The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2

Date: April 28, 2025 Course: EE 445S Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	Point Value	Your score	Topic
1	24		Baseband PAM System
2	30		QAM Communication Performance
3	26		Improving Communication Performance
4	20		Communication System Tradeoffs
Total	100		

Problem 2.1. *Baseband PAM System. 24 points.*

PAM System Parameters

- $a[n]$ symbol amplitude
- $2d$ constellation spacing
- f_s sampling rate
- f_{sym} symbol rate
- $g[m]$ pulse shape
- $h[m]$ matched filter impulse response
- J bits/symbol
- L samples/symbol period
- M levels, i.e. $M = 2^J$
- m sample index

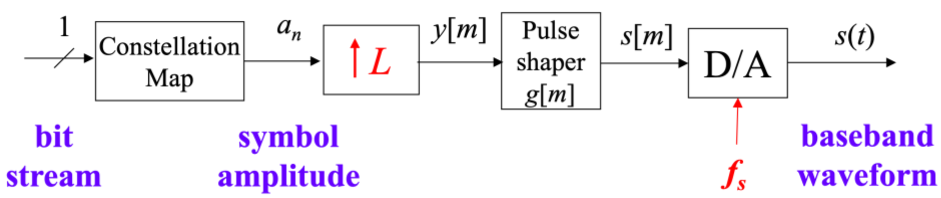
Consider a two-level pulse amplitude modulation (2-PAM) system, a.k.a. binary phase shift keying.

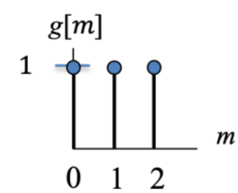
The system parameters are described on the right:

- $J = 1$ bits/symbol; i.e., $M = 2$ levels/symbol
- $L = 3$ samples per symbol period
- Pulse shape $g[m]$ is a rectangular pulse of 3 samples in duration as shown below.
- Constellation map: input '0' maps to 1 Volt and input '1' maps to -1 Volt.

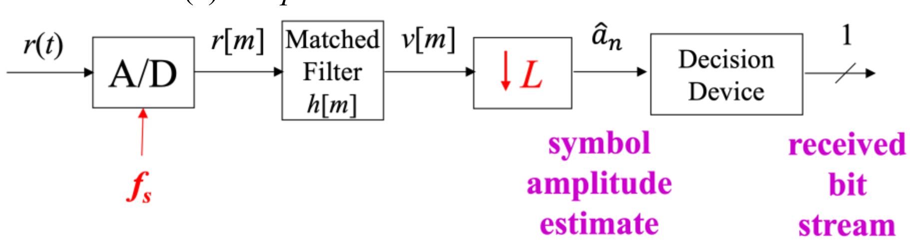
Assume all filters are linear and time-invariant (LTI).

(a) For the **2-PAM transmitter** below, input bit stream is 0101. Plot the discrete-time signals $a[n]$, $y[m]$ and $s[m]$. *12 points*.



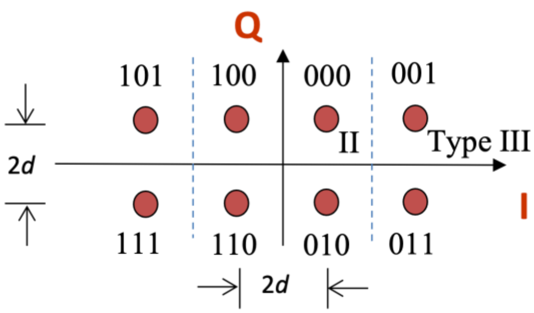


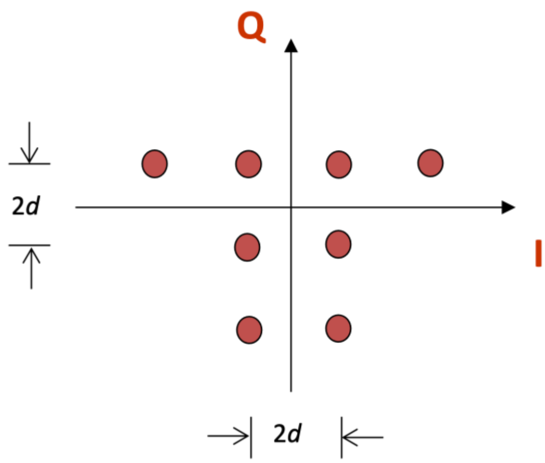
(b) For the **2-PAM receiver** below, assume there is no channel distortion or additive noise, and assume $r[m] = s[m]$ and $h[m] = g[m]$**.** The Decision Device compares the input value against 0. Plot the discrete-time signals $v[m]$, $\hat{a}[n]$ and give the received bit stream based on the 2-PAM transmitter in (a). *12 points*.



Problem 2.2 *QAM Communication Performance. 30 points.*

Consider the two 8-QAM constellations below. Constellation spacing is 2*d*.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**
(a) Peak transmit power	$10d^2$	
(b) Average transmit power	$6d^2$	
(c) Peak-to-average power ratio	$\frac{10d^2}{6d^2} = \frac{5}{3} \approx 1.67$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***		
(e) Number of type I QAM regions	0	
(f) Number of type II QAM regions	4	
(g) Number of type III QAM regions	4	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2.	$P_e = \frac{5}{2} Q\left(\frac{d}{\sigma}\right) - \frac{3}{2} Q^2\left(\frac{d}{\sigma}\right)$	
(i) Express the argument of the *Q* function as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{6d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{6}}$	

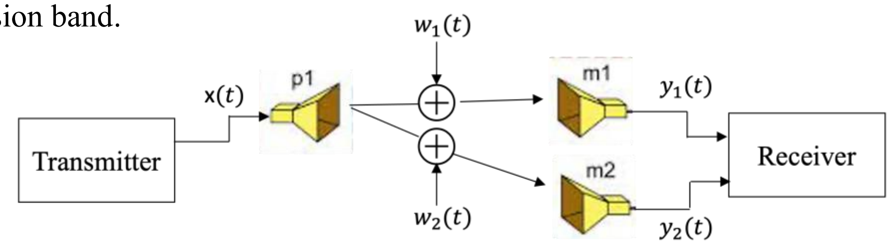
(j) Give a Gray coding for the right constellation or show that one does not exist. *3 points*.

Problem 2.3. *Improving Communication Performance*. *26 points*.

One way to improve communication performance is to use two antennas to process a transmitted signal sent by a single antenna.

For this problem, assume the transmitter is sending a pulse amplitude modulation (PAM) signal over the air in a radio frequency (RF) transmission band.

(a) First, we'll assume the only impairment in the system is additive thermal noise as shown on the right:

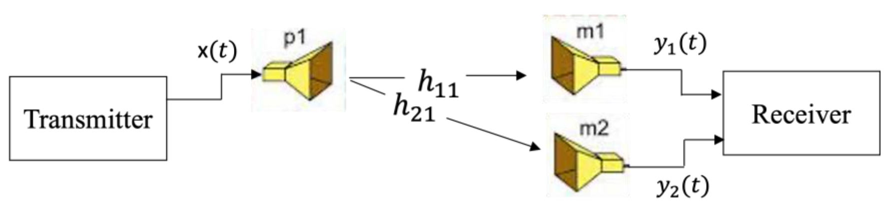


i. The model for additive thermal noise is in the communication channel, but where does the additive thermal noise physically occur in the system? *3 points.*

ii. What is a good statistical model for the additive thermal noise? Why? Explain what the statistical model parameters mean. *4 points.*

iii. How would you recommend combining the receive antenna outputs $y_1(t)$ and $y_2(t)$ to produce a single PAM signal that would have a lower probability of symbol error than either $y_1(t)$ or $y_2(t)$ by itself? Assume the receiver knows the values of the statistical model parameters for $w_1(t)$ and $w_2(t)$. *6 points.*

(b) Second, we'll assume the only impairment in the system is a gain in each path from the transmitter to the receiver as shown on the right.



i. What is each gain modeling in terms of a physical phenomenon? *3 points.*

ii. Propose a method for the receiver to estimate the gains h_{11} and h_{21}. *4 points.*

iii. How would you recommend combining the receive antenna outputs $y_1(t)$ and $y_2(t)$ to produce a single PAM signal that would have a lower probability of symbol error than either $y_1(t)$ or $y_2(t)$ by itself? Assume the receiver knows gains h_{11} and h_{21}. *6 points.*

Problem 2.4. *Communication System Tradeoffs. 20 points.*

Two-way communication systems have a data channel and a control channel in each direction.

The data channel supports high bit rates such as for streaming audio or video whereas the control channel has low bit rates for configuration and feedback information (e.g. received SNR). The received SNR is used by the transmitter to determine the number of bits per symbol, J.

The bit rate is $J\ f_{sym}$ and the parameters are explained on the right.

For the remainder of this problem, consider the data rate on the data channel only.

QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
N_g	number of symbol periods in a pulse shape
n	symbol index

(a) One way to increase the bit rate is to increase J which is the number of bits per symbol.

i. Give one transmitter method to increase J. *3 points.*

ii. What is the tradeoff in run-time implementation complexity? *2 points.*

iii. Give one receiver method to increase J. *3 points.*

iv. What is the tradeoff in run-time implementation complexity? *2 points.*

(b) The other way to increase the bit rate is to increase the symbol rate, f_{sym}.

i. How does an increase in f_{sym} affect transmission bandwidth? Give a formula. *4 points.*

ii. What is the tradeoff in transmitter run-time implementation complexity when increasing f_{sym}? *3 points.*

iii. What is the tradeoff in receiver run-time implementation complexity when increasing f_{sym}? *3 points.*