The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Version 1.0***

Date: April 27, 2026 Course: EE 445S Evans

Name: **Solutions**
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the particular content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	**Point Value**	**Your score**	**Topic**
1	24		Baseband PAM Repeater
2	27		QAM Communication Performance
3	27		Compensating for Impairments
4	22		Communication System Tradeoffs
Total	100		

Lectures: 7 Pulse Shaping, 13 Digital PAM and 14 Matched Filtering; JSK Ch. 8 and 9; Labs 3, 5 & 6; HW 1.2, 2.1, 2.2, 4.2, 4.3, 5.2 & 6.2; Midterm 2: F21 Prob. 2.1, F24 Prob. 2.1, Sp25 Prob. 2.1, F25 Prob. 2.1

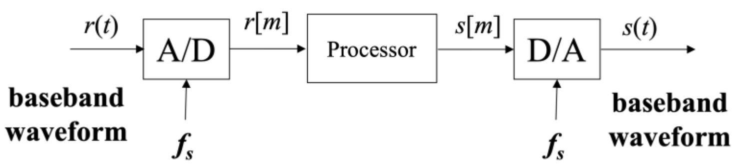
PAM System Parameters

a_n	symbol amplitude
$\hat{a}_n$	estimated symbol amplitude
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index

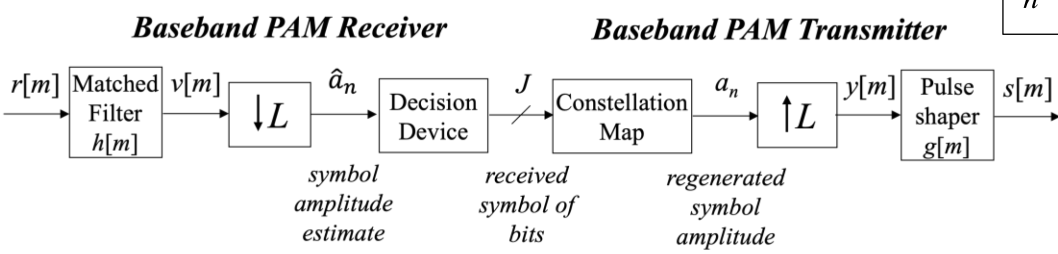
Problem 2.1. *Baseband PAM Repeater. 24 points.*

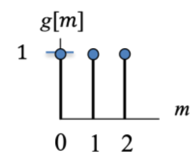
A repeater boosts a signal to extend its range.

Consider a baseband PAM repeater using your board in lab:



The baseband PAM repeater will be implemented on the processor:



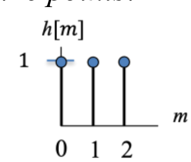


Assume the repeater is repeating a 2-PAM signal, i.e., $J = 1$.

A bit of value '1' maps to symbol amplitude d and '0' maps to symbol amplitude $-d$, with $d = 1$.

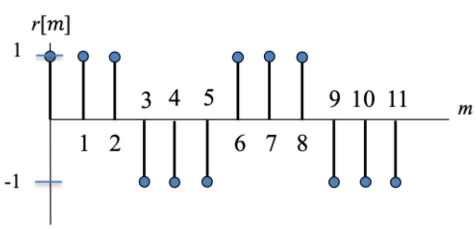
(a) For the pulse shape $g[m]$ above, plot the optimal matched filter impulse response $h[m]$. *6 points.*

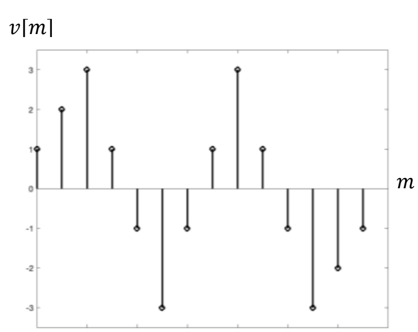
The optimal matched filter impulse response is $h[m] = k\, g^*[L - m]$.
Here, $L = 3$ which is the number of samples in $g[m]$.

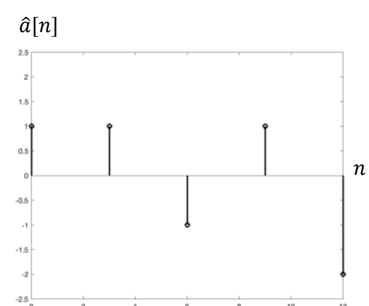


(b) For the following received signal $r[m]$, plot $\hat{a}_n$ vs. the symbol index n. *6 points.*

$v[m] = h[m] * r[m]$
has 3+12-1=14 samples







(c) What is the received bit stream? *6 points*. **{ 1, 1, 0, 1, 0 }**

Compare $\hat{a}_n$ against 0
Map positive values to a bit of '1'
Map negative values to '0'.

(d) How many bits (if any) should we remove at the beginning of the received bit stream before retransmitting them? *6 points.*

The transmitted bit stream is { 1, 0, 1, 0 }.
Remove the first bit before transmitting.

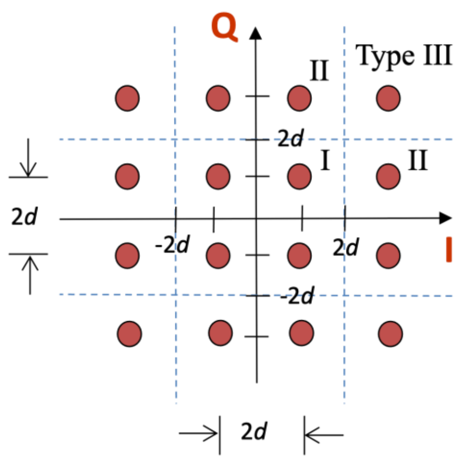
```
%% Matlab code for part (b)
h = [1 1 1];
r = [1 1 1 -1 -1 -1 1 1 1 -1 -1 -1];
v = conv(h, r);
m = 0 : length(v) - 1;
figure;
stem(m, v, 'k', 'LineWidth',2);
ylim( [-3.5 3.5] );
figure;
n = m(1:3:end);
ahat = v(1:3:end);
stem(n, ahat, 'k', 'LineWidth',2);
ylim( [-2.5 2.5] );
```

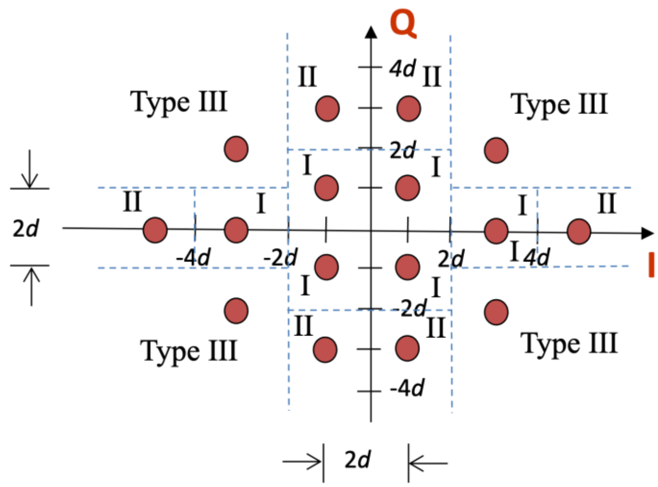
As an alternative to the approach in part (d), we could have removed the first L-1=2 samples from $v[m]$ to obtain {3, -3, 3, -3} for $\hat{a}_n$ and a bit stream of { 1, 0, 1, 0 }. This approach has the advantage of maximizing power in $\hat{a}_n$ and hence the SNR for reduce bit errors in the presence of additive noise.

Lectures 13-16; JSK Ch. 16; Labs 5 & 6; HW 4.1, 4.2, 4.3, 5.2, 6.3 & 7.3; Handout P PAM vs. QAM; Midterm 2.2: F14, Sp15, F15, Sp16, F16, Sp17, F17, Sp18, F18, Sp19, F19, Sp20, F20, Sp21, F21, Sp22, Sp25, F25

Problem 2.2 *QAM Communication Performance. 27 points.*

Consider the two 16-QAM constellations below. Constellation spacing is $2d$.





Energy in the pulse shape is 1. Symbol time T_{sym} is 1s. The constellation on the left includes the decision regions with boundaries shown by the in-phase (I) axis, quadrature (Q) axis and dashed lines.

Each part below is worth 3 points. **Please fully justify your answers. Show intermediate steps.**

	Left Constellation	**Right Constellation**	
(a) Peak transmit power	$18d^2$	$\boldsymbol{25d^2}$	
(b) Average transmit power	$10d^2$	$\mathbf{10.5}\boldsymbol{d^2}$	**See below**
(c) Peak-to-average power ratio	$\frac{18d^2}{10d^2} = \frac{9}{5} = 1.8$	$\frac{25d^2}{10.5d^2} = \frac{50}{21} \approx 2.38$	
(d) Draw the type I, II and/or III decision regions for the right constellation on top of the right constellation ***that will minimize the probability of symbol error using such decision regions.***			
(e) Number of type I regions	4	**6**	
(f) Number of type II regions	8	**6**	
(g) Number of type III regions	4	**4**	
(h) Probability of symbol error for additive Gaussian noise with zero mean & variance σ^2	$3Q\left(\frac{d}{\sigma}\right) - \frac{9}{4}Q^2\left(\frac{d}{\sigma}\right)$	$\frac{25}{8}Q\left(\frac{d}{\sigma}\right) - \frac{5}{2}Q^2\left(\frac{d}{\sigma}\right)$	**See below**
(i) Express d/σ as a function of the Signal-to-Noise Ratio (SNR) in linear units	$\text{SNR} = \frac{10d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{10}}$	$\text{SNR} = \frac{10.5d^2}{\sigma^2}$ $\frac{d}{\sigma} = \sqrt{\frac{\text{SNR}}{10.5}}$	

(b) Because d is a voltage, the average transmit power is proportional to

$$\frac{1}{8}\left(2(d^2 + d^2) + 2(d^2 + (3d)^2) + 2((3d)^2 + (2d)^2) + (3d)^2 + (5d)^2\right) = \frac{84d^2}{8} = 10.5\,d^2$$

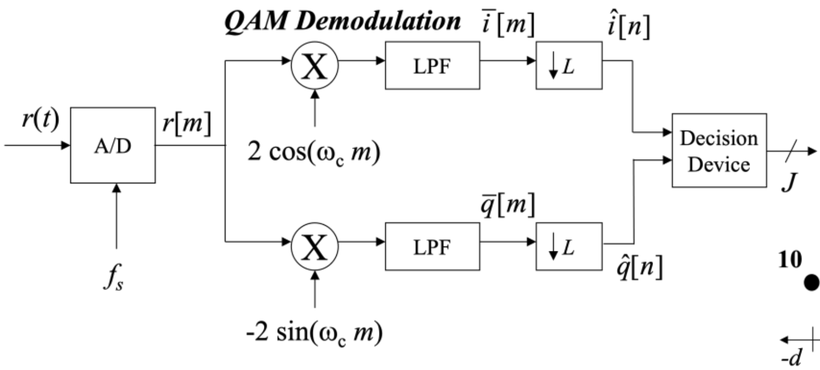
(h) Let $q = Q\left(\frac{d}{\sigma}\right)$. $P_c = \frac{6}{16}P_c^{I} + \frac{6}{16}P_c^{II} + \frac{4}{16}P_c^{III} = \frac{3}{8}(1-2q)^2 + \frac{3}{8}(1-q)(1-2q) + \frac{2}{8}(1-q)^2$

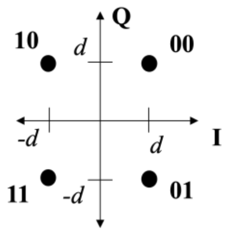
and $P_e = 1 - P_c$.

JSK Sec. 3.7, 6.5-6.8, 9.3; HW 5.1, 6.1, 6.2, 7.2, 7.3; HW 7.3 Sp14; In-Lecture Assignments 3 & 4; PAM Lectures 12-14; QAM Lectures 15-16; AGC Lecture Slide 16-4; AGC Midterm #2 Prob. 2.3 in Sp15 & Sp19; Decision Directed Midterm #2 Prob. 2.3 in F17, F21, Sp 23, F23, F25

Problem 2.3. *Compensating for Impairments. 27 points.*

Consider the baseband Quadrature Amplitude Modulation (QAM) receiver





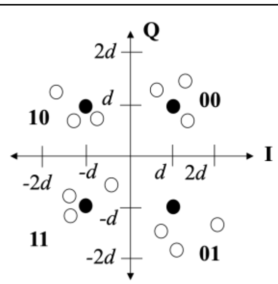
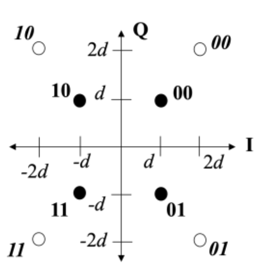
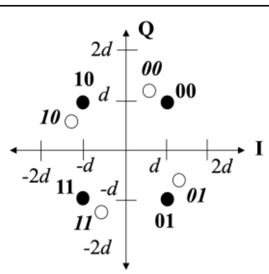
Assume the baseband QAM receiver is synchronized in symbol timing with the baseband QAM transmitter.

QAM System Parameters

$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$i[n]$	in-phase component of nth symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
n	symbol index
$q[n]$	quadrature component of nth symbol amplitude

During training, transmitted 4-QAM baseband symbol amplitudes (solid black dots) and received baseband 4-QAM symbol amplitudes (black circles) are shown below. **Please fill out the table below.**

In each case, all transmitted/received symbol amplitudes in the same quadrant correspond to the same symbol.

Symbol amplitudes: Transmitted (black solid dots) and received (black circles)	**The primary impairment (circle only one)** *2 pts.*	**Location(s) of impairment (circle all that apply)** *2 pts.*	**Subsystem to add or modify in baseband Rx to compensate impairment.** *2 pts.*	**Give an equation governing how you would design the subsystem.** *3 pts.*
	additive noise carrier phase offset magnitude distortion	Tx analog/RF front end Channel Rx analog/RF front end	**Use a matched filter for lowpass filter (LPF) with impulse response $h[m]$ -OR-** **Add adaptive decision directed equalizer after A/D**	$h[m] = k\, g^*[L - m]$ **k is non-zero gain -OR-** **Adapt FIR coefficients with objective $J(m)$ of** $(\hat{i}[n] - i[n])^2 + (\hat{q}[n] - q[n])^2$
	additive noise carrier phase offset magnitude distortion	Tx analog/RF front end Channel Rx analog/RF front end	**Add automatic gain control after A/D** **-OR-** **Add adaptive decision directed equalizer after A/D**	**Use algorithm** on lecture slide 16-4 -OR-** **Adapt FIR coefficients with above objective $J(m)$ using steepest descent algorithm**
	additive noise carrier phase offset magnitude distortion	Tx analog/RF front end Channel Rx analog/RF front end	**Add phase locked loop to estimate phase offset for carrier signals -OR-** **Add adaptive decision directed equalizer after A/D**	**Use Costas loop HW 6.1 to estimate phase offset** $\theta[n+1] = \theta[n] - \mu\, \hat{i}[n]\, \hat{q}[n]$ **-OR- Adapt FIR coeffs as described above**

**** Automatic gain control algorithm on lecture slide 16-4 updates gain $c(t)$ applied prior to A/D converter $c(t) = (1 + 2 f_0 - f_{-128} - f_{127})\, c(t - \tau)$ to keep A/D input signal in the A/D voltage range.**

Many more answers

Lectures 13-16; JSK Sec. 14.4; JSK Ch. 2, 3, 16; Labs 3, 5 & 6; HW 3.3, 5.3, 6.3, 7.1, 7.2;
Reading from Ch. 4 of Simon Haykin's Communication Systems book;
Midterm Prob. 2.4 in F20, 2.3 in Sp21, 2.4 in F23, 2.4 in Sp24, 2.4 in F24, 2.4 in Sp25, 2.4 in F25

Problem 2.4. *Communication System Tradeoffs. 22 points.*

Claude Shannon derived the following upper bound on the capacity, C, for a communication channel in units of bits/s for a QAM system:

$$C = B_T \log_2(1 + \text{SNR})$$

where B_T is the transmission bandwidth in Hz and

SNR is the Signal-to-Noise Ratio at the receiver in linear units (not in decibels) which is

$$\text{SNR} = \frac{\textit{Signal Power}}{\textit{Noise Power}}$$

The upper bound on the number of bits/symbol, J, is $\log_2(1 + \text{SNR})$ rounded down to make it an integer.

We seek to increase the channel capacity in a QAM system.

- Assume the transmission bandwidth, B_T, remains constant.
- The received SNR is evaluated in the baseband QAM receiver where the in-phase and quadrature symbol amplitude values are estimated.

QAM System Parameters

B	number of bits in the D/A converter in transmitter
$2d$	constellation spacing
f_s	sampling rate
f_{sym}	symbol rate
$g[m]$	pulse shape
$h[m]$	matched filter impulse resp.
$i[n]$	in-phase symbol amplitude
$q[n]$	quadrature symbol amplitude
J	bits/symbol
L	samples/symbol period
M	levels, i.e. $M = 2^J$
m	sample index
N_g	number of symbol periods in a pulse shape
n	symbol index

(a) Describe two approaches in the transmitter that can improve the received SNR. *6 points.*

Since transmission bandwidth B_T is constant and $B_T = f_{sym}$, symbol rate f_{sym} is constant.

↑ J or d causes ↑ in transmit signal power and ↑ B.

↑ transmitted signal power leads to ↑ Rx signal power. (*Note: A power amp is in Tx front end.*)

↑ B reduces the additive quantization noise power at the D/A converter output
↑ L increases number of samples Rx uses to make decision about what symbol was Tx
↑ N_g increases frequency selectivity of pulse shaping filter to remove more out-of-band noise (sinc or raised cosine pulses truncated to $L\,N_g$ samples; more samples, more selectivity)
Use training signals allow the receiver to adapt subsystems to improve SNR including automatic gain control, equalizer, timing recovery, and carrier recovery

(b) Assuming both of your transmitter approaches in part (a) are implemented, how does improvement in the receiver SNR effect the following measures? Does the measure increase, decrease, or stay the same. Justify your answer. *4 points for each item below.*

i. Bit rate **is $J\,f_{sym}$ and $J \le \log_2(1 + \text{SNR})$ rounded down. Increasing SNR will ↑ bit rate if the SNR increase is enough for $\log_2(1 + \text{SNR})$ to give additional bit(s).**

ii. Probability of symbol error (also known as the symbol error rate).

$P_e = C_0\, Q(C_1\sqrt{\text{SNR}}) - C_2\, Q^2(C_3\sqrt{\text{SNR}})$. **First term larger than second term because Q function returns values on [0, 1] and $C_0 > C_2 > 0$. Because Q is monotonically decreasing, every approach increasing SNR decreases Probability of Symbol Error.**

iii. Baseband transmitter run-time implementation computational complexity.

↑ in computational complexity (a) in the matched filter with ↑ L or ↑ N_g and (b) in the symbol amplitude quantizer (constellation demapping) with ↑ J and (c) for training.

iv. Power consumption in the D/A converter in the transmitter analog/RF front end.
$P_{D/A} \propto f_s\, 2^B$ **and** $f_s = L\, f_{sym}$ **. ↑ in power consumption with ↑ L or ↑ B, and hence also with ↑ J or ↑ d which in turn ↑ B. (*D/A output is later boosted by a power amp.*)**