The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 Solutions ***Version 3.0***

Date: April 28, 2025 Course: EE 445S Evans

Name: **Severance**
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

	Problem	Point Value	Your score	Topic
Dylan G.	1	24		Baseband PAM System
Helly R.	2	30		QAM Communication Performance
Mark S.	3	26		Improving Communication Performance
Irving B.	4	20		Communication System Tradeoffs
	Total	100		

Note: Character names are in the Optima font which is one of the fonts used in Severance.

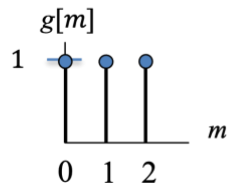
***Prologue*: Lectures 7 Pulse Shaping, 13 Digital PAM and 14 Matched Filtering; JSK Ch. 8 and 9; Labs 3 and 5; HW 2.1, 4.2, 4.3, 5.2 & 6.2; Fall 2021 Midterm Prob. 2.1**

Problem 2.1. *Baseband PAM System. 24 points.*

Consider a two-level pulse amplitude modulation (2-PAM) system, a.k.a. binary phase shift keying.

The system parameters are described on the right:

- $J = 1$ bits/symbol; i.e., $M = 2$ levels/symbol
- $L = 3$ samples per symbol period
- Pulse shape $g[m]$ is a rectangular pulse of 3 samples in duration as shown on the right.
- Constellation map: input '0' maps to 1 Volt and input '1' maps to -1 Volt.



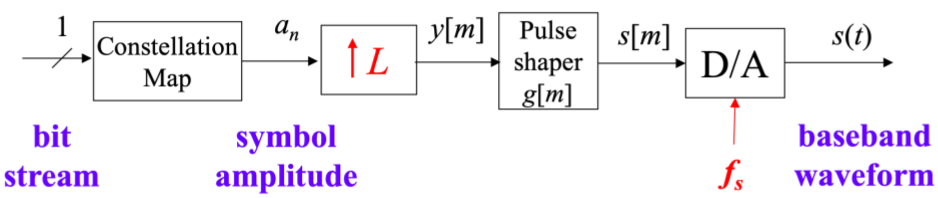
PAM System Parameters

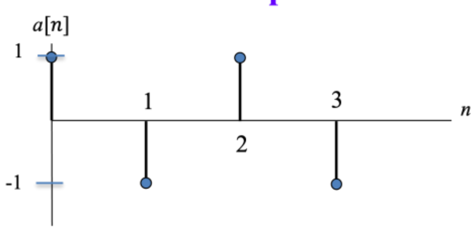
$a[n]$	symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse response
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index

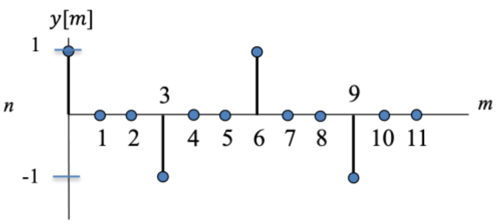
Assume all filters are linear and time-invariant (LTI).

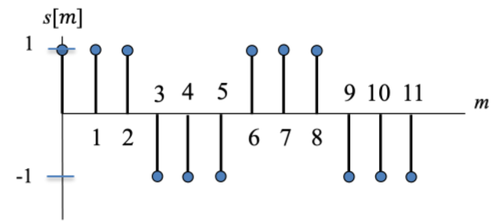
(a) For the **2-PAM transmitter** below, input bit stream is 0101. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points.*

```
a = [1 -1 1 -1]; L = 3;
y = zeros(1, L*length(a));
y(1:L:end) = a;
g = ones(1, L); %%% [1 1 1]
s = filter(g, 1, y);
m = 0 : length(s)-1;
figure; stem(m, s, 'black');
ylim( [-1.2, 1.2] );
```



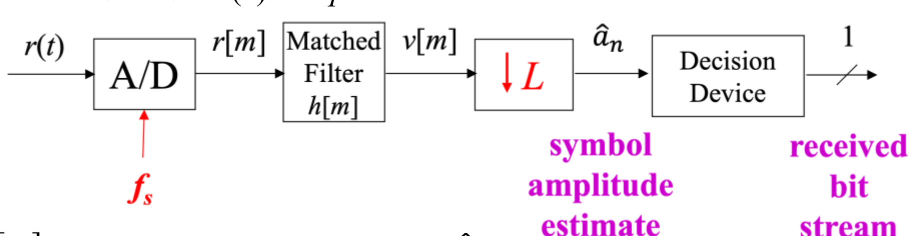






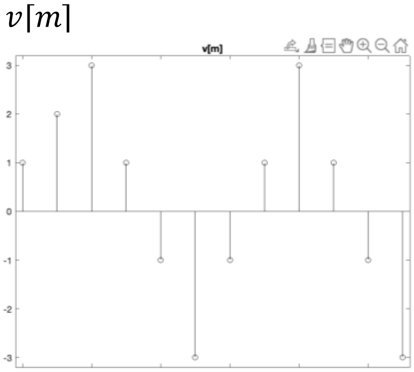
The upsampler copies each input sample to the output and appends $L - 1 = 2$ zeros. The upsampler output has L times the number of samples ($L = 3$). Matlab code for $s[m]$ is above.

(b) For the **2-PAM receiver** below, assume there is no channel distortion or additive noise, and assume $r[m] = s[m]$ and $h[m] = g[m]$. The Decision Device compares the input value against 0. Plot the discrete-time signals $v[m]$, $\hat{a}[n]$ and give the received bit stream based on the 2-PAM transmitter in (a). *12 points.*

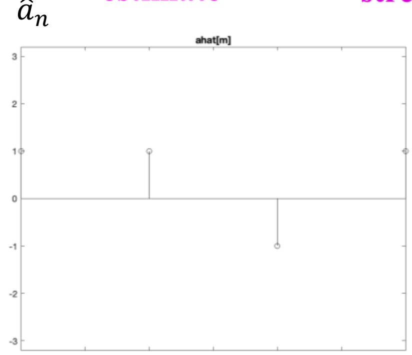



```
r = s; h = g;
v = filter(h, 1, r);
m = 0 : length(v)-1;
figure;
stem(m, v, 'black');
title( 'v[m]' );
xlim( [-0.2, length(v)-0.8] );
ylim( [-L-0.2, L+0.2] );
%%% v = v(L:end);
ahat = v(1:L:end);
n = 0 : length(ahat)-1;
figure;
stem(n, ahat, 'black');
title( 'ahat[m]' );
ylim( [-L-0.2, L+0.2] );
bits = (ahat < 0) %%% 0 0 1 0
```

$v[m]$



$\hat{a}_n$

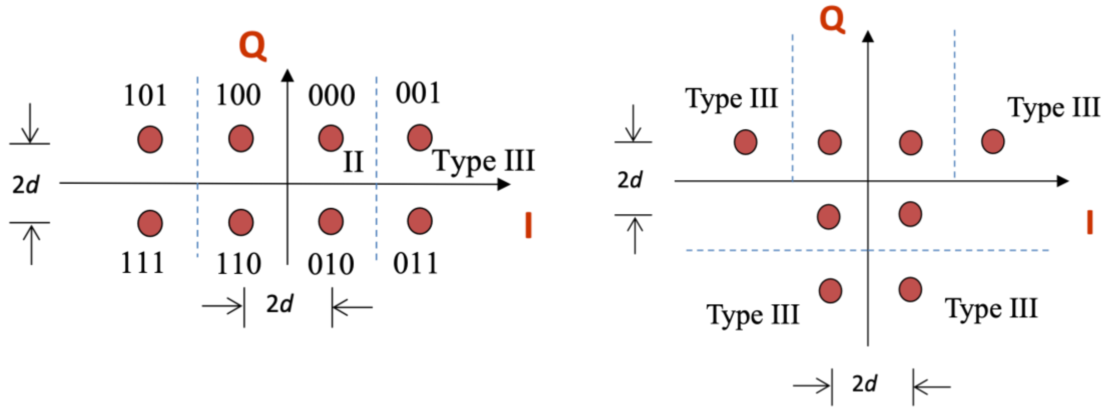


Although not asked, bit errors can be fixed by accounting for delay through matched filter. Discard L-1 samples at output of the matched filter.

***Prologue*: Lectures 13-16; JSK Ch. 16; Labs 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3; Handout P PAM vs. QAM; Midterm 2.2: F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20, F20, Sp21, F21 & Sp22**

Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 8-QAM constellations below. Constellation spacing is $2d$.



Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10d^2$	$\mathbf{10d^2}$
(b) Average transmit power	$6d^2$	$\mathbf{6d^2}$
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	$\boldsymbol{\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67}$
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	**0**
(f) Number of type II QAM regions	4	**4**
(g) Number of type III QAM regions	4	**4**
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	$\boldsymbol{P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)}$
(i) Express the argument of the Q function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	$\boldsymbol{\text{SNR} = \frac{6d^2}{\sigma^2}}$ $\boldsymbol{\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}}$

(j) Give a Gray coding for the right constellation or show that one does not exist. *3 points*.

Gray coding means that the bit pattern for each symbol of bits among any pair of neighboring constellation regions differs by one bit; this minimizes the number of bits errors when there is a symbol error. Gray coding is possible for the constellation on the upper left. Based on the constellation regions on the upper right, the constellation region for the point $\boldsymbol{d - jd}$ has four nearest neighbors, and there are only three bits in an 8-QAM constellation. Same is true for the point at $\boldsymbol{-d - jd}$. <u>So, Gray coding is not possible for the constellation on the right</u>.

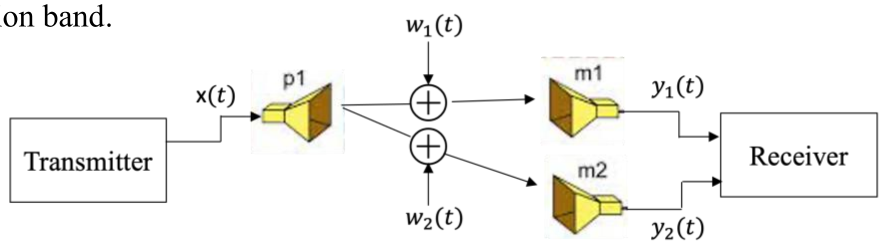
Prologue*: Primarily Lecture 12 Channel Impairments (additive noise) but also Lectures 13 Digital PAM and 14 Matched Filtering; JSK Sections 2.1 and 9.1; JSK Ch. 10; Labs 4, 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3; Midterm 2.4 in Sp 22

Problem 2.3. *Improving Communication Performance. 26 points.*

One way to improve communication performance is to use two antennas to process a transmitted signal sent by a single antenna.

For this problem, assume the transmitter is sending a pulse amplitude modulation (PAM) signal over the air in a radio frequency (RF) transmission band.

(a) First, we'll assume the only impairment in the system is additive thermal noise as shown on the right:



i. The model for additive thermal noise is in the communication channel, but where does the additive thermal noise physically occur in the system? *3 points.*

The additive thermal noise physically occurs in the analog/RF front end of the receiver. It is due to the random motion of electrons due to temperature.

ii. What is a good statistical model for the additive thermal noise? Why? Explain what the statistical model parameters mean. *4 points.*

The additive thermal noise consists of the aggregate contribution of random motion of electrons due to temperature. Each electron moves according to a statistical distribution. Assuming that the electron motions are statistically independent, the additive thermal noise is approximated well as a Gaussian distribution due to the Central Limit Theorem. The mean is zero, and the variance σ^2 is the noise power.

iii. How would you recommend combining the receive antenna outputs $y_1(t)$ and $y_2(t)$ to produce a single PAM signal that would have a lower probability of symbol error than either $y_1(t)$ or $y_2(t)$ by itself? Assume the receiver knows the values of the statistical model parameters for $w_1(t)$ and $w_2(t)$. *6 points.*

Answer #1: Antenna selection. **Select the antenna with the higher SNR, i.e., lower variance in this case since we're assuming that each antenna receives the same transmit power. This only partially answers the question, but it is commonly used.**

Answer #2: Equal gain combining **by averaging $y_1(t)$ and $y_2(t)$. This approach assumes the variances σ_1^2 and σ_2^2 have the same value. It's also a good approach when we do not know the values of the variances σ_1^2 and σ_2^2 or the SNR at each antenna.**

***Answer #3: Weighted combination*. The higher the variance, the less reliable the received signal is. So, we could give a higher weighting to the more reliable signal, e.g.**

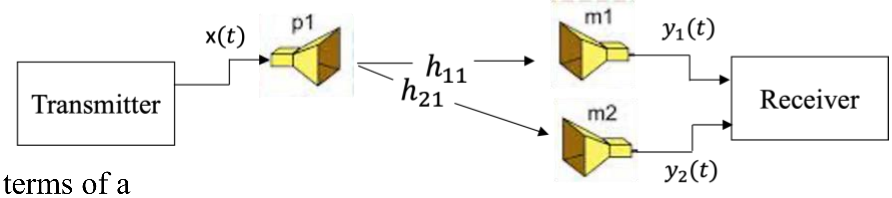
$$y(t) = \frac{\sigma_2}{\sigma_1 + \sigma_2} y_1(t) + \frac{\sigma_1}{\sigma_1 + \sigma_2} y_2(t)$$

This comes from normalize the weighting by the standard deviations:

$$y(t) = \frac{1/\sigma_1}{1/\sigma_1 + 1/\sigma_2} y_1(t) + \frac{1/\sigma_2}{1/\sigma_1 + 1/\sigma_2} y_2(t)$$

***Prologue*: Primarily Lectures 12 Channel Impairments (LTI modeling) and 16 QAM Receiver (Ideal Channel) and Handout on Designing Averaging Filters; Labs 4, 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3; Midterm 2.4 in Sp 22**

(b) Second, we'll assume the only impairment in the system is a gain in each path from the transmitter to the receiver as shown on the right.



i. What is each gain modeling in terms of a physical phenomenon? *3 points.*

The attenuation due to distance that the electromagnetic wave propagates.

ii. Propose a method for the receiver to estimate the gains h_{11} and h_{21}. *4 points.*

Here are the mathematical relationships: $y_1(t) = h_{11}\, x(t)$ and $y_2(t) = h_{21}\, x(t)$.
Transmitter sends training signal $x(t)$ known to the receiver. Gain h_{11} is best linear fit of $y_1(t)$ vs. $x(t)$. By observing the received training signal $y_1(t)$ and treating the h_{11} as a constant, we take the expectation (average value) w/r to time of both sides:

$$E\{\, y_1(t)\, \} = E\{\, h_{11}\, x(t)\, \} = \; h_{11}\, E\{\, x(t)\, \} \text{ which means}$$

$$h_{11} = \frac{E\{y_1(t)\}}{E\{\, x(t)\, \}} = \frac{\textit{Average value of } y_1(t)}{\textit{Average value of } x(t)}$$

Same for h_{21} in terms of $y_2(t)$ vs. $x(t)$.

iii. How would you recommend combining the receive antenna outputs $y_1(t)$ and $y_2(t)$ to produce a single PAM signal that would have a lower probability of symbol error than either $y_1(t)$ or $y_2(t)$ by itself? Assume the receiver knows gains h_{11} and h_{21}. *6 points.*

At the receiver, we have two estimates of $x(t)$ which are $\frac{y_1(t)}{h_{11}}$ and $\frac{y_2(t)}{h_{11}}$. We could average the two estimates for higher reliability:

$$y(t) = \frac{1}{h_{11}} y_1(t) + \frac{1}{h_{21}} y_2(t)$$

or we could normalize the weighted combination of these coefficients as

$$y(t) = \frac{1/h_{11}}{1/h_{11} + 1/h_{21}} y_1(t) + \frac{1/h_{21}}{1/h_{11} + 1/h_{21}} y_2(t)$$

$$y(t) = \frac{h_{21}}{h_{11} + h_{21}} y_1(t) + \frac{h_{11}}{h_{11} + h_{21}} y_2(t)$$

Figures are modified from Lars Reichardt, Juan Pontes, Yoke Leen Sit, and Thomas Zwick, "Antenna Optimization for Time-Variant MIMO Systems", EuCap, 2011.

Prologue: Lectures 13 Digital PAM, 14 Matched Filtering, 15 QAM Transmitters and 16 QAM Receivers; JSK Ch. 8, 9, 10, 11 & 16; Labs 4, 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3

Problem 2.4. *Communication System Tradeoffs. 20 points.*

Two-way communication systems have a data channel and a control channel in each direction.

The data channel supports high bit rates such as for streaming audio or video whereas the control channel has low bit rates for configuration and feedback information (e.g. received SNR). The received SNR is used by the transmitter to determine the number of bits per symbol, J.

The bit rate is $J\ f_{sym}$ and the parameters are explained on the right.

For the remainder of this problem, consider the data rate on the data channel only.

QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
N_g	number of symbol periods in a pulse shape
n	symbol index

(a) One way to increase the bit rate is to increase J which is the number of bits per symbol.

The Shannon Capacity, C, for the communication of information over an additive noise channel is

$$C = \frac{1}{2}\, D\, B\, \log_2(1 + SNR)$$

in bits/s/Hz where D is modulation dimension (1 for PAM and 2 for QAM), B is bandwidth, and SNR is the Signal-to-Noise Ratio at the receiver in linear units. In order to increase the bound on the number of bits per symbol, $\log_2(1 + SNR)$, we'll need to increase the SNR.

i. Give one transmitter method to increase J. *3 points.*

Answer #1. **Increase the power of the transmitted signal, which will in turn increase the power of the received signal.**

Answer #2: **Use a training signal to help the receiver to adapt its subsystems to compensate for impairments experienced by the transmitted signal through the analog/RF front ends and the communication channel. It's important to keep the training signal short enough because training is overhead that reduces the bit rate.**

Answer #3: **Pause transmission for a short time between symbols or add a cyclic prefix to each symbol to reduce the inter-symbol interference in the receiver. Need to keep the pause duration / cyclic prefix length short, because each reduces the bit rate.**

ii. What is the tradeoff in run-time implementation complexity? *2 points.*

Answer #1. **Increases the power consumption in the transmitter.**

Answer #2: **Need to add a subsystem to generate the training signal, e.g. a pseudonoise signal (linear shift feedback register) or a chirp signal.**

Answer #3: **Insert a pause saves power in the transmitter; adding a cyclic prefix copies the last few samples of a symbol to the front of the symbol.**

iii. Give one receiver method to increase J. *3 points.*

Answer #1: **Add a matched filter. It maximizes the SNR of the received symbol amplitude if the only impairment in additive noise.**

Answer #2: **Use a channel equalizer to compensate for impairments experienced by the transmitted signal through the analog/RF front ends and the communication channel. Needs a training signal from the transmitter.**

iv. What is the tradeoff in run-time implementation complexity? *2 points.*

Answer #1: **Matched filter is a finite impulse response (FIR) filter, which has the same run-time implementation complexity as the pulse shaping FIR filter in the transmitter, which is $L^2\, N_g\, f_{sym}$ multiplications/s. We can implement the cacade of the matched filter and the downsampling by L in a polyphase decimation filter bank to save a factor of L in computations.**

Answer #2: **Use a channel equalizer to compensate for impairments experienced by the transmitted signal in the analog/RF front ends and the communication channel. Needs a training signal from the transmitter and need to regenerate the training signal in the receiver. An adaptive FIR equalizer takes at least twice the complexity of an FIR filter with N coefficients, which is $N\, f_s$ multiplications/s.**

The other way to increase the bit rate is to increase the symbol rate, f_{sym}.

v. How does an increase in f_{sym} affect transmission bandwidth? Give a formula. *4 points.*

The baseband bandwidth is $\frac{1}{2}\, f_{sym}\,(1+\ \alpha)$ where $\alpha\epsilon[0, 1]$ is the rolloff factor for a raised cosine pulse, and the transmission bandwidth is double that, $f_{sym}\,(1+\ \alpha)$.

vi. What is the tradeoff in transmitter run-time implementation complexity when increasing f_{sym}? *3 points.*

Sampling rate $f_s = L\, f_{sym}$ increases linearly with f_{sym}. Power consumption in the digital-to-analog (D/A) converter is proportional to the sampling rate and all the discrete-time signal processing in the baseband transmitter runs at a rate proportional to f_{sym}.

vii. What is the tradeoff in receiver run-time implementation complexity when increasing f_{sym}? *3 points.*

Similar to the answer in part vi. Sampling rate $f_s = L\, f_{sym}$ increases linearly with f_{sym}. Power consumption in the analog-to-digital (A/D) converter is proportional to the sampling rate and all the discrete-time signal processing in the baseband receiver runs at a rate proportional to f_{sym}.