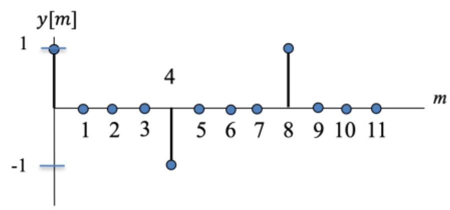
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

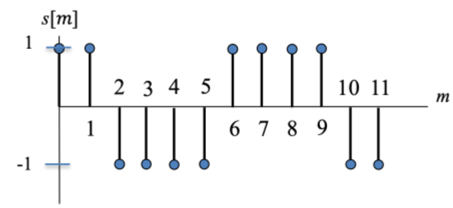
(a) For the **BPSK transmitter** below, the input bit stream is 101. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points*.





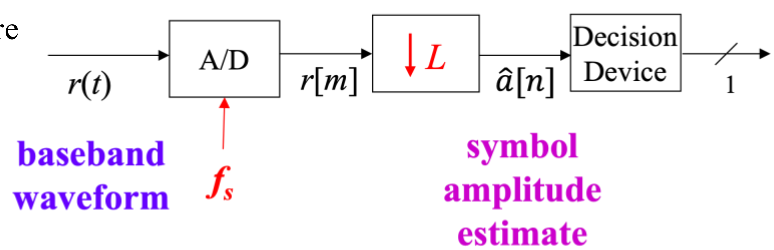


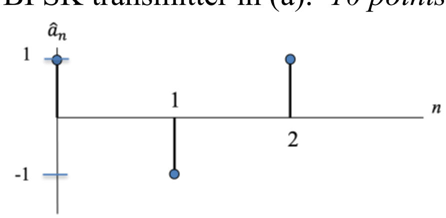




See the next page for a Matlab solution.

(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *10 points*.





Received bit stream: {1, 0, 1}

See the next page for a Matlab solution.

Problem 2.1. *Baseband PAM System. 22 points.* **Using MATLAB.**

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

The system parameters are described on the right:

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PAM System Parameters

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$2d$	constellation spacing
f_s	sampling rate
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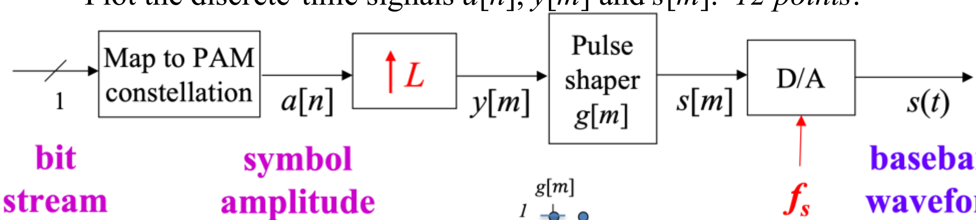
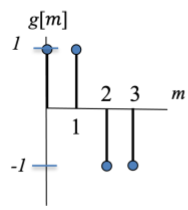
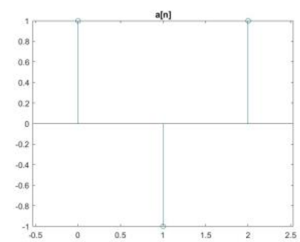
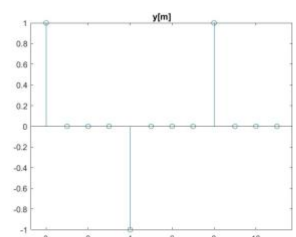
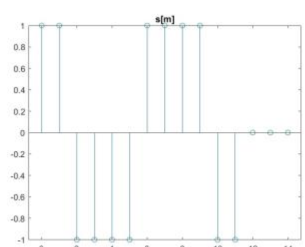




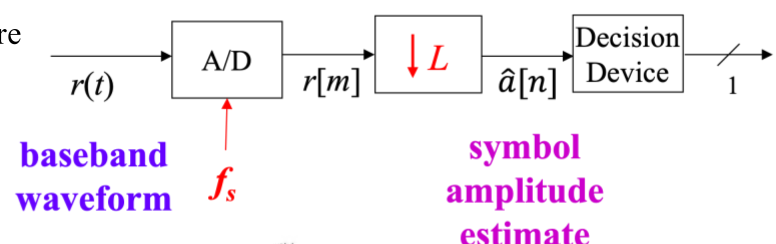

```
J=1; L=4;
g = [1, 1, -1, -1];

a = [1, -1, 1];
n = 0:2;
figure; stem(n,a); title('a[n]');

y = upsample(a, L);
m = 0:length(y)-1;
figure; stem(m, y); title('y[m]');

s = conv(y,g);
m = 0:length(s)-1;
figure; stem(m, s); title('s[m]');
```



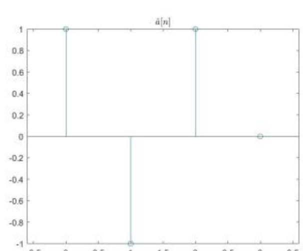




(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *10 points.*



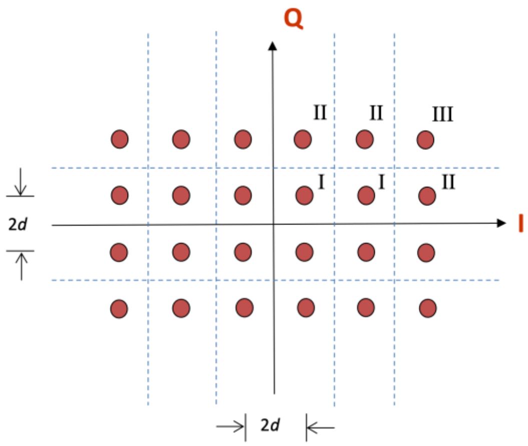

```
a_hat = downsample(s,L);
n = 0:length(a_hat)-1;
figure; stem(n,a_hat);
title('$\hat{a}[n]$', 'Interpreter', 'latex');
received_bits = (a_hat > 0);
disp(received_bits);
received_bits = received_bits(1:end-1);
sprintf('%d,', received_bits);
```

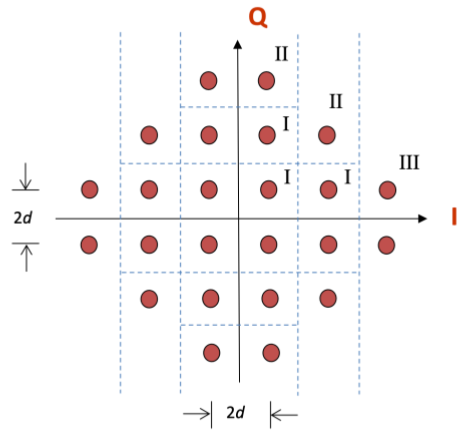


Received bits: {1, 0, 1, 0}

Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 24-QAM constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$34\, d^2$	$\mathbf{26\, d^2}$
(b) Average transmit power	$\frac{50}{3} d^2$	$\mathbf{\frac{46}{3} d^2}$
(c) Peak-to-average power ratio	$\frac{34\, d^2}{\frac{50}{3} d^2} = \frac{51}{25} = 2.04$	$\mathbf{\frac{26\, d^2}{\frac{46}{3} d^2} = \frac{78}{46} = 1.70}$
(d) For the left constellation, IQ axes are also boundaries of decision regions. For the right constellation, draw type I, II and/or III decision regions.		
(e) Number of type I QAM regions	8	**12**
(f) Number of type II QAM regions	12	**8**
(g) Number of type III QAM regions	4	**4**
(h) Probability of symbol error in presence of additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{19}{6} Q\left(\frac{d}{\sigma}\right) - \frac{5}{2} Q^2\left(\frac{d}{\sigma}\right)$	$\mathbf{P_e = \frac{20}{6} Q\left(\frac{d}{\sigma}\right) - \frac{17}{6} Q^2\left(\frac{d}{\sigma}\right)}$
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{\left(\frac{50}{3}\right) d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{3}{50}\text{SNR}}$	$\mathbf{SNR = \frac{\left(\frac{46}{3}\right) d^2}{\sigma^2}}$ $\mathbf{\frac{d}{\sigma} = \sqrt{\frac{3}{46}SNR}}$

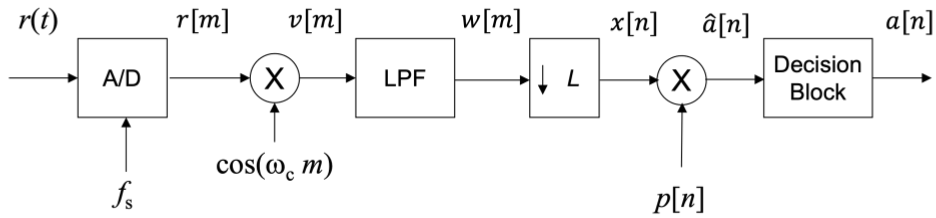
(j) Give a Gray coding for right constellation or show that one does not exist. *3 points*.

Gray coding does not exist. We need 5 bits to encode 24-QAM. Use the first two bits to encode a quadrant, which leaves three bits to encode the six constellation points in the quadrant. For lower right quadrant, which is shown on the right, constellation region #5 has four nearest neighbors: regions 2, 3, 4, and 6. There are only three bits (degrees of freedom) to work with.

1 2 3
4 5
6

Problem 2.3. *Symbol Timing Recovery. 24 points.*

For the upconverted pulse amplitude modulation (PAM) receiver,



you'll develop an adaptive algorithm to generate the symbol timing offset τ to be used in the symbol timing offset correction signal

$$p[n] = \cos(2\,\pi\, f_{sym}\,\tau)$$

which does not depend on n. Lowpass filter (LPF) is the matched filter

PAM System Parameters	
$a[n]$	Tx symbol amplitude
$\hat{a}[n]$	Rx symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
$p[n]$	symbol timing offset correction signal
ω_c	discrete-time carrier freq.

(a) What training sequence for the symbol amplitude values $a[n]$ would you use? Why? *3 points.*

PN sequences have a non-zero magnitude for all frequencies, which allows the receiver to estimate the frequency distortion over all frequencies experienced by the discrete-time baseband transmitted signal $s[m]$ until it arrives at $r[m]$.

(b) Derive the steepest descent algorithm to update $\tau[n]$ using the objective function (*15 points*)

$$J_{DD}[n] = e^2[n]$$

where $e[n]$ is the decision-directed (DD) error signal

$$e[n] = \hat{a}[n] - a[n]$$

and

$$\hat{a}[n] = p[n]\, x[n]$$

Steepest descent algorithm: $\boldsymbol{\tau[n+1] = \tau[n] - \mu \left. \frac{d}{d\tau} J_{DD}[n] \right|_{\tau=\tau[n]}}$

During training, $\boldsymbol{p[n] = \cos(2\,\pi\, f_{sym}\,\tau)}$ **is independent of** $\boldsymbol{a_n}$ **whereas** $\boldsymbol{\hat{a}[n] = p[n]\, x[n]}$**.**

$$\frac{d}{d\tau} J_{DD}[n] = 2\, e[n] \frac{d}{d\tau} e[n] = 2\, e[n] \frac{d}{d\tau} \hat{a}[n] = 2\, e[n]\, x[n] \frac{d}{d\tau} p[n]$$

$$\frac{d}{d\tau} p[n] = -\sin(2\,\pi\, f_{sym}\,\tau)\,(2\,\pi\, f_{sym}) = -2\,\pi\, f_{sym} \sin(2\,\pi\, f_{sym}\,\tau)$$

(c) What is initial value of $\tau[n]$ would you use? *3 points.*

If we use *zero* as the initial value of $\tau[n]$, the initial value of $\left. \frac{d}{d\tau} J_{DD}[n] \right|_{\tau=\tau[n]}$ is zero and steepest descent algorithm does not update. One could choose $\tau[n] = 1/(4\, f_{sym})$ to put $\frac{d}{d\tau} p[n]$ at its maximum value.

(d) What value of the step size (learning rate) μ would you use? *3 points.*

A small positive value to help ensure convergence. A value of zero would cause the steepest descent algorithm not to update. A negative value would cause the steepest descent algorithm to maximize instead of minimizing the objective function.

Problem 2.4. *Potpourri*. *24 points*.

(a) One version of the MP3 (MPEG layer III) audio compression standard divides an audio signal into 32 equally spaced frequency bands. The direct approach uses a set of 32 bandpass FIR filters in parallel and each FIR filter output is downsampled by 32. Each FIR filter has 512 coefficients.

i. How many multiplication operations are needed for the direct approach? *4 points*.

512 multiplications per filter x 32 filters = 16384 multiplications

ii. How many multiplication operations are needed if each of the 32 cascades of a bandpass FIR filter and downsampling by 32 are implemented as a polyphase filter bank? *4 points*.

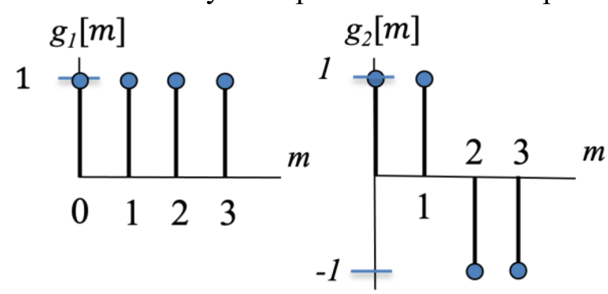
A polyphase filter bank converts the direct approach of FIR filtering using 512 filter coefficients followed by downsampling by 32 into a bank of 32 polyphase FIR filters with 512/32 = 16 coefficients. The polyphase filter bank only computes the FIR outputs that will be kept by the downsampler, which requires 16384/32 = 512 multiplications.

iii. What is the amount of savings for the polyphase approach? *4 points.*

Savings of a factor of 32 for a savings of 16384 – 16384/32 = 15872 multiplications.

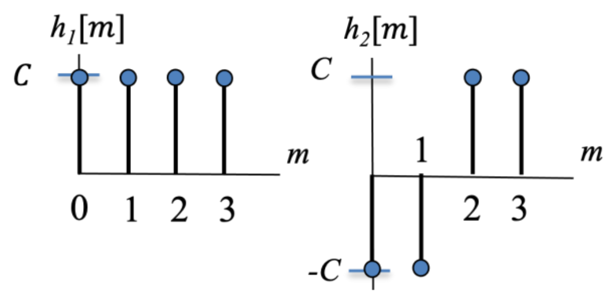
(b) You're evaluating two candidate pulse shapes for a baseband pulse amplitude modulation (PAM) system – rectangular pulse $g_1[m]$ and Manchester non-return-to-zero pulse $g_2[m]$. The system is like problem 2.1 but with a matched filter after the A/D converter. Symbol period is $L = 4$ samples.

i. The pulse shape is the impulse response of the pulse shaping filter. How many multiplications does each pulse shaping filter for $g_1[m]$ and $g_2[m]$ require? *4 points*.

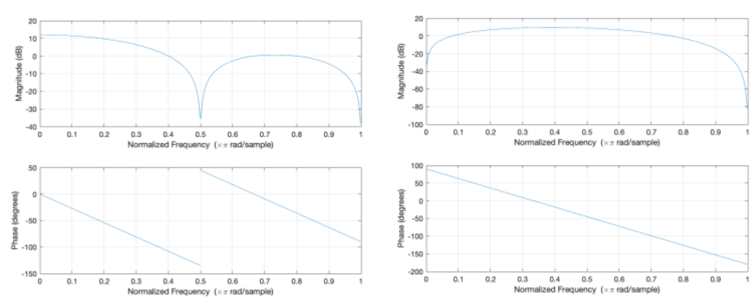


Zero in both cases. In the input-output equation, a coefficient of 1 becomes an addition, and a coefficient of -1 becomes a subtraction.

ii. For a pulse shape $g[m]$, the optimal choice of the impulse response of the matched filter is $h[m] = C\, g^*[L - m]$ where C is a constant and * is complex conjugation. Plot the optimal matched filter impulse responses $h_1[m]$ and $h_2[m]$ for pulse shapes $g_1[m]$ and $g_2[m]$, respectively. *4 points*. **On right.**



iii. Here are the magnitude and phase plots in dB and degrees for $g_1[m]$ and $g_2[m]$ by using the freqz command in Matlab. Which pulse shape would you choose for your system and why? *4 points*.



Rectangular Pulse

Manchester Pulse

Both have linear phase. Rectangular pulse is lowpass with bandwidth 0.5π. The Manchester pulse is bandpass with bandwidth 0.9π. Choose rectangular pulse because bandwidth is scarce in communication systems.