Pulse Amplitude Modulation (PAM)

- **Communications**:

  1 Analog communications: transmit and receive analog waveform (*e.g.,* AM, PM, FM).

  2 Digital communications: transmit and receive discrete information (*e.g.,* PAM, PSK, QAM, spread spectrum, ...).

- **Pulse Modulation**:

Modulate $M = 2^J$ discrete messages or J bits of information into the amplitude of a signal.

If the amplitude mapping changes at a rate of f_s, then the bit rate is Jf_s.

The conventional mapping of the discrete messages to amplitude is

$$l_i = d(2i-1), \quad i = -\frac{M}{2}+1, \ldots, 0, \ldots, \frac{M}{2}.$$

The PAM modulated signal can be expressed

$$s^*(t) = \sum_{k=-\infty}^{\infty} a_k \delta(t-kT).$$

- **Pulse Shaping**:

The signal $s^*(t)$ has infinite bandwidth and cannot be sent by a real transmitter. Thus, we need to limit its bandwidth by a pulse shaping filter whose impulse response is $g_T(t)$. Then the transmit signal is

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g_T(t-kT).$$

- **Eye Diagram and Intersymbol Interference**

Figure 6 shows an eye diagram. The wider the "eye" opens, the better the signal quality is. An eye diagram is a empirical measure of the quality of the received signal.

$$x(n) = \sum_{k=-\infty}^{\infty} a_k g(nT-kT) = g(0)\left(\underbrace{a_n}_{signal} + \underbrace{\sum_{k=-\infty, k\neq n}^{\infty} a_k \frac{g(nT-kT)}{g(0)}}_{ISI}\right).$$

The intersymbol interference (ISI):

$$D \leq (M-1)d \sum_{k=-\infty, k\neq n}^{\infty} \left|\frac{g(nT-kT)}{g(0)}\right| = (M-1)d \sum_{k=-\infty, k\neq 0}^{\infty} \left|\frac{g(kT)}{g(0)}\right|.$$

The ISI is the interference to the current symbol by preceding symbols. The raised-cosine filter has zero ISI at the *correct* sample point.

- **Symbol Clock Recovery**

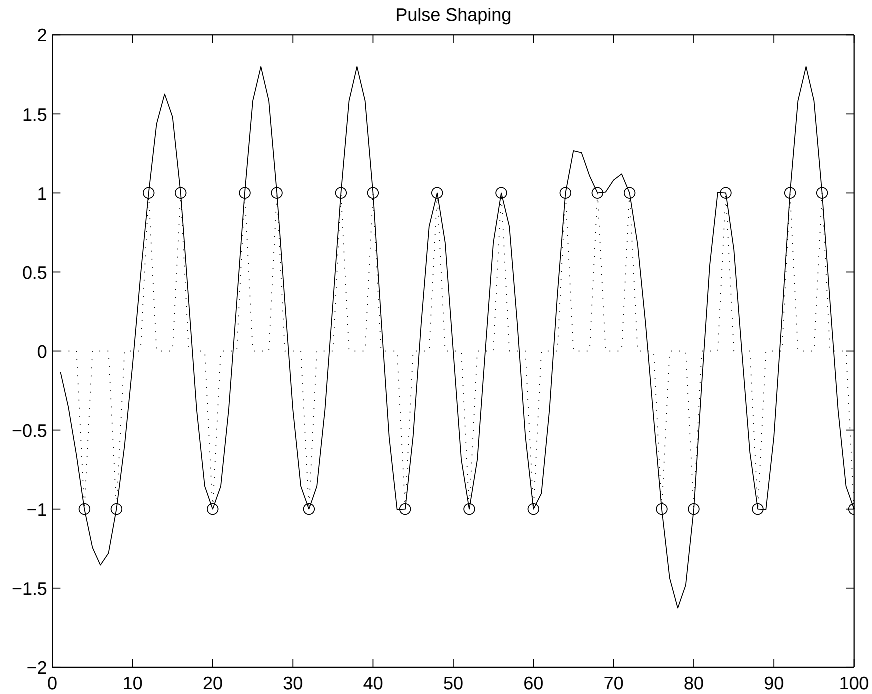

Figure 1: An Example of Pulse Shaping

In typical digital communication system, it is critical to sample a correct time instance to have the maximum signal power and minimum ISI. Since the transmitter and receiver normally have different crystal oscillators, a digital receiver should try its best to synchronize with the transmitter clock. In other words, the receiver must extract the clock information from the received signal and then adjust its A/D timing.

Let $g_1(t)$ is the impulse response of the *composite* channel including pulse shaping, real communication media, and the receive low-pass filter. Then the received signal $q(t)$ becomes

$$q(t) = s^*(t) \odot g_1(t) = \sum_{k=-\infty}^{\infty} a_k g_1(t - kT).$$

Then

$$p(t) = q^2(t) = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} a_k a_m g_1(t - kT) g_1(t - mT).$$

and

$$E\{p(t)\} = \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underbrace{E\{a_k a_m\}} - a^2 \delta(k - m) g_1(t - kT) g_1(t - mT) = a^2 \sum_{k=-\infty}^{\infty} g_1^2(t - kT).$$

Since we can easily show that $E\{p(t+T)\} = E\{p(t)\}$, $E\{p(t)\}$ is a periodic function, *i.e.*,

$$E\{p(t)\} = \sum_{k=-\infty}^{\infty} p_k e^{jk\omega_s t}$$

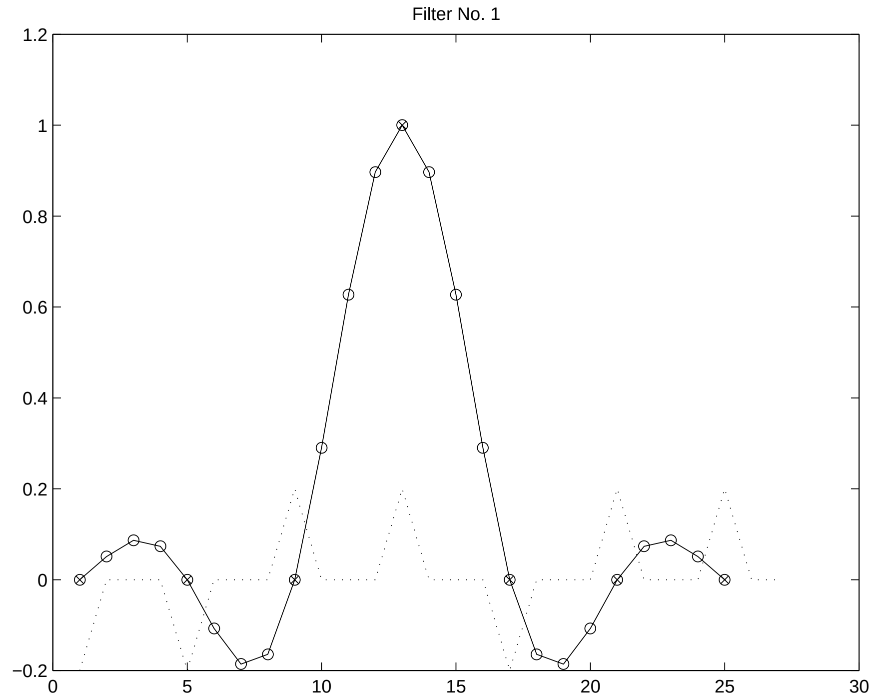

Figure 2: Implementation of Pulse Shaping via a Filter Bank: Filter No. 1

where

$$p_k = \frac{1}{T}\int_0^T E\{p(t)\}e^{-jk\omega_s t}dt.$$

Clearly, if we let $E\{p(t)\}$ passes through a low-pass filter that covers the fundamental frequency ω_s and filters out the harmonics, $2\omega_s$, $3\omega_s$, $\ldots$ We can extract a sin wave that has the same frequency as the symbol rate. This signal can be used as a reference signal to a digital phase locked loop or reshaped as a sampling clock.

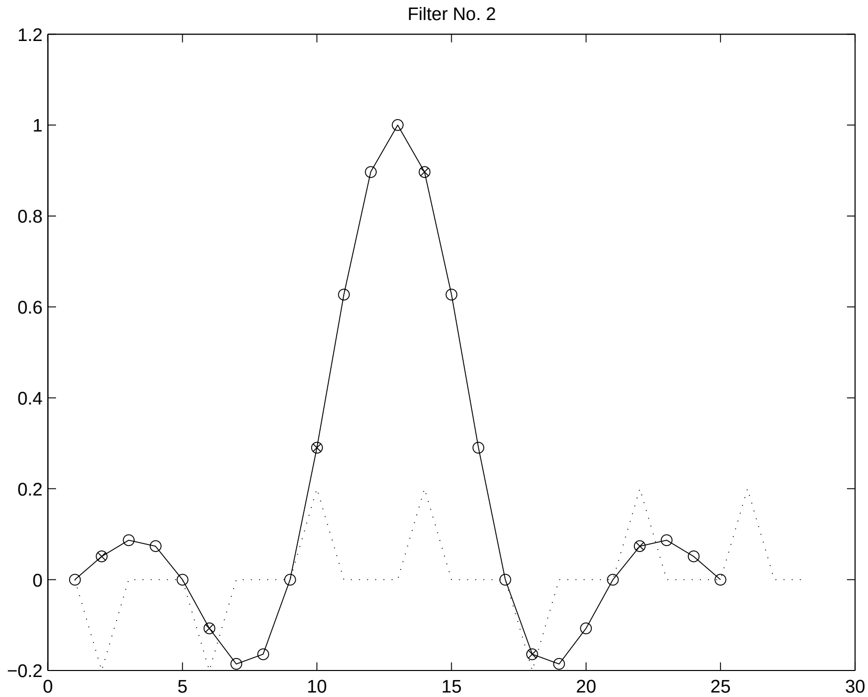

Figure 3: Implementation of Pulse Shaping via a Filter Bank: Filter No. 2

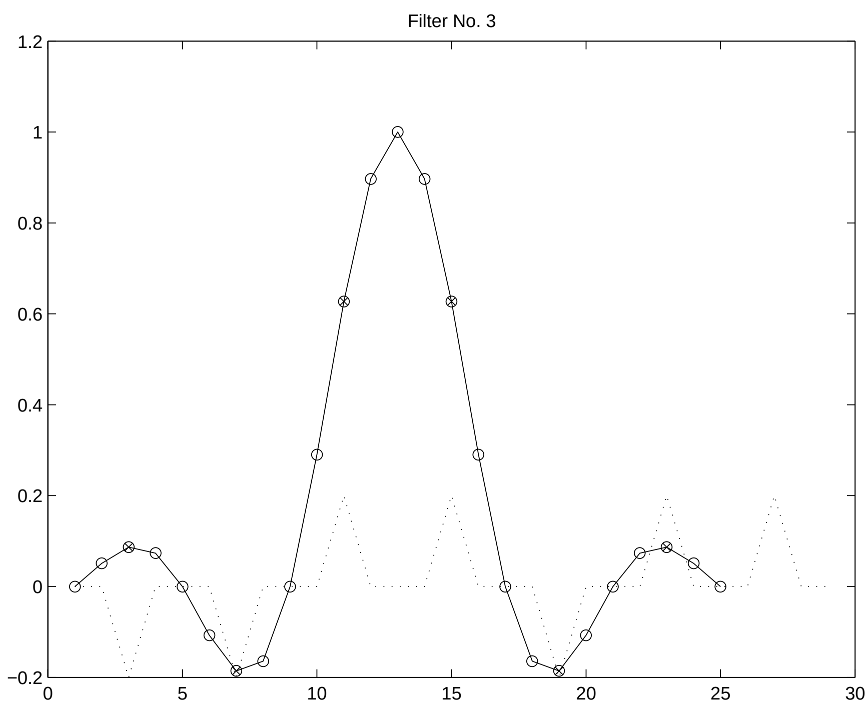

Figure 4: Implementation of Pulse Shaping via a Filter Bank: Filter No. 3

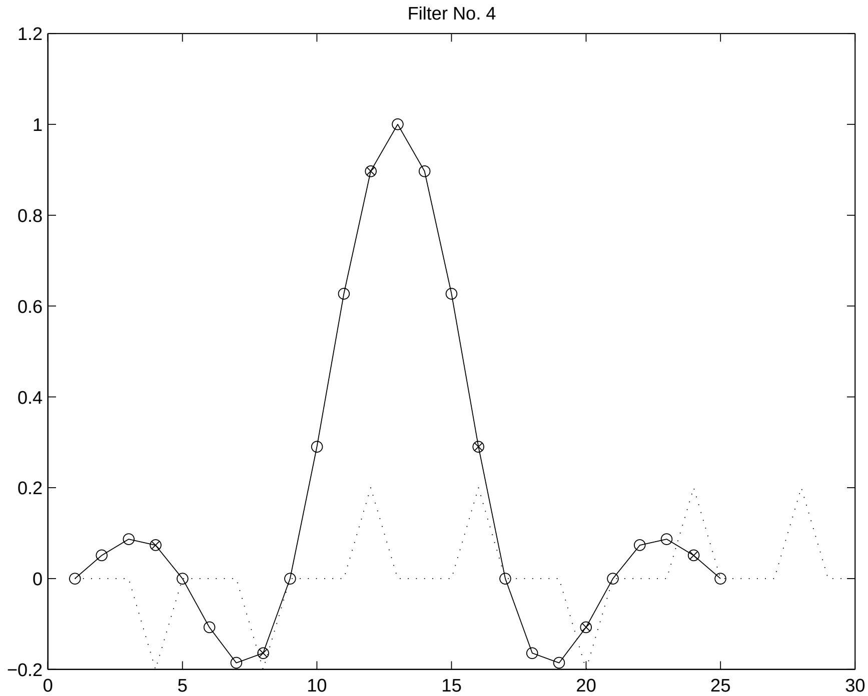

Figure 5: Implementation of Pulse Shaping via a Filter Bank: Filter No. 4

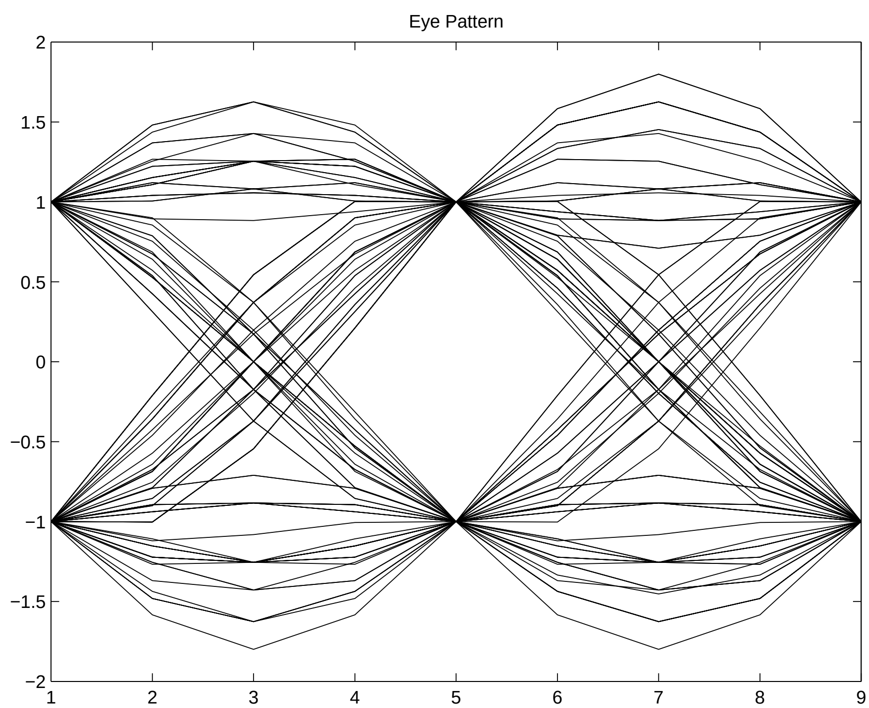

Figure 6: An Example of Eye Pattern