The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: April 24, 2023 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except your instructor, Prof. Evans, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	**Point Value**	**Your score**	**Topic**
1	22		Baseband PAM System
2	30		QAM Communication Performance
3	24		Symbol Timing Recovery
4	24		Potpourri
Total	100		

Problem 2.1. *Baseband PAM System. 22 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

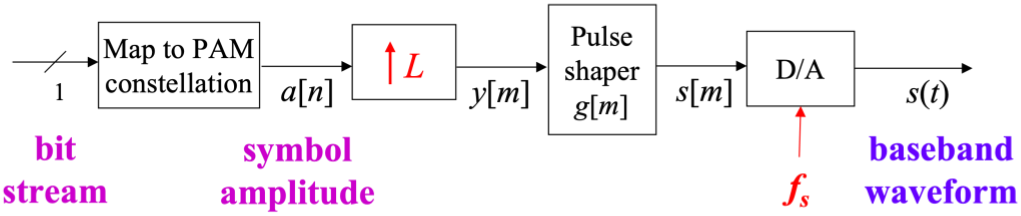
The system parameters are described on the right:

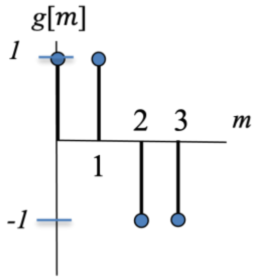
- $J = 1$ bit/symbol
- $L = 4$ samples per symbol period
- Pulse shape $g[m]$ is a **Manchester non-return-to-zero** pulse of 4 samples in duration plotted below on the right.
- A bit of value 0 is mapped to symbol amplitude *-1*, and a bit of value 1 is mapped to symbol amplitude *1*.

PAM System Parameters

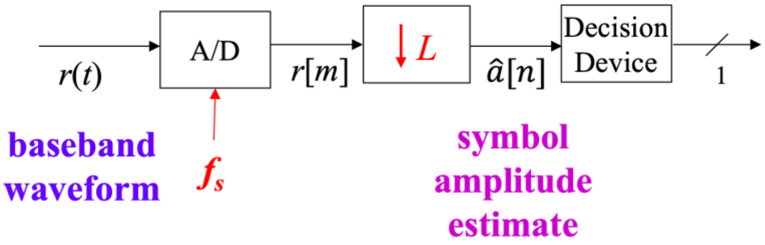
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

(a) For the **BPSK transmitter** below, the input bit stream is 101. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points*.



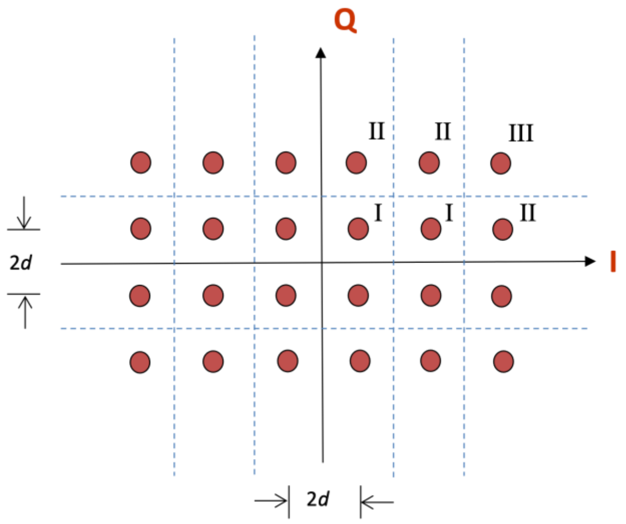


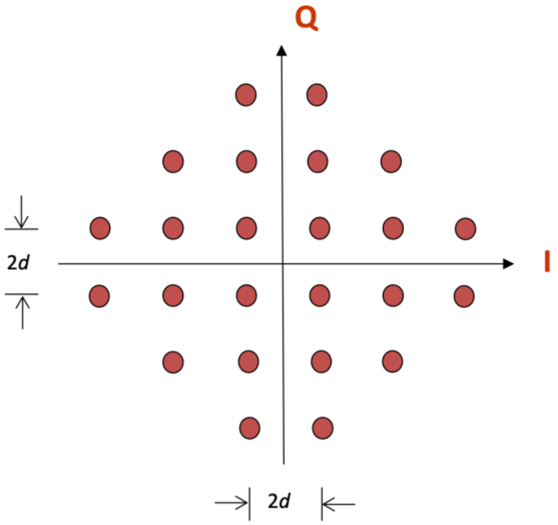
(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *10 points*.



Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 24-QAM constellations below. Constellation spacing is 2*d*.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

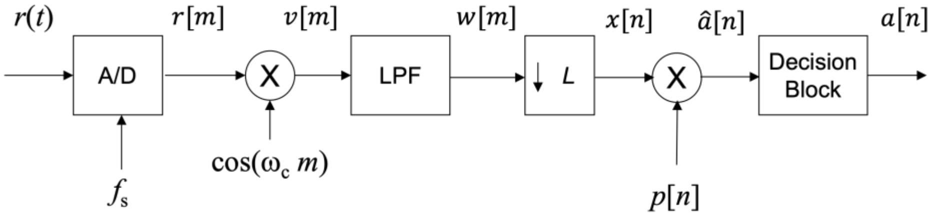
Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$34\,d^2$	
(b) Average transmit power	$\frac{50}{3}d^2$	
(c) Peak-to-average power ratio	$\frac{34\,d^2}{\frac{50}{3}d^2} = \frac{51}{25} = 2.04$	
(d) For the left constellation, IQ axes are also boundaries of decision regions. For the right constellation, draw type I, II and/or III decision regions.		
(e) Number of type I QAM regions	8	
(f) Number of type II QAM regions	12	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error in presence of additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{19}{6}Q\left(\frac{d}{\sigma}\right) - \frac{5}{2}Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the *Q* function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{\left(\frac{50}{3}\right)d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{3}{50}\text{SNR}}$	

(j) Give a Gray coding for right constellation or show that one does not exist. *3 points*.

Problem 2.3. *Symbol Timing Recovery. 24 points.*

For the upconverted pulse amplitude modulation (PAM) receiver,



you'll develop an adaptive algorithm to generate the symbol timing offset τ to be used in the symbol timing offset correction signal

$$p[n] = \cos\left(2\, \pi\, f_{sym}\, \tau\, n\right)$$

The lowpass filter (LPF) is the matched filter.

PAM System Parameters

$a[n]$	Tx symbol amplitude
$\hat{a}[n]$	Rx symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
$p[n]$	symbol timing offset correction signal
ω_c	discrete-time carrier freq.

(a) What training sequence for the symbol amplitude values $a[n]$ would you use? Why? *3 points.*

(b) Derive the steepest descent algorithm to update $\tau[n]$ using the objective function

$$J_{DD}[n] = e^2[n]$$

where $e[n]$ is the decision-directed (DD) error signal

$$e[n] = \hat{a}[n] - a[n]$$

and

$$\hat{a}[n] = p[n]\, x[n]$$

15 points.

(c) What is initial value of $\tau[n]$ would you use? *3 points.*

(d) What value of the step size (learning rate) μ would you use? *3 points.*

Problem 2.4. *Potpourri*. *24 points*.

(a) One version of the MP3 (MPEG layer III) audio compression standard divides an audio signal into 32 equally spaced frequency bands. The direct approach uses a set of 32 bandpass FIR filters in parallel and each FIR filter output is downsampled by 32. Each FIR filter has 512 coefficients.

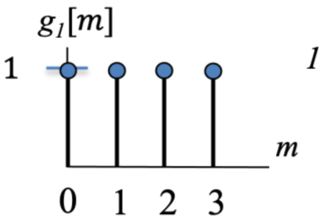
i. How many multiplication operations are needed for the direct approach? *4 points*.

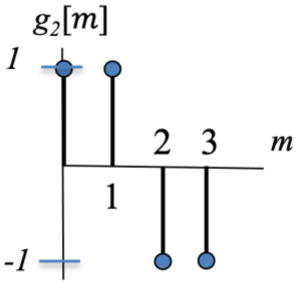
ii. How many multiplication operations are needed if each of the 32 cascades of a bandpass FIR filter and downsampling by 32 are implemented as a polyphase filter bank? *4 points*.

iii. What is the amount of savings for the polyphase approach? *4 points.*

(b) You're evaluating two candidate pulse shapes for a baseband pulse amplitude modulation (PAM) system – rectangular pulse $g_1[m]$ and Manchester non-return-to-zero pulse $g_2[m]$. The system is like problem 2.1 but with a matched filter after the A/D converter. Symbol period is $L = 4$ samples.

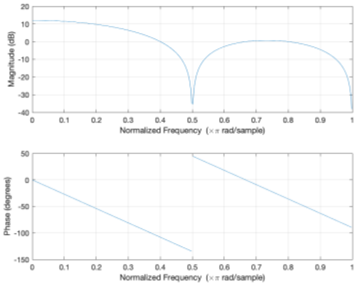
i. The pulse shape is the impulse response of the pulse shaping filter. How many multiplications does each pulse shaping filter for $g_1[m]$ and $g_2[m]$ require? *4 points*.



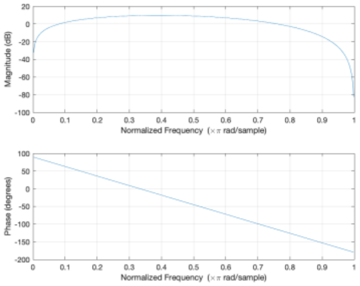


ii. For a pulse shape $g[m]$, the optimal choice of the impulse response of the matched filter is $h[m] = C\,g^*[L-m]$ where C is a constant and * is complex conjugation. Plot the optimal matched filter impulse responses $h_1[m]$ and $h_2[m]$ for pulse shapes $g_1[m]$ and $g_2[m]$, respectively. *4 points*.

iii. Here are the magnitude and phase plots in dB and degrees for $g_1[m]$ and $g_2[m]$ by using the freqz command in Matlab. Which pulse shape would you choose for your system and why? *4 points*.



Rectangular Pulse



Manchester Pulse