

Homework #7

***Discrete-Time Filtering***

Assigned on Thursday, November 7, 2024

Due on Monday, November 11, 2024, by 11:59 pm via Gradescope submission

*Late homework is subject to a penalty of two points per minute late.*

**Reading:** McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Chapters 8 and 9.  
 Companion Web site with demos and other supplemental information: <http://dspfirst.gatech.edu/>  
 Web site contains solutions to selected homework problems from *DSP First*.

Office hours for Mr. Balti and Prof. Evans follow:

<i>Time Slot</i>	<i>Monday</i>	<i>Tuesday</i>	<i>Wednesday</i>	<i>Thursday</i>	<i>Friday</i>
11:00 am		Evans (ECJ 1.204)		Evans (ECJ 1.204)	
11:30 am		Evans (ECJ 1.204)		Evans (ECJ 1.204)	
12:00 pm		Evans (ECJ 1.204)		Evans (ECJ 1.204)	
12:30 pm					
1:00 pm			Balti (EER 5.652)		
1:30 pm			Balti (EER 5.652)		
2:00 pm	Evans (EER 6.882; <a href="#">Zoom</a> )		Balti (EER 5.652)		Balti (EER 4.650)
2:30 pm	Evans (EER 6.882; <a href="#">Zoom</a> )		Balti (EER 5.652)		Balti (EER 4.650)
3:00 pm	Evans (EER 6.882; <a href="#">Zoom</a> )				Balti (EER 4.650)
3:30 pm			Evans (EER 6.882; <a href="#">Zoom</a> )		Balti (EER 4.650)
4:00 pm			Evans (EER 6.882; <a href="#">Zoom</a> )		
4:30 pm			Evans (EER 6.882; <a href="#">Zoom</a> )		
5:00 pm				Balti (EER 4.702)	
5:30 pm				Balti (EER 4.702)	
6:00 pm				Balti (EER 4.702)	
6:30 pm				Balti (EER 4.702)	

Prof. Evans holds coffee/advising hours on Fridays 12:00-2:00pm in the EER café.

[ECE 313 tutoring](#) is available 7-10pm on Mondays and Thursdays in person.

### 1. Transfer Functions in the $z$ domain. 27 points.

For each of the following linear time-invariant (LTI) systems, derive the transfer function, compute the poles and zeros, and plot the poles and zeros using `zplane`:

- First-order unnormalized averaging filter (lowpass filter):  $y[n] = x[n] + x[n - 1]$  for  $n \geq 0$  and the initial condition  $x[-1] = 0$  to satisfy LTI properties.
- First-order difference filter (highpass filter):  $y[n] = x[n] - x[n - 1]$  for  $n \geq 0$  and the initial condition  $x[-1] = 0$  to satisfy LTI properties. This is a discrete-time approximation to a first derivative in continuous time with respect to time.
- Second-order difference filter (highpass filter):  $y[n] = x[n] - 2x[n - 1] + x[n - 2]$  for  $n \geq 0$  and the initial conditions  $x[-1] = 0$  and  $x[-2] = 0$  to satisfy LTI properties. This is a discrete-time approximation to a second derivative in continuous time with respect to time.

This is a sequel to homework problem 6.1.

### 2. Discrete-Time Averaging Filters. 41 points.

For a discrete-time LTI system with input signal  $x[n]$  and impulse response  $h[n]$ , the output signal  $y[n]$  is the convolution of  $h[n]$  and  $x[n]$ :

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n - k]$$

- Compute the output  $y[n]$  when the input  $x[n]$  is a rectangular pulse of amplitude 1 for  $n = 0, 1, \dots, N_x - 1$  and amplitude 0 otherwise, and  $x[n]$  is filtered by an LTI unnormalized averaging filter whose impulse response  $h[n]$  is a rectangular pulse of amplitude 1 for  $n = 0, 1, \dots, N_h - 1$  and amplitude 0 otherwise. Assume  $N_x \neq N_h$ .
  - Write the difference equation relating output  $y[n]$  and input  $x[n]$ . 3 points.
  - What are the initial conditions and what values should they be set to? 3 points.
  - Develop a formula for  $y[n] = h[n] * x[n]$  using the convolution definition in terms of  $N_x$  and  $N_h$ . Show the intermediate steps in computing the convolution. 6 points.
  - Validate the formula for  $y[n]$  by using Matlab to compute the convolution for  $N_x = 9$  samples and  $N_h = 4$  samples. 5 points.
  - Use the  $z$ -transform to find  $y[n] = h[n] * x[n]$ . Track the region of convergence. 6 points.
- An LTI system outputs the weighted average of the previous output value  $y[n - 1]$  and the current input value  $x[n]$  using the difference equation  $y[n] = 0.9y[n - 1] + 0.1x[n]$  for  $n \geq 0$ .
  - What are the initial conditions and what value should they be set to? 3 points.
  - Compute a formula for the impulse response  $h[n]$  for the system. 3 points.
  - Develop a formula for  $y[n] = h[n] * x[n]$  using the convolution definition when the input signal is  $x[n] = 0.8^n u[n]$ . 6 points.
  - Use the  $z$ -transform to find  $y[n] = h[n] * x[n]$ . Track the region of convergence. 6 points.

### 3. Transfer Function and Frequency Response Connections. 32 points.

*Signal Processing First*, problem P-8.16, page 242. Assume that all filters are LTI.

In addition, for each of the four filters,

- i. give a formula for the transfer function in the z-domain including the region of convergence,
- ii. give a formula for the frequency response from the transfer function in the z-domain in part i. Why can convert
- iii. plot the magnitude response in the frequency domain
- iv. indicate the frequency selectivity as lowpass, highpass, bandpass, or bandstop.

**Connecting filter poles/zeros to its frequency selectivity:** Please see lecture slides 11-6 through 11-11 and watch the recording from our lecture on Oct. 31, 2023, from 1:42 to 39:03, which is available on Canvas.. I have another recording of the same demos on YouTube [video in spring 2014](#) for the Real-Time Digital Signal Processing Lab course from the 1:29 to 22:25 and from 43:01 to the end (50:51). Takeaways from either videorecording:

- When poles and zeros are separated in angle, the angles of the poles near the unit circle indicate the frequencies in the passband(s) and the angles of the zeros near or on the unit circle indicate the frequencies in the stopband(s). Please see lecture slide 11-7.
- Poles must be inside the unit circle for bounded-input bounded-output (BIBO) stability. Please see lecture slides 11-12 and 11-13.