The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: April 29, 2024 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	25		Baseband PAM System
2	30		QAM Communication Performance
3	27		Channel Estimation
4	18		Communication System Tradeoffs
Total	100		

Problem 2.1. *Baseband PAM System. 25 points.*

Consider a binary phase shift keying (BPSK) system, a.k.a. a two-level pulse amplitude modulation (2-PAM) system.

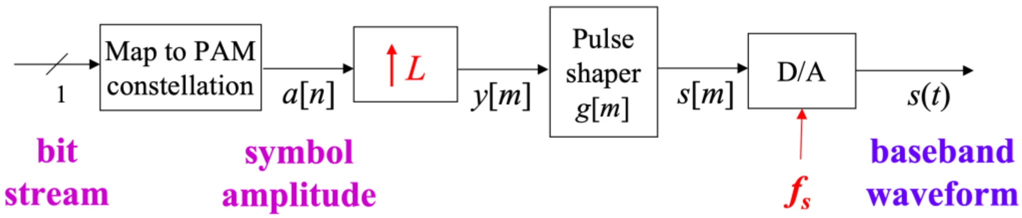
The system parameters are described on the right:

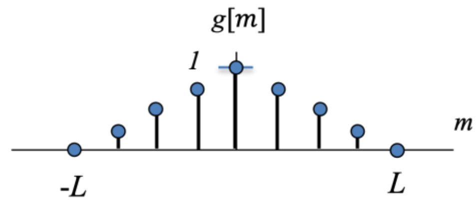
- $J = 1$ bit/symbol
- $L = 4$ samples per symbol period
- Pulse shape $g[m]$ is a triangular pulse of 8 samples in duration from $m = -4$ to $m = 3$ inclusive of both and plotted below.
- A bit of value 0 is mapped to symbol amplitude -1, and a bit of value 1 is mapped to symbol amplitude 1.

PAM System Parameters

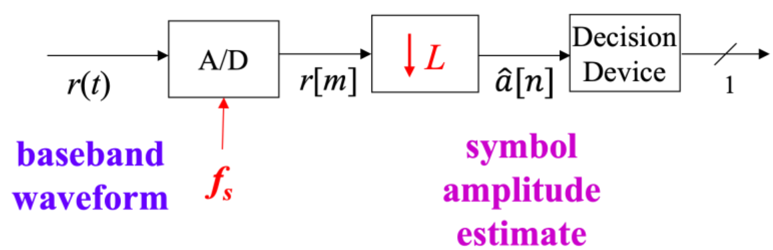
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

(a) For the **BPSK transmitter** below, the input bit stream is 10. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points.*





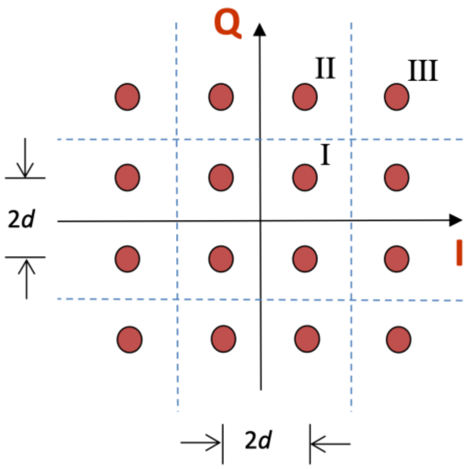
(b) For the **BPSK receiver** to the right, assume there is no channel distortion or additive noise and assume that $r[m] = s[m]$. **There is no matched filter.** Plot the discrete-time signal $\hat{a}[n]$ and give the received bit stream based on the BPSK transmitter in (a). *7 points.*

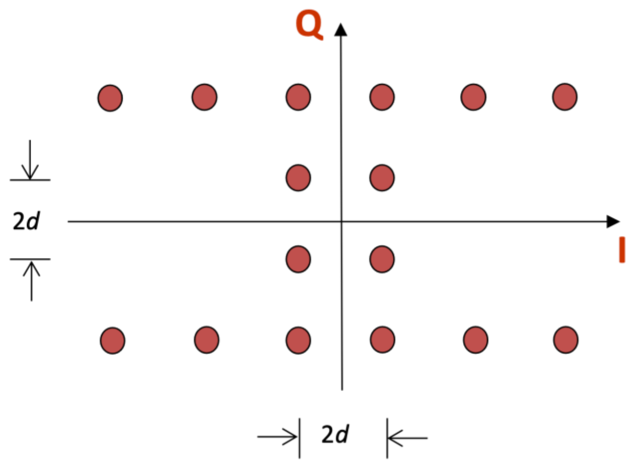


(c) What is the formula for the optimal matched filter $h[m]$? Plot the optimal matched filter impulse response $h[m]$ given the pulse shape $g[m]$ in part (a). *6 points.*

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

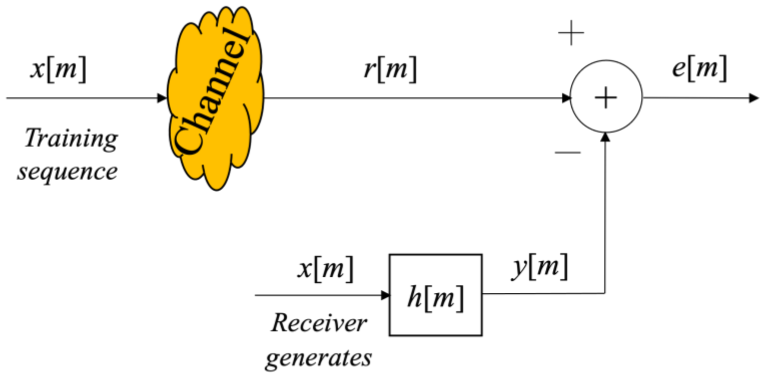
Each part below is worth 3 points. **Please fully justify your answers.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$18d^2$	
(b) Average transmit power	$10d^2$	
(c) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(d) Number of type I regions	4	
(e) Number of type II regions	8	
(f) Number of type III regions	4	
(g) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right)-\frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	
(h) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{10d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{10}}$	

(i) For the right constellation, will using your type I, II, and III rectangular decision regions lead to Gray coding for symbols? Either give a Gray coding for the right constellation, or show that it is not possible. *3 points*.

Problem 2.3. *Channel Estimation*. *27 points*.

In a communication system, we model impairments between the baseband transmitter output $x[m]$ and baseband receiver input $r[m]$ as the baseband equivalent channel shown as "Channel" below:



The baseband equivalent channel is a nonlinear, time-varying system.

A common step in many communication systems is to model the baseband equivalent channel as a linear-time invariant (LTI) system and estimate its impulse response $h[m]$.

We will adapt the baseband equivalent channel impulse response $h[m]$ estimate during training.

We will model the LTI system $h[m]$ as an FIR filter with M coefficients: $\vec{h} = [\ h_0 \ \ h_1 \ldots h_{M-1}\ ]$

(a) What training sequence $x[m]$ would you use? Why? *3 points*.

(b) Give an objective function $J[m]$. Why did you choose it? *6 points*.

(c) What is the initial value of the adaptive FIR coefficients you would use? Why? *3 points*.

(d) Derive an update equation for the adaptive FIR coefficients vector at iteration m

$$\vec{h}[m] = [\ h_0[m] \ \ h_1[m] \ldots h_{M-1}[m]\ ]$$

Compute all derivatives. Simplify the result. *12 points*

(e) What range of values would you recommend for the step size μ? Why? *3 points*.

Problem 2.4. *Communication System Tradeoffs*. *18 points*.

Two-way communication systems have a data channel and a control channel in each direction.

The data channel conveys high speed data such as streaming audio or video whereas the control channel conveys configuration and feedback information. For example, a phone will send the received signal strength to the basestation on a control channel.

The control channel provides lower bit rates at lower symbol error rates and higher delays relative to the data channel.

(a) For an increase in each parameter below, indicate whether the pulse amplitude modulation (PAM) transmitter performance measure in each column increases, decreases, or stays the same. As you consider increasing one of the parameters in the table, assume the other two parameters in the table are not changing in value. Assume the bits/symbol, J, and the number of samples in a symbol period, L, do not change.

(b) Justify your answers.

Increasing Parameter	Transmitter Total Power Consumption	Transmitter Run-Time Complexity	Transmitter Bit Rate
B bits in D/A converter			
$2d$ constellation spacing in Volts			
f_{sym} symbol rate in Hz			

PAM System Parameters

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