

Mini Project #1: Digital Audio Effects

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Introduction

In this mini project, we explore various audio effects that can be realized digitally with the use of signal generation and synthesis techniques learned throughout the course thus far. We focus on implementing two types of effects, modulation and time shifting, to induce the vibrato, tremolo, and echo audio effects.

Vibrato and tremolo are both implemented through modulation of a signal: vibrato through frequency modulation and tremolo through amplitude modulation. On the other hand, an echo is implemented through a combination of effects including, but not limited to, delay with signal damping.

In each of the following experiments, we begin with a ‘pure tone’, a sinusoidal signal that has constant amplitude, frequency, and phase shift as shown below:

$$x(t) = A\cos(2\pi f_0 t + \theta)$$

We then modify or build upon the pure tone in order to generate the desired audio effects and discuss how well our digital recreation of the audio effect performs.

Modulation Effects

For modulation effects, we focus on two types of effects: frequency modulation and amplitude modulation. In this section, we examine how modulation of a signal’s frequency or amplitude changes the pitch and volume of a signal. First, we implement frequency modulation by periodically changing the phase of a sinusoidal signal, and then we implement amplitude modulation by periodically changing the amplitude of a sinusoidal signal.

Vibrato – Frequency Modulation

Vibrato, a changing in the pitch that is heard, is implemented through frequency modulation. We modulate the frequency of a pure tone by replacing the constant phase shift, θ , in a sinusoidal signal with a periodic phase shift, $\theta(t)$, where:

$$\theta(t) = \frac{\Delta f}{f_\tau} \cos(2\pi f_\tau t)$$

Thus, the resulting frequency modulated pure tone is:

$$y(t) = \cos(2\pi f_0 t + \theta(t)) = \cos(2\pi f_0 t + \frac{\Delta f}{f_\tau} \cos(2\pi f_\tau t))$$

From these equations used to generate the vibrato effect from a pure tone, we can see that the frequency modulation is dependent on two main parameters, Δf and f_τ , which we will examine in greater detail in the following experiments.

For our experiment, we choose to generate the initial pure tone from a note on the Western scale in the fourth octave, such as A4, which gives us a frequency of $f_0 = 440 \text{ Hz}$. To generate the samples for the tone, we use the given sampling rate of $f_s = 48 \text{ kHz}$, which satisfies the Sampling Theorem for our chosen pure tone.

We tested out various values for Δf and f_τ to see their effects on the resulting frequency modulated signal in audio. For a fixed f_τ , as we increase Δf , we hear greater variation in the pitch. Conversely, if Δf is small, then we hear less variation in the pitch, meaning that the signal with vibrato sounds similar in pitch to the signal without vibrato. Now, if we keep Δf constant and adjust f_τ , we hear that for larger f_τ , the change in pitch occurs much more frequently. And conversely, for a smaller f_τ , the pitch changes slowly over time.

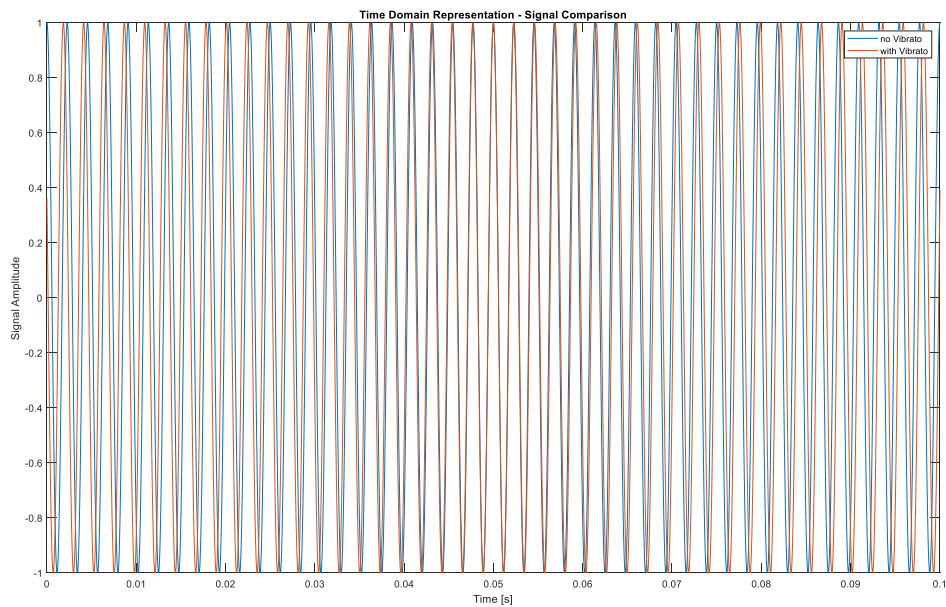
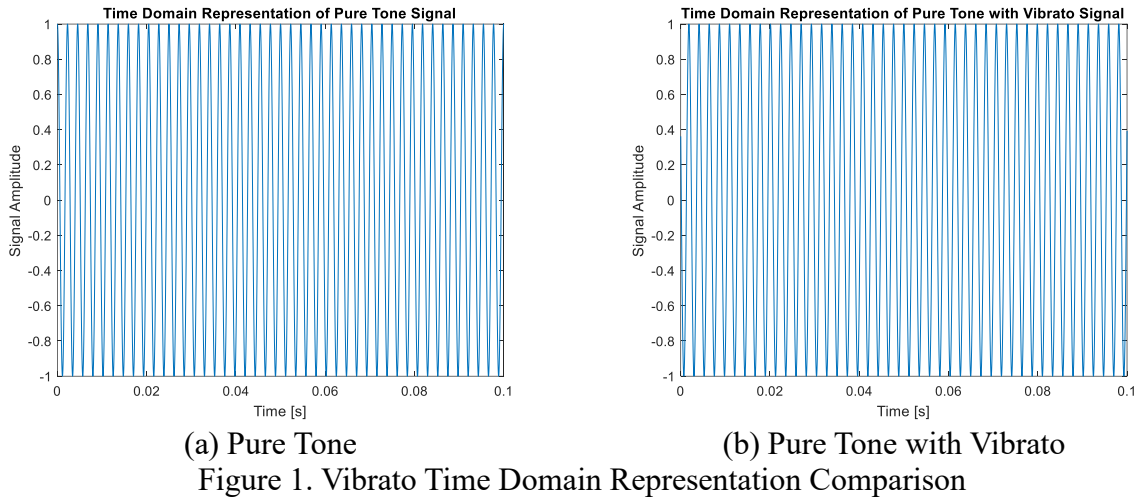
These observed audio changes can be derived from the instantaneous frequency of the vibrato signal. We compute the instantaneous frequency, $f_I(t)$, using the following steps below:

$$\begin{aligned} f_I(t) &= \frac{1}{2\pi} \frac{d}{dt} (2\pi f_0 t + \theta(t)) \\ f_I(t) &= \frac{1}{2\pi} \frac{d}{dt} (2\pi f_0 t + \frac{\Delta f}{f_\tau} \cos(2\pi f_\tau t)) \\ f_I(t) &= \frac{1}{2\pi} \frac{d}{dt} (2\pi f_0 t) + \frac{1}{2\pi} \frac{d}{dt} (\frac{\Delta f}{f_\tau} \cos(2\pi f_\tau t)) \\ f_I(t) &= \frac{1}{2\pi} (2\pi f_0) + \frac{1}{2\pi} \frac{\Delta f}{f_\tau} (\frac{d}{dt} \cos(2\pi f_\tau t)) \\ f_I(t) &= f_0 - \frac{1}{2\pi} \frac{\Delta f}{f_\tau} 2\pi f_\tau \sin(2\pi f_\tau t) \\ f_I(t) &= f_0 - \Delta f \sin(2\pi f_\tau t) \end{aligned}$$

In our derivation, we introduce a multiplicative constant $\frac{1}{2\pi}$ in order to get the instantaneous frequency in Hz rather than in radians. From the equation for $f_I(t)$, we see that Δf controls the maximum range of frequencies deviating from f_0 and that f_τ controls how fast the frequency deviates over time. The derived analytical results match the observations which we heard.

Now, we compare our original pure tone signal against the frequency modulated vibrato signal in the time, frequency, and spectrogram analyses. For these analyses, we examine the vibrato effect for $\Delta f = 6 \text{ Hz}$ and $\Delta f = 5 \text{ Hz}$.

When comparing the time domain plots for the signals in Figure 1, we can't really see significant differences between the two signals. However, if we overlay the plots for the pure tone in Figure 1(a) and the vibrato signal in Figure 1(b) together, we can see in Figure 2 how the frequency of the vibrato signal changes with time in contrast to the constant spacing of the pure tone signal.



To compare the frequency domain plots, we examine only the positive frequencies since the negative frequencies contain the same content, but mirrored. In this analysis, we see that the pure tone signal contains just one frequency component at $f_0 \cong 440 \text{ Hz}$ in Figure 3(a). However, for the signal with vibrato effect in Figure 3(b), we see that multiple frequency components are now present, centered around $f_0 \cong 440 \text{ Hz}$ and spaced apart roughly by $\Delta f \cong 6 \text{ Hz}$. These additional frequency components result from the phase shift that induces the frequency modulation.

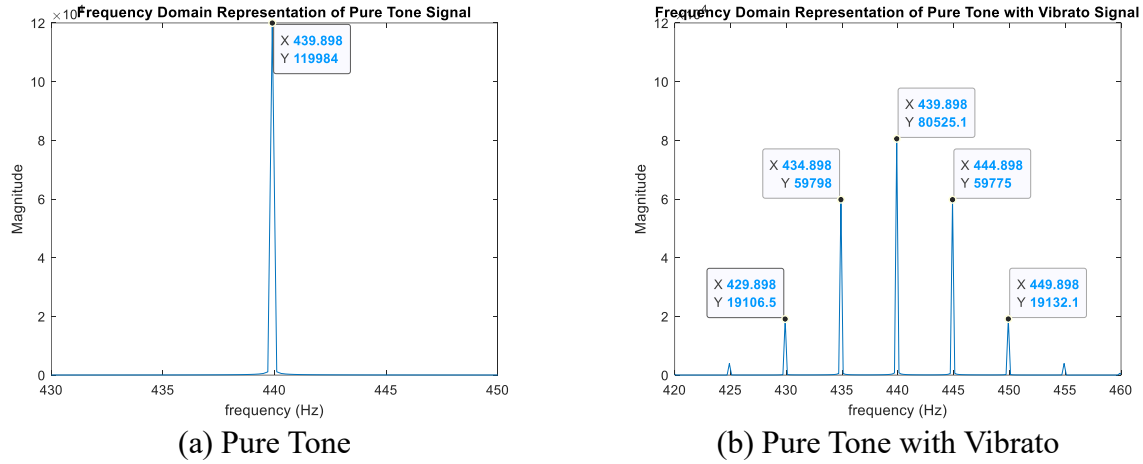


Figure 3. Vibrato Positive Frequency Domain Representation Comparison

And for the spectrogram comparison, we see similar characteristics that we observed for the frequency domain analysis. For the pure tone signal in Figure 4(a), we see a constant frequency at $f_0 = 440 \text{ Hz}$ for all 5 seconds of the signal duration. Likewise, for the frequency modulated signal spectrogram in Figure 4(b), we see that the frequency content spreads around $f_0 = 440 \text{ Hz}$. Furthermore, by improving the resolution of the spectrogram analysis as shown in Figure 5, we can see the frequency of the vibrato signal change over time according to f_t , which we derived the instantaneous frequency for earlier.

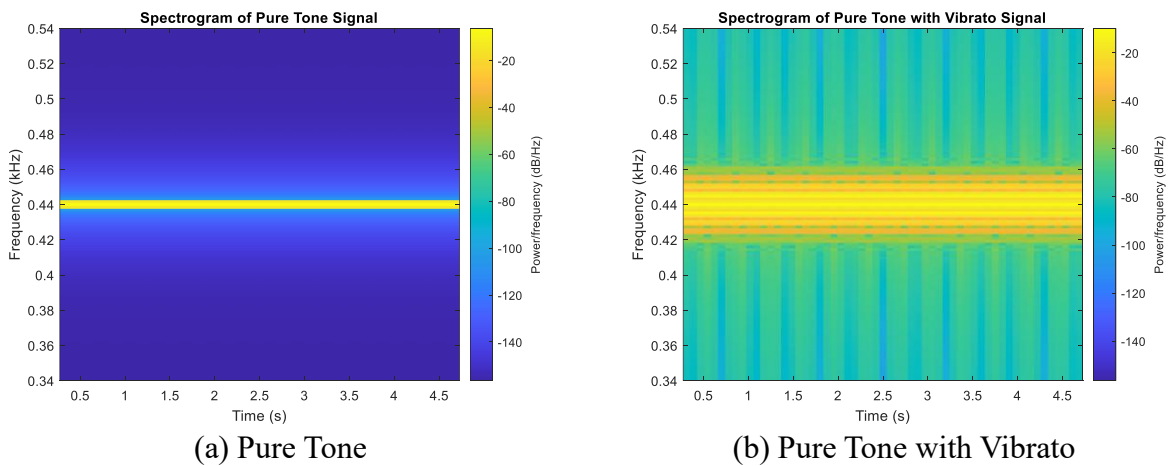


Figure 4. Vibrato Spectrogram Comparison

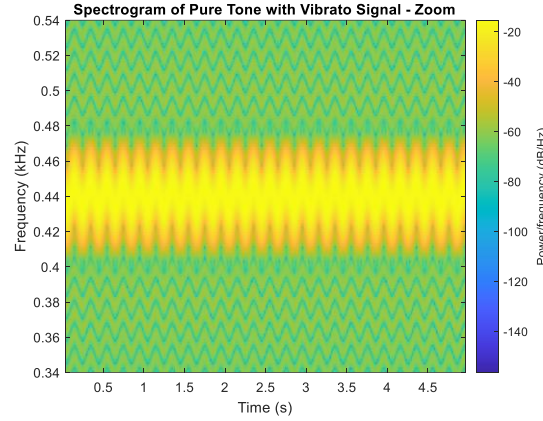


Figure 5. Vibrato Spectrogram – Higher Resolution

Tremolo – Amplitude Modulation

We now examine our second modulation audio effect, tremolo. In this mini project, tremolo refers to the oscillation of the volume of a signal, which can be achieved through amplitude modulation. We modulate the amplitude by multiplying the signal of interest, $x(t)$, with a pulse-shaping signal, $p(t)$. For our implementation, we use the sinusoidal pulse-shaping signal below:

$$p(t) = A \cos(2\pi f_t t)$$

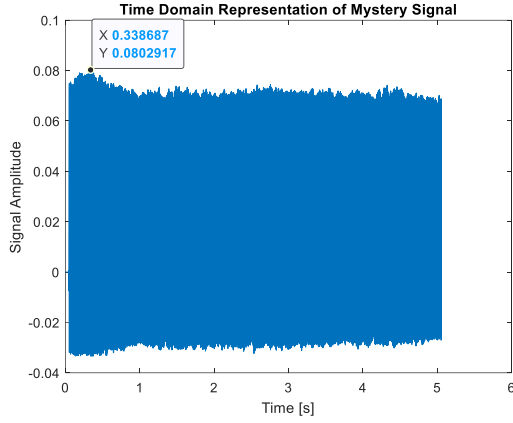
Then, we combine $x(t)$ and $p(t)$ to obtain the amplitude modulated signal:

$$y(t) = p(t)x(t) = A \cos(2\pi f_t t) \cos(2\pi f_0 t)$$

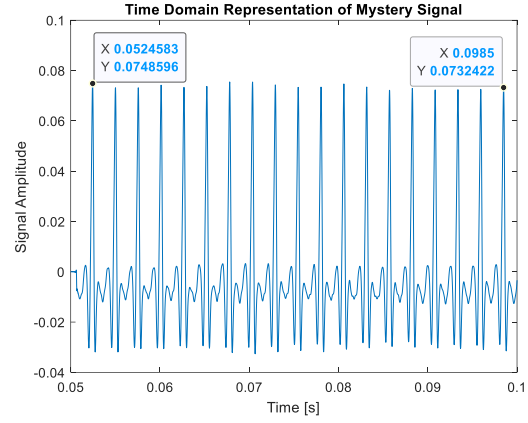
To better understand the implementation and effects of tremolo, we apply the amplitude modulation onto a pure tone signal as well as a mystery signal. With these comparisons, we can also compare how a generated tone is different from a recorded audio signal.

First, we analyze our unknown mystery signal to determine its frequency components that we will later use to implement our pure tone signal.

From Figure 6(a), we determine the signal lasts about 5 seconds and reaches a maximum amplitude value of 0.0803. If we zoom into the time domain representation shown in Figure 6(b) and estimate the period at a certain chunk of the signal, we compute that the mystery signal has a period of about $T_0 = 0.0026$ s and a corresponding fundamental frequency of about $f_0 = 390.96$ Hz. When we compare our time domain analysis against the frequency domain representation of the signal in Figure 7(a), we see that the strongest frequency component lies at $f_0 = 391.53$ Hz, which closely matches our time domain estimate. Thus, we can conclude that the note played in the mystery signal is likely G4 on the Western scale, that is: $f_0 = 392$ Hz.



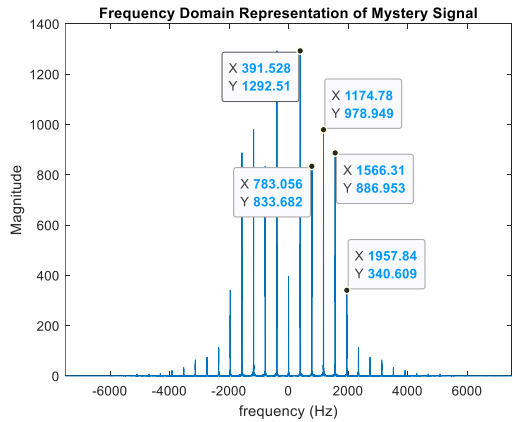
(a) Full Plot



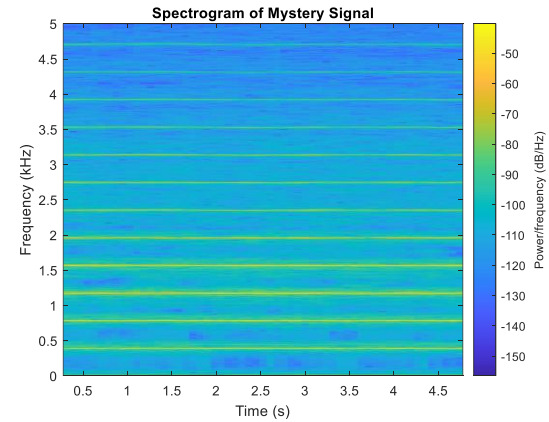
(b) Zoom-in Plot

Figure 6. Time Representation Analysis of Mystery Signal for Tremolo

Both the frequency and spectrogram representations in Figures 7(a) and 7(b) show frequency components equally spaced apart, indicating that the additional frequency components likely occur at integer multiples of the fundamental frequency and are harmonic frequencies. We verify our findings by comparing against integer multiples of the estimated fundamental frequency, f_0 , in Table 1. From our results, we see that the additional frequency components are indeed the next four harmonics: 783.06 Hz, 1174.59 Hz, 1566.12 Hz, and 1957.65 Hz.



(a) Frequency Domain Plot



(b) Spectrogram

Figure 7. Frequency Representation Analysis of Mystery Signal for Tremolo

Table 1 Comparison of Frequency Components in Mystery Signal for Tremolo

Identified Frequencies	Computed Frequencies ($f_k = k f_0$)
$f_1 = 391.53 \text{ Hz}$	$f_1 = 1f_0 = 391.53 \text{ Hz}$
$f_2 = 783.06 \text{ Hz}$	$f_2 = 2f_0 = 783.06 \text{ Hz}$
$f_3 = 1174.78 \text{ Hz}$	$f_3 = 3f_0 = 1174.59 \text{ Hz}$
$f_4 = 1566.31 \text{ Hz}$	$f_4 = 4f_0 = 1566.12 \text{ Hz}$
$f_5 = 1957.85 \text{ Hz}$	$f_5 = 5f_0 = 1957.65 \text{ Hz}$

Having analyzed the mystery signal, we can now generate our pure tone signal using the determined fundamental frequency of $f_0 = 392 \text{ Hz}$ and the obtained sampling rate $f_s = 48 \text{ kHz}$:

$$x(t) = \cos(2\pi f_0 t) = \cos(2\pi(392 \text{ Hz})t)$$

We generate a sinusoidal pulse-shaping signal with an amplitude of $A = 0.0803$ to match with maximum amplitude observed in the mystery signal and an arbitrarily chosen $f_\tau = 4 \text{ Hz}$:

$$p(t) = A \cos(2\pi f_\tau t) = 0.0803 \cos(2\pi(4 \text{ Hz})t)$$

We did also vary the values chosen for A and f_τ to examine their effects on the audio signal. The value of $|A|$ is directly tied to the amplitude of the signal, or the volume of the audio. As $|A|$ increases, the volume heard also increases, and vice versa. However, if $|A|$ becomes too large, such that $|A| > 1$, the audio begins to distort due to clipping.

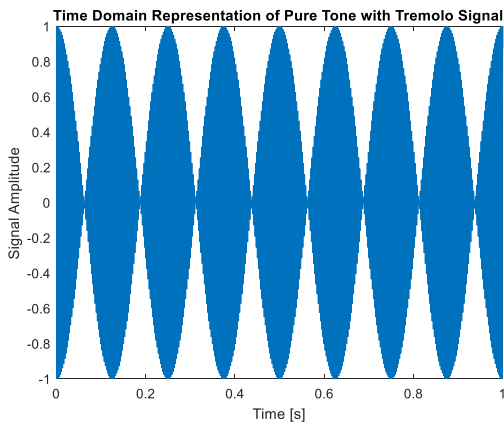
After applying our pulse-shaping signal to the pure tone signal, we analyzed the effects in the time domain representations. From the plot in Figure 8(a), it seems like the modulated pure tone signal oscillates with a fundamental period of 0.125 seconds . However, when we compute the theoretical fundamental period, we find that $T_{\text{fundamental,pure}} = 0.25 \text{ seconds}$.

The theoretical fundamental period is computed from the greatest common divisor of the present frequency components. In our pure tone signal with tremolo effect, we identify two frequency components after rewriting our amplitude modulated signal, $y_{\text{pure}}(t)$, as:

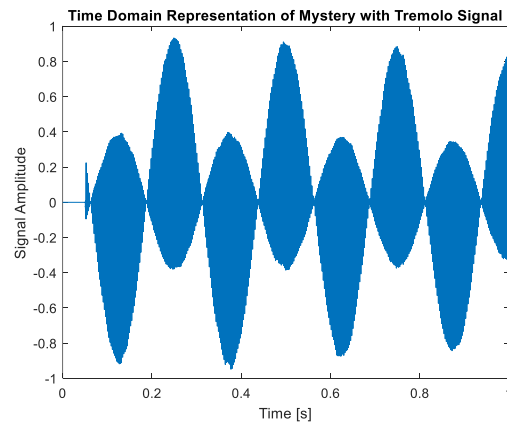
$$y_{\text{pure}}(t) = A \cos(2\pi f_\tau t) \cos(2\pi f_0 t) = \frac{A}{2} \cos(2\pi(f_0 + f_\tau)t) + \frac{A}{2} \cos(2\pi(f_0 - f_\tau)t)$$

$$y_{\text{pure}}(t) = 0.0401 \cos(2\pi(396 \text{ Hz})t) + 0.0401 \cos(2\pi(388 \text{ Hz})t)$$

Thus, the greatest common divisor of 388 and 396 is 4, which gives our signal a fundamental frequency of 4 Hz and a fundamental period of 0.25 seconds . This is different from what I expected, because I was misled by the prominent envelope of the amplitude modulation.



(a) Pure Tone with Tremolo



(b) Mystery Signal with Tremolo

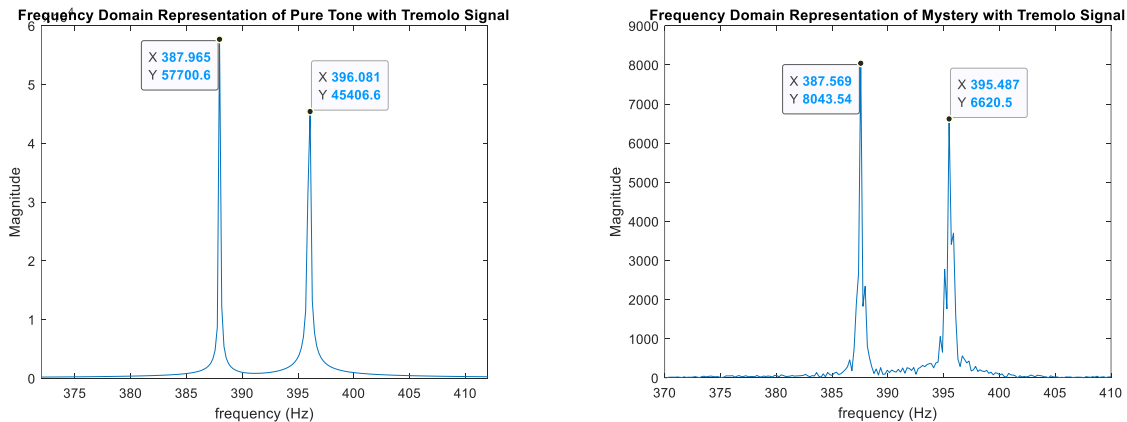
Figure 8. Tremolo Time Domain Representation Comparison

Having applied the tremolo effect to a pure tone signal, we now do the same for the mystery signal. In order to normalize the amplitude to 1 and ensure no audio distortion, we set the pulse-shaping signal for the mystery signal to have an amplitude of $A = 12$, and use the same $f_\tau = 4 \text{ Hz}$. We can now compare the similarities and differences of the performance for the pure tone signal with tremolo against the mystery signal with tremolo.

When we play the modulated pure tone and mystery signals, although we hear that both signals oscillate in volume, we note that the mystery signal sounds much richer compared to the more electronic sounding pure tone. This difference is due to the pure tone having only a single frequency component that is modulated by amplitude, whereas the mystery signal has many frequencies, fundamental and harmonic, that are modulated.

In the time domain analysis, we see that the amplitude of both signals with tremolo oscillate at the same rate of f_τ . The pure tone signal spans the full range of amplitude values from $A = [-1, 1]$. However, we see in Figure 8(b) that the mystery signal with tremolo does not span the full range of amplitude values and is asymmetric about the y-axis because of the variation present in the original recording.

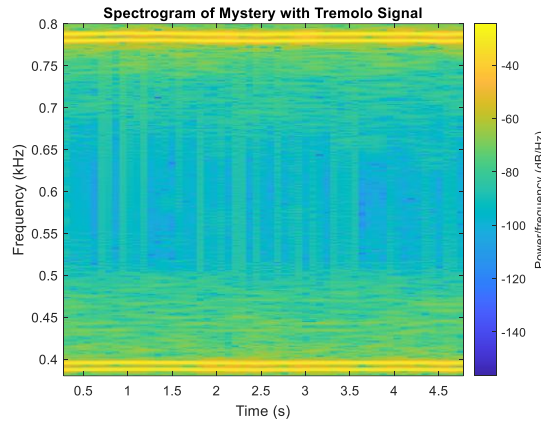
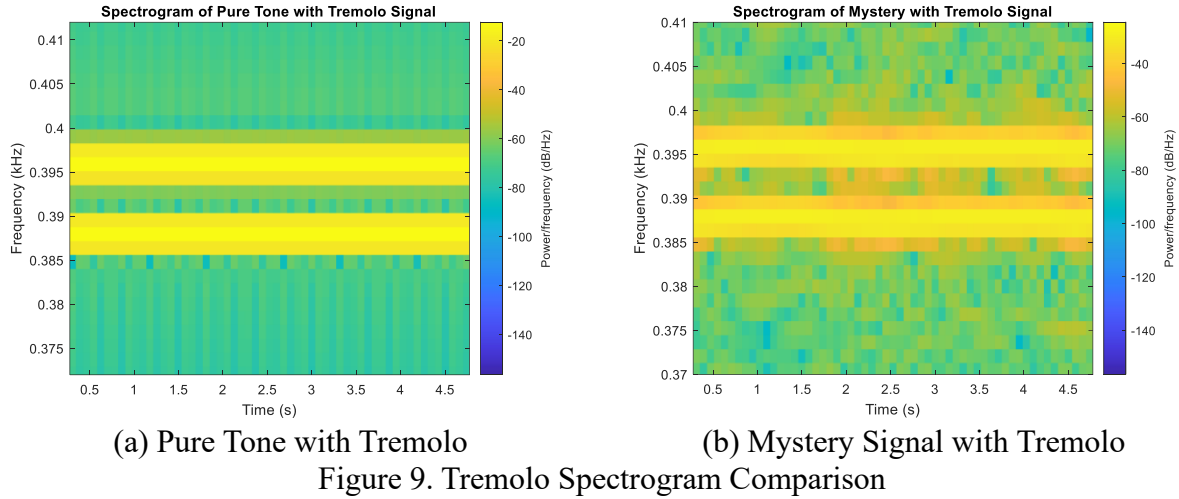
In the positive frequency domain analysis, we see in Figure 9 that both signals generate 2 frequency components centered around 392 Hz spaced about 4 Hz away. However, the modulated pure tone in Figure 9(a) shows clear peaks at 388 Hz and 396 Hz , but the modulated mystery signal shows artifacts surrounding those peaks. This behavior is also seen in the spectrogram, where the modulated pure tone shows a clean spectrum in Figure 10(a) but the modulated mystery signal shows a noisier spectrum in Figure 10(b). This difference is due to the harmonics of the fundamental frequency present in the original mystery signal. Our pure tone only has a singular frequency component, so even after amplitude modulation, the frequency representation remains very clean due to the lack of additional frequency components. In Figure 11, we can zoom out our spectrogram analysis of the modulated mystery signal to show that each of its harmonics also are affected by the amplitude modulation.



(a) Pure Tone with Tremolo

(b) Mystery Signal with Tremolo

Figure 9. Tremolo Positive Frequency Domain Representation Comparison

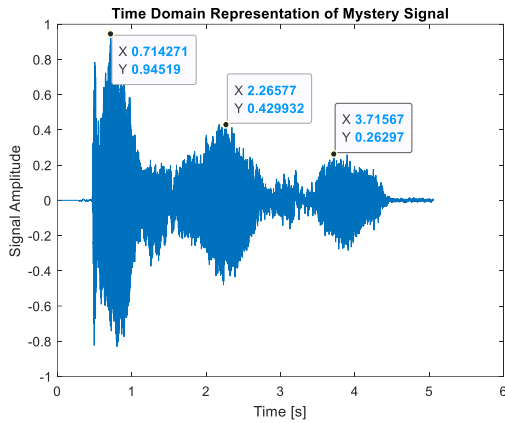


Time Shift Effects

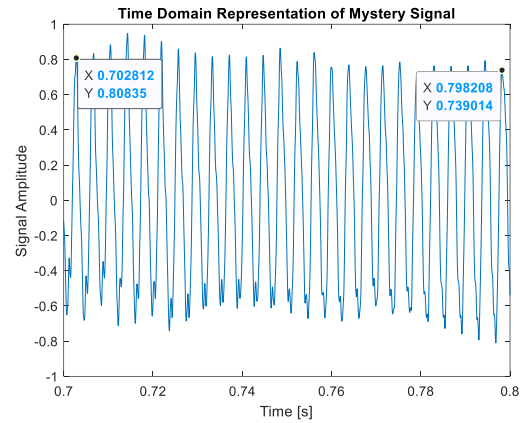
Previously we implemented and examined the effects of two types of modulation to generate different audio effects. In this section, we consider another effect: echo, which utilizes time shifting and signal damping to manipulate a pure tone signal.

For this experiment, we analyze a realistic echoing mystery signal, which we then use as a model for our simulated echoing pure tone.

From Figure 11(a), we can see that the echoing mystery signal contains three distinct pulses with peaks at 0.71s, 2.27s, and 3.72s. The first pulse is the original signal, and the remaining two pulses are echoes that we can also hear in the audio playback. If we zoom into the time domain representation in Figure 11(b), we can estimate the fundamental frequency of the mystery signal to be 259.35 Hz. However, if we examine the frequency domain plot of the mystery signal in Figure 12(a), we can more clearly pinpoint the fundamental frequency of the signal to be $f_0 = 261.48$ Hz, which corresponds to the note C4 on the Western scale, that is: $f_0 = 261$ Hz.



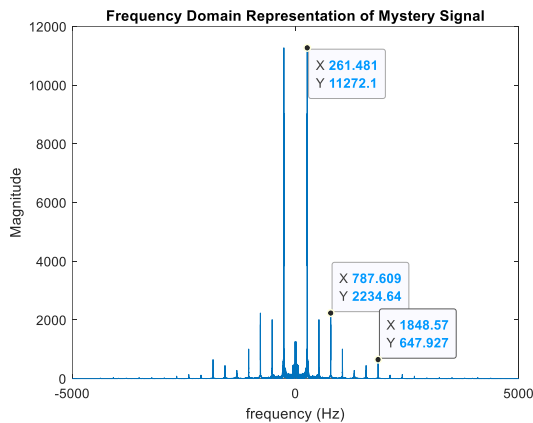
(a) Full Plot



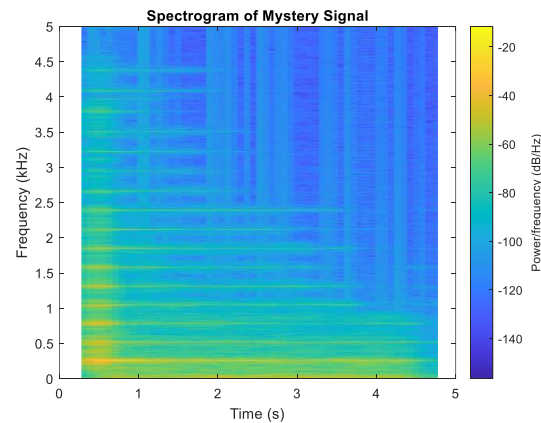
(b) Zoom-in Plot

Figure 11. Time Representation Analysis of Mystery Signal for Echo

The frequency and spectrogram plots in Figure 12(a) and Figure 12(b) both show the spread of frequencies up to ± 5 kHz, but it seems that most frequency components die out after ± 2 kHz. Based on the constant spacing between each frequency component, it is fair to assume that these additional frequency components are likely integer multiples of the fundamental frequency and thus harmonic frequencies. For a more rigorous analysis, we could compare the identified frequency components in Figure 12(a) against integer multiples of the fundamental frequency.



(a) Frequency Domain Plot



(b) Spectrogram

Figure 12. Frequency Representation Analysis of Mystery Signal for Echo

We will now build our simple model of the echoing mystery signal from a pure tone in two steps: first we use time shifting to generate a delayed signal that serves as an echo to the original pure tone, and second, we apply a damping signal so that the resulting time shifted pure tone decays over time just as an echo would.

Before we can obtain a delayed copy of the pure tone, we need to generate the pure tone itself. We use the determined fundamental frequency of $f_0 = 261 \text{ Hz}$ and the obtained sampling rate $f_s = 48 \text{ kHz}$ for our pure tone:

$$x(t) = \cos(2\pi f_0 t) = \cos(2\pi(261 \text{ Hz})t)$$

Now, we can generate a delayed, or time shifted, copy of our original pure tone signal. We choose a delay of $t_1 = 1.55 \text{ seconds}$ for our “echo” in order to match with the second pulse at its maximum amplitude. To implement this delay in the MATLAB code, we need to convert our delay from seconds to samples using the sampling rate and rounding to the nearest integer sample. Thus, our delay of 1.55 seconds becomes 74472 samples . We then combine our delayed signal with the original pure tone to obtain our first attempt at generating an echo effect from a pure tone:

$$y_1(t) = x(t) + x(t - t_1) = x(t) + x(t - 1.55s)$$

Our first attempt sounds nothing like an echo, much less like the echoing mystery signal example. Instead of hearing the initial signal fade away with softer repetitions, we in fact hear the opposite: a sustained tone with a louder, distorted repetition. The distortion in the audio occurs because we have chosen a time shift that constructively combines with the initial pure tone and causes our signal amplitude to increase above the clipping threshold value of 1, which can be seen in Figure 13. The time domain plot also supports our claim that the simulated echoing signal does not decay because we see that the amplitude is sustained throughout the 5 seconds. The poor results in our first attempt demonstrate the need to apply signal damping.

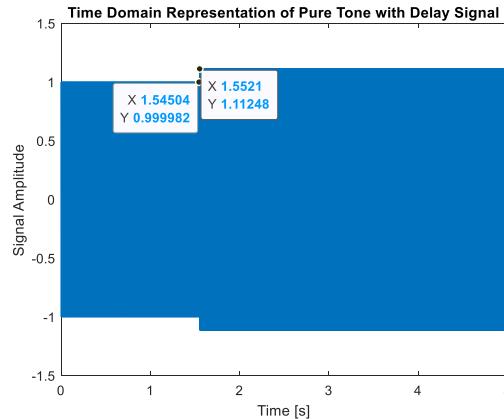


Figure 13. Echo Recreation – Pure Tone with Delay Time Domain Plot

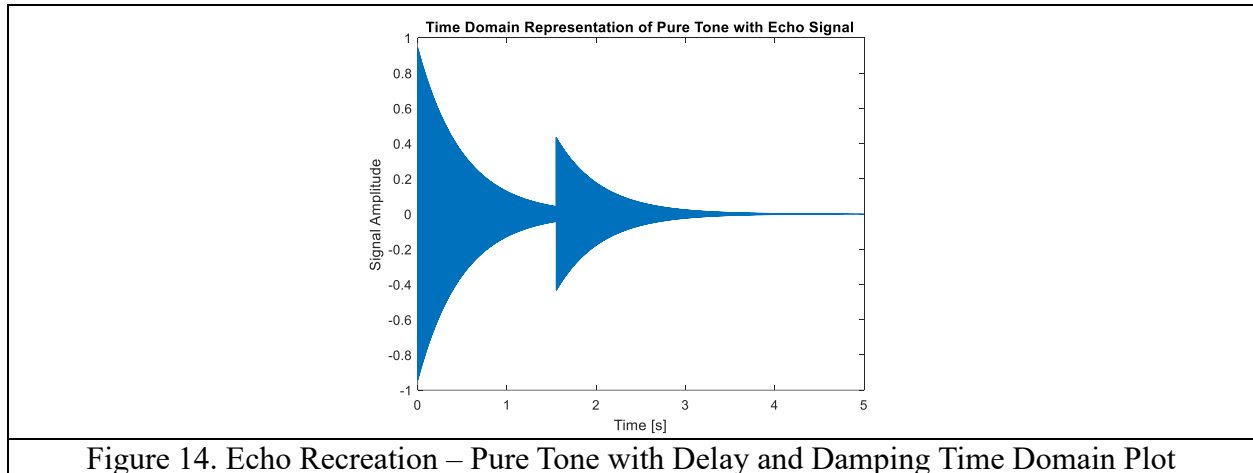
In our second step, we build upon our first attempt of the echoing pure tone by generating and applying a damping signal to each of the signal copies present in $y_1(t)$. To roughly model the signal fading away, we use the exponentially decaying function below:

$$p(t) = Ae^{-kt}$$

To improve our initial attempt at the echoing pure tone, we generate damping signals customized to each copy of the pure tone signal. For the initial copy, we choose $A = 0.95$ to match with the maximum amplitude of the first pulse from the mystery signal, and for the time shifted copy we choose $A = 10$ to account for the signal decay at time t_1 and to reach the maximum amplitude of the second pulse (first echo pulse) from the mystery signal. We chose $k = 2$ to model the signal amplitude decay for both copies of the signal because it generated a more realistic sounding “echo” in the audio playback. By carefully choosing the values for A and k that account for signal decay over time and potential constructive combining of the signal, we can avoid the audio distortion present in our first attempt. With these improvements, we see our second attempt at an echoing pure tone:

$$y_2(t) = p(t)x(t) + p_1(t)x(t - t_1) = 0.95e^{-2t}x(t) + 10e^{-2t}x(t - 1.55s)$$

The second attempt sounds much more like I expect for an “echo”. The audio signal decays with time, and the repetitions of the initial signal come in softer. Our observations are supported by the time domain representations of the signal’s amplitude in Figure 14(a). However, there are still some stark differences between our echoing pure tone signal and the echoing mystery signal. One main difference is that the mystery signal contains two echoes, but our simulated signal only contains a singular echo. Additionally, the mystery signal echo sounds much richer because it contains multiple harmonic frequencies, whereas our echoing pure tone is generated with only a single frequency component.



The differences discussed in our comparison highlight that our simple model for generating an echoing pure tone could be much improved. To enhance our simulated “echo” signal and better match it to the mystery signal, I would recommend the following:

1. Adding harmonic frequencies that were present in the mystery signal, rather than relying on a single frequency component.
2. Adding a third “echo”.

3. Delaying the start of the simulated signal until 0.5 seconds, which is when the mystery signal starts to play.
4. Use different damping or pulse-shaping signals that are fitted to the shape of the mystery signal pulses, rather than using an exponential decay which makes for a very abrupt “echo”.

Conclusion

Through the course of this mini project, we have used techniques learned in the course such as time and frequency domain analysis, sinusoidal generation, signal synthesis and more to implement various audio effects digitally. We explored vibrato, a frequency modulation audio effect that changes the pitch of a signal over time with a phase shift. This frequency modulation technique depends on two key parameters, Δf and f_t , which control how much the pitch deviates from the center frequency and how quickly the pitch changes. For the tremolo effect, we implemented amplitude modulation by applying a pulse shaping signal to the signal of interest. The pulse shaping signal that amplitude modulation relies on is defined by the A and f_t parameters that control how much the amplitude changes and how quickly. We also discovered that should the amplitude be set too high, audio distortion will occur due to clipping of the signal values. Our last audio effect of interest focused on generating an echo from a pure tone that utilized time shifts and damping signals. By combining delayed copies of a pure tone and ensuring the signal copies decay over time by applying a damping signal, we model a simple echoing signal. We found that our audio effects could be much improved by incorporating additional harmonics to make our simulated sound richer. At the end of this mini project, we have gained hands-on experience generating and modifying signals to produce a variety of audio effects.

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