

Homework #2 Fourier Series

Assigned on Sunday, September 3, 2023

Due on Friday, September 8, 2021, by 11:59 pm via Canvas submission

Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer and Yoder, *Signal Processing First*, 2003, Ch. 3. [Errata](#).

Companion Web site with demos and other supplemental information: <http://dspfirst.gatech.edu/>

Web site contains solutions to selected homework problems from *DSP First*.

E-mail address for Mr. Elyes Balti (TA) is ebalti@utexas.edu. Please consider posting questions on the class [Ed Discussion site](#), which can be answered by anyone in the class. You can post anonymously to the other students and the instructors would still be able to see your name.

Lecture and office hours for Mr. Balti and Prof. Evans follow:

<i>Time Slot</i>	<i>Monday</i>	<i>Tuesday</i>	<i>Wednesday</i>	<i>Thursday</i>	<i>Friday</i>
11:00 am		Evans (ECJ 2.104)		Evans (ECJ 2.104)	
11:30 am		Evans (ECJ 2.104)		Evans (ECJ 2.104)	
12:00 pm		Evans (ECJ 2.104)		Evans (ECJ 2.104)	
12:30 pm					
1:00 pm					
1:30 pm					
2:00 pm			Evans (EER 6.882 and Zoom)	Evans (EER 6.882 and Zoom)	Balti (EER 3.648)
2:30 pm			Evans (EER 6.882 and Zoom)	Evans (EER 6.882 and Zoom)	Balti (EER 3.648)
3:00 pm			Evans (EER 6.882 and Zoom)	Evans (EER 6.882 and Zoom)	Balti (EER 3.648)
3:30 pm		Balti (EER 3.648)			
4:00 pm		Balti (EER 3.648)			
4:30 pm		Balti (EER 3.648)			
5:00 pm				Balti (EER 3.648)	
5:30 pm				Balti (EER 3.648)	
6:00 pm				Balti (EER 3.648)	

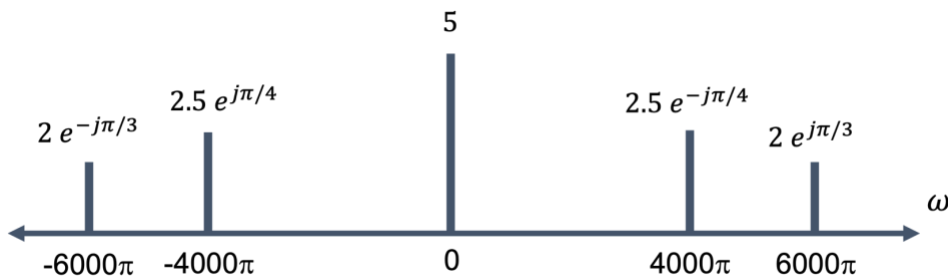
As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see the guidelines for writing your solutions, which are available on the last page of this assignment and on the [homework page](#).

Takeway: Fourier analysis reveals what frequency components are in a time-domain signal and how strong they are, and Fourier synthesis converts the frequency components into a time-domain signal. The representation of the frequency components has a natural visual representation, and drawing the frequency components can help guide the interpretation and analysis of the signal.

1. Fourier Synthesis. 25 points.

Signal Processing First, problem P-3.2, page 65, but use the following spectrum:



2. Amplitude Modulation. 24 points.

In AM radio, the transmitted signal (voice or music) is modulated by a sinusoid at the assigned broadcast frequency of the AM station. For example, the Zone Austin Sports Talk in Austin has a broadcast (carrier) frequency f_c of 1300 kHz. In the US, the AM radio band is 540 kHz to 1700 kHz.

If $x(t)$ is the voice/music signal, then the transmitted signal would be:

$$y(t) = (x(t) + A) \cos(2 \pi f_c t)$$

where A is a constant. When A is chosen so that $x(t) + A \geq 0$, the receiver design can be simplified to a squaring block (single transistor) plus accompanying circuitry.

Suppose the signal to be transmitted is an 'E' in the fifth octave on the Western scale:

$$x(t) = 2 \cos(2\pi(660)t + \pi/2)$$

Draw the spectrum for $y(t)$ assuming a carrier frequency of 1300 kHz with $A = 2$. Hint: Substitute for $x(t)$ and expand $y(t)$ into a sum of cosine terms of three different frequencies.

3. Chirp Signal. 27 points.

This problem analyzes the chirp signal, which is used in sonar and radar systems, IEEE 802.11az Wi-Fi indoor positioning, test and measurement, and 4G/5G cellular communications. The chirp signal is a sinusoid whose principal frequency increases (or decreases) over time. A chirp signal has the form

$$c(t) = \cos(\theta(t)) \text{ where } \theta(t) = 2\pi(f_0 + \frac{1}{2}f_{\text{step}}t)t = 2\pi f_0 t + \pi f_{\text{step}} t^2$$

The principal frequency is f_0 when $t = 0$ and then changes over time at a rate of f_{step} in units of Hz/s. The principal frequency of a sinusoid at a given point in time is called the *instantaneous frequency*, and it is defined as

$$\frac{d\theta(t)}{dt} = \frac{\text{Change in angle (radians)}}{\text{Change in time (seconds)}} = 2\pi f_0 + 2\pi f_{\text{step}} t = 2\pi(f_0 + f_{\text{step}} t)$$

and has units of rad/s which is a frequency measure. We can divide by 2π to convert from rad/s to Hz.

- (a) Generate a chirp signal that lasts 10s with $f_0 = 220$ Hz and $f_{\text{step}} = 400$ Hz/s. Use sampling rate f_s of 44100 Hz. The chirp will sweep from ‘A’ in the third octave (220 Hz) to the highest note (4186 Hz) on an 88-key piano https://en.wikipedia.org/wiki/Piano_key_frequencies. We are starting at 220 Hz because many playback systems on laptops do not play frequencies below 220 Hz.

Here’s Matlab code to help get started:

```
%%% Generate a chirp signal with frequency increasing
%%% from f0 to (f0 + fstep time) over time seconds
time = 10;
f0 = 220;
fstep = 400;
fs = 44100;
Ts = 1 / fs;
t = 0 : Ts : time;
%%% Add code here to define the chirp signal y = cos( angle(t) )
```

- (b) Play the chirp signal as an audio signal. Describe what you hear. The Matlab command is

```
sound(y, fs);
```

- (c) Plot the spectrogram of the chirp signal using the `spectrogram` function in Matlab and describe the visual representation. Here’s Matlab code to get started:

```
figure;
spectrogram(y, hamming(256), 128, 256, fs, 'yaxis');
```

Provide all the Matlab code that you used in solving this problem.

4. Audio Effects. 24 points.

Consider the signal $x(t) = \cos(2\pi f_0 t)$ where $f_0 = 440$ Hz is an ‘A’ in the 4th octave on the Western scale.

Write MATLAB code to implement the following audio effects. Play each audio signal for 5 seconds over $0 \leq t \leq 5$ and use a sampling rate of 8000 Hz:

- (a) $y(t) = x^2(t)$. Used in power calculations, image display, communication systems.
 (b) $y(t) = \exp(x(t))$. Used as a penalty function in machine learning training algorithms.

For each part,

- (i) give a mathematical analysis to determine what frequencies are present in $y(t)$

- (ii) plot the spectrogram and describe the principal frequency components and how they compare to your analysis in part (i)
- (iii) play $y(t)$ as an audio signal by subtracting the average value (because we cannot hear DC) and then using the `soundsc` command on the remaining signal, and describe what you hear vs. $x(t)$.

Please submit the MATLAB code that you have written.

Homework Guidelines

Amount of work to show:

1. An explanation should be given for every single answer. Answers written without explanation will lose two-thirds of the points allotted for that part.
2. Only "standard" formulas (like Euler's formula, trigonometric formulas, etc.) can be used without a reference. If you're using something non-standard, then please put a reference to the formula number in the book, or whatever source you got it from. Just using the final result of a similar problem done in the class, and omitting the intermediate steps, is not okay. You have to show your work.
3. There shouldn't be big jumps in logic from one step to the next.
4. For everything, expect to show at least one intermediate step between the first line and answer. Even if it seems unnecessary to you, please err on the side of caution. Things that seem obvious to you when writing the solution are not quite so obvious for someone reading it.
5. If you're in any doubt about how much work to show, please ask the instructor or the teaching assistant.

MATLAB source code guidelines:

1. Put a comment before the solution of each part, telling the question number of the solution.
2. If you're using complicated logic, leave a comment about what the code is supposed to do.
3. Use variable names that related to their meaning/use.
4. Avoid using two different variables for the same thing.
5. Try to avoid using "magic numbers" in the code. If you're using a number, write a comment telling me how you derived it.
6. Make sure that your code will compile & run in a clean workspace; i.e., one without any variables present. Use a clear all; at least once before submitting it.
7. No marks will be deducted based on the efficiency of the code unless the problem asks you to write efficient code.

Technical points:

1. Merge all the files together into one PDF file.
2. Please adjust the contrast, exposure etc., to get a good scan quality so that the TA can easily read what you write. Take extra care to get a good scan for parts written in pencil.
3. For the MATLAB code you write for an assignment, please copy the code into Word or include a screenshot showing the code. Do not submit handwritten code.

Other things:

1. All plots must have axis labels, with units.
2. Final answers must be boxed, underlined or otherwise differentiated from the rest of solution.
3. All final answers must have units, if they exist.
4. Read the questions carefully.
5. Try to answer all parts of a question together. If the solution to some parts of a question is written elsewhere, then leave a note telling the reader where to find it.