Fall 2025 EE 313 Linear Systems and Signals Prof. Evans

Homework #8

Continuous-Time Frequency Response and Intro to Fourier Transform

Assigned on Monday, November 17, 2024
Due on Friday, November 21, 2024, by 11:59 pm via Gradescope submission

Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Sections 10.1-11.4.
Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from *DSP First*.

Lecture and office hours for Mr. Jacobellis and Prof. Evans follow. Prof. Evans also holds office hours in person in EER 6.882 and online on Zoom.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
11:00 am		**Evans (EER 1.516)**		**Evans (EER 1.516)**	
11:30 am		**Evans (EER 1.516)**		**Evans (EER 1.516)**	
12:00 pm		**Evans (EER 1.516)**		**Evans (EER 1.516)**	
12:30 pm		**Jacobellis (EER 1.810)**			
1:00 pm		**Jacobellis (EER 1.810)**			
1:30 pm		**Jacobellis (EER 1.810)**			
2:00 pm	**Evans (EER 6.882)**	**Jacobellis (EER 1.810)**			
2:30 pm	**Evans (EER 6.882)**				
3:00 pm	**Evans (EER 6.882)**			**Jacobellis (Zoom)**	
3:30 pm			**Evans (EER 6.882)**	**Jacobellis (Zoom)**	
4:00 pm			**Evans (EER 6.882)**	**Jacobellis (Zoom)**	
4:30 pm			**Evans (EER 6.882)**	**Jacobellis (Zoom)**	
5:00 pm				**Jacobellis (Zoom)**	**Jacobellis (EER 1.810)**
5:30 pm					**Jacobellis (EER 1.810)**
6:00 pm					**Jacobellis (EER 1.810)**

ECE 313 tutoring is available on Tuesdays, Thursdays, and Sundays 7-10pm in person.

1. Continuous-Time System Properties. ***48 points.***

Signal Processing First, problem P-9.2, page 279.

2. Continuous-Time Averaging Filters. ***52 points.***

For a continuous-time LTI system with input signal $x(t)$ and impulse response $h(t)$, the output signal $y(t)$ is the convolution of $h(t)$ and $x(t)$:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

(a) Compute the output $y(t)$ when the input $x(t)$ is a rectangular pulse of amplitude 1 for $t \in [0, T_x]$ and amplitude 0 otherwise and $x[n]$ is filtered by an LTI unnormalized averaging filter whose impulse response $h(t)$ is a rectangular pulse of amplitude 1 for $t \in [0, T_h]$ and amplitude 0 otherwise. Assume $T_x \neq T_h$.

i. Write an equation relating output $y(t)$ and input $x(t)$. *6 points*

ii. What is(are) the initial condition(s) and what value should it(they) be set to? *5 points*

iii. Develop a formula for $y(t) = h(t) * x(t)$ using the convolution definition in terms of T_x and T_h. Show the intermediate steps in computing the convolution. *9 points*

iv. Validate the formula for $y(t)$ to compute the convolution for $T_x = 9$ seconds and $T_h = 4$ seconds. *6 points*

(b) When an input signal has an average value of zero, i.e. the DC component is zero, an LTI integrator can be used as an averaging filter. The differential equation governing the input-output relationship is

$$y(t) = \int_{0}^{t} x(\tau)\, d\tau \quad \text{for } t \geq 0$$

i. What is(are) the initial condition(s) and what value should it(they) be set to? *5 points*

ii. What is the impulse response? *6 points*

iii. Develop a formula for $y(t) = h(t) * x(t)$ using the convolution definition when the input signal is $x(t) = u(t)$. Note that $x(t)$ has bounded amplitude. *9 points*

iv. Is the LTI integrator bounded-input bounded-output (BIBO) stable? Your work in part iii might be helpful. *6 points*

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional formatting and content guidelines on the homework page.

Please read the homework guidelines.