Fall 2026 EE 313 Linear Systems and Signals Prof. Evans

Homework #7

Discrete-Time Filtering

Assigned Thursday, November 5, 2026
Due Monday, November 9, 2026, by 11:59 pm via Gradescope submission

Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Chapters 8 and 9.
Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from *DSP First*.

Lecture and office hours for Mr. Jacobellis and Prof. Evans follow. Prof. Evans also holds office hours in person in EER 6.882 and online on Zoom.

Time Slot	*Monday*	*Tuesday*	*Wednesday*	*Thursday*	*Friday*
11:00 am		**Evans (ECJ 1.214)**		**Evans (ECJ 1.214)**	
11:30 am		**Evans (ECJ 1.214)**		**Evans (ECJ 1.214)**	
12:00 pm		**Evans (ECJ 1.214)**		**Evans (ECJ 1.214)**	
12:30 pm					
1:00 pm					
1:30 pm					
2:00 pm		**Evans (EER 6.882 and Zoom)**			**Liu (EER 6.702 and Zoom)**
2:30 pm		**Evans (EER 6.882 and Zoom)**			**Liu (EER 6.702 and Zoom)**
3:00 pm		**Evans (EER 6.882 and Zoom)**	**Liu (EER 5.704 and Zoom)**	**Liu (EER 5.704 and Zoom)**	**Liu (EER 6.702 and Zoom)**
3:30 pm		**Evans (EER 6.882 and Zoom)**	**Liu (EER 5.704 and Zoom)**	**Liu (EER 5.704 and Zoom)**	**Liu (EER 6.702 and Zoom)**
4:00 pm			**Liu (EER 5.704 and Zoom)**	**Liu (EER 5.704 and Zoom)**	
4:30 pm			**Liu (EER 5.704 and Zoom)**	**Liu (EER 5.704 and Zoom)**	

Coffee hours will be held Friday 12:00-2:00pm in EER 2.512, but Prof. Evans will be out of town.

ECE 313 tutoring is available 7-10pm on Mondays and Thursdays in person.

1. Transfer Functions in the z domain. ***27 points.***

For each of the following linear time-invariant (LTI) systems, derive the transfer function, compute the poles and zeros, and plot the poles and zeros using `zplane`:

a) First-order unnormalized averaging filter (lowpass filter): $y[n] = x[n] + x[n-1]$ for $n \geq 0$ and the initial condition $x[-1] = 0$ to satisfy LTI properties.

b) First-order difference filter (highpass filter): $y[n] = x[n] - x[n-1]$ for $n \geq 0$ and the initial condition $x[-1] = 0$ to satisfy LTI properties. This is a discrete-time approximation to a first derivative in continuous time with respect to time.

c) Second-order difference filter (highpass filter): $y[n] = x[n] - 2\,x[n-1] + x[n-2]$ for $n \geq 0$ and the initial conditions $x[-1] = 0$ and $x[-2] = 0$ to satisfy LTI properties. This is a discrete-time approximation to a second derivative in continuous time with respect to time.

This is a sequel to homework problem 6.1.

2. Discrete-Time Averaging Filters. ***41 points.***

For a discrete-time LTI system with input signal $x[n]$ and impulse response $h[n]$, the output signal $y[n]$ is the convolution of $h[n]$ and $x[n]$:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

(a) Compute the output $y[n]$ when the input $x[n]$ is a rectangular pulse of amplitude 1 for $n = 0, 1, \ldots N_x - 1$ and amplitude 0 otherwise, and $x[n]$ is filtered by an LTI unnormalized averaging filter whose impulse response $h[n]$ is a rectangular pulse of amplitude 1 for $n = 0, 1, \ldots N_h - 1$ and amplitude 0 otherwise. Assume $N_x \neq N_h$.

i. Write the difference equation relating output $y[n]$ and input $x[n]$. *3 points.*

ii. What are the initial conditions and what values should they be set to? *3 points.*

iii. Develop a formula for $y[n] = h[n] * x[n]$ using the convolution definition in terms of N_x and N_h. Show the intermediate steps in computing the convolution. *6 points.*

iv. Validate the formula for $y[n]$ by using Matlab to compute the convolution for $N_x = 9$ samples and $N_h = 4$ samples. *5 points.*

v. Use the z-transform to find $y[n] = h[n] * x[n]$. Track the region of convergence. *6 points.*

(b) An LTI system outputs the weighted average of the previous output value $y[n-1]$ and the current input value $x[n]$ using the difference equation $y[n] = 0.9\,y[n-1] + 0.1\,x[n]$ for $n \geq 0$.

i. What are the initial conditions and what value should they be set to? *3 points.*

ii. Compute a formula for the impulse response $h[n]$ for the system. *3 points.*

iii. Develop a formula for $y[n] = h[n] * x[n]$ using the convolution definition when the input signal is $x[n] = 0.8^n\,u[n]$. *6 points.*

iv. Use the z-transform to find $y[n] = h[n] * x[n]$. Track the region of convergence. *6 points.*

3. Transfer Function and Frequency Response Connections. ***32 points*****.**

Signal Processing First, problem P-8.16, page 242. Assume that all filters are LTI.

In addition, for each of the four filters,

i. give a formula for the transfer function in the z-domain including the region of convergence,
ii. give a formula for the frequency response from the transfer function in the z-domain in part i.
iii. plot the magnitude response in the frequency domain
iv. indicate the frequency selectivity as lowpass, highpass, bandpass, or bandstop**.**

Connecting filter poles/zeros to its frequency selectivity: Please see lecture slides 11-6 through 11-11. I have another recording of the same demos on YouTube video in spring 2014 for the Real-Time Digital Signal Processing Lab course from the 1:29 to 22:25 and from 43:01 to the end (50:51). Takeaways from either videorecording:

- When poles and zeros are separated in angle, the angles of the poles near the unit circle indicate the frequencies in the passband(s) and the angles of the zeros near or on the unit circle indicate the frequencies in the stopband(s). Please see lecture slide 11-7.
- Poles must be inside the unit circle for bounded-input bounded-output (BIBO) stability. Please see lecture slides 11-12 and 11-13.