Blind Estimation of FIR Channels in CDMA Systems with Aperiodic Spreading Sequences

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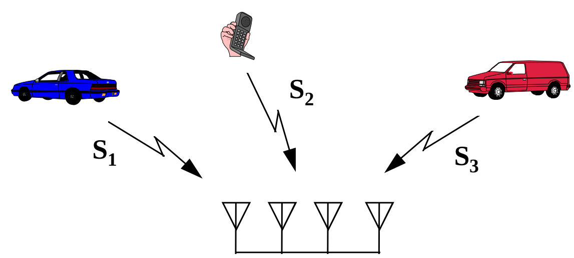
Motivations

- In multipath environment, CDMA receivers often suffer from interference.



- Blind signal estimation schemes cannot be used because they require periodic spreading sequences.
- RAKE receivers are often used, but they cannot fully exploit the rich structure of CDMA signals to minimize interference.

Background: Channel Model

Multipath channel model between the ith user and the M-element antenna array at the base station

$$\mathbf{h}_i(t) = \begin{bmatrix} h_{1,i}(t) \\ \vdots \\ h_{M,i}(t) \end{bmatrix} = \sum_{l=1}^{L_i} \mathbf{a}_i(\theta_l) p(t - \tau_i(l))$$

- $p(t)$ is the pulse shaping function
- $\tau_i(l)$ is the delay
- $\mathbf{a}_i(\theta_l)$ is the array response vector of the lth multipath signal
- L_i is the total number of paths of ith user

Background: Data Model

The baseband signals from the antenna outputs of an asynchronous CDMA system with P users

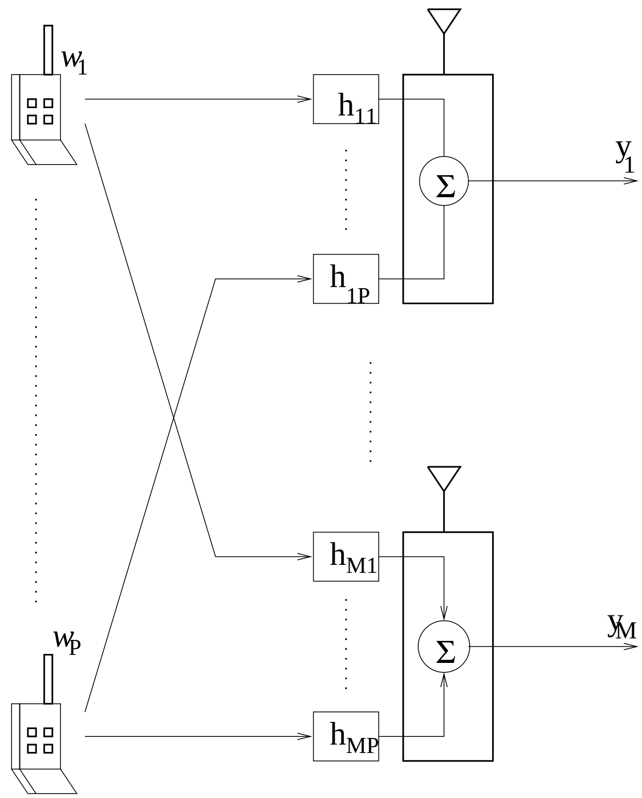
$$\mathbf{y}(t) = \sum_{i=1}^{P} \sum_{n=-\infty}^{\infty} w_i(n)\, \mathbf{h}_i(t - nT) + \mathbf{v}(t)$$

- T is the chip period
- $\mathbf{v}(t)$ is the noise vector
- $w_i(k) = s_i(n) c_i(k - nL_c - k_i)$
- $n = \lfloor \frac{k-k_i}{L_c} \rfloor;\ k_i$ ($0 \leq k_i < L_c$) is the chip delay index assumed to be known.

We know

- channels are FIR
- Source symbols are drawn from a finite alphabet
- Pseudo-noise (PN) spreading codes

Channel Model in a P-User CDMA System with an M-element Antenna Array



Blind Estimation Problem in CDMA Systems

- The blind estimation problem is to estimate FIR channel parameters without the use of training sequences.
- Many subspace-based algorithms have been successfully developed on multiuser FIR channel estimation in CDMA systems with periodic spreading sequences.
- These algorithms rely on the periodicity of spreading sequences in order to estimate the channel parameters.
- Furthermore, periodicity also simplify the use of multi-user detection techniques.
- **However** these algorithms are only applicable to CDMA systems with periodic spreading sequences.

Why Aperiodic Spreading Sequences?

Many practical systems such as IS-95 use aperiodic spreading sequences to

- achieve uniform signal spectrum
- identify cell cites uniquely
- obtain other desirable properties.

A few algorithms have been developed on channel estimation to be used in such systems

Here, we propose a method which provides promising signal estimates using the inherent structure information of CDMA signals.

Channel Estimation Methods

Current Methods in CDMA sytems with aperiodic spreading sequences

- 1-D RAKE receivers
- 2-D RAKE receivers (with an antenna array)
- Principal Component (PC) Algorithm proposed by Liu and Zoltowski

Our method

- Uses two different frameworks to capture the rich structure of CDMA signals

First Framework at Chip Level

In the first framework, we construct the data matrix of the signal sampled at the chip rate

$$\mathbf{Y} = \mathbf{H}\mathbf{W} = \begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{h}_P \end{bmatrix} \begin{bmatrix} \mathbf{w}_1(N) \\ \vdots \\ \mathbf{w}_P(N) \end{bmatrix}$$

- $\mathbf{h}_i = [\mathbf{h}_i(L-1)\ \mathbf{h}_i(L-2)\ \cdots\ \mathbf{h}_i(0)]$
- $\mathbf{w}_i$ is constructed as

$$\begin{bmatrix} w_i(1) & w_i(2) & \cdots & w_i(NL_c - L + 1) \\ w_i(2) & w_i(3) & \cdots & w_i(NL_c - L + 2) \\ \vdots & \vdots & \cdots & \vdots \\ w_i(L) & w_i(L+1) & \cdots & w_i(NL_c) \end{bmatrix}$$

Solution for the above equation

$$\boxed{\mathbf{H} = \mathbf{Y}\mathbf{W}^{\dagger}}$$

Most of $\mathbf{W}$ is known due to the known PN spreading sequence.

Second Framework at Symbol level

In the second framework, we stack the spatial data samples so that the data matrix

$$\mathcal{Y} = \left[\begin{array}{cccc} \mathbf{y}^1 & \mathbf{y}^2 & \cdots & \mathbf{y}^M \end{array} \right]^{\mathcal{T}}$$

can be represented as

$$\mathcal{Y} = \underbrace{\left[\begin{array}{cccc} \mathcal{G}_1 & \mathcal{G}_2 & \cdots & \mathcal{G}_P \end{array} \right]}_{\mathcal{G}} \underbrace{\left[\begin{array}{c} \mathbf{s}_1^T \\ \mathbf{s}_2^T \\ \vdots \\ \mathbf{s}_P^T \end{array} \right]}_{\mathbf{S}}$$

- $\mathbf{s}_i = [s_i(1) \ \cdots \ s_i(N)]$.
- $\mathcal{G}_i = \underbrace{\left[\begin{array}{ccc} \mathcal{C}_i & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathcal{C}_i \end{array} \right]}_{M \text{ blocks}} \left[\begin{array}{c} \mathcal{H}_i^1 \\ \vdots \\ \mathcal{H}_i^M \end{array} \right]$

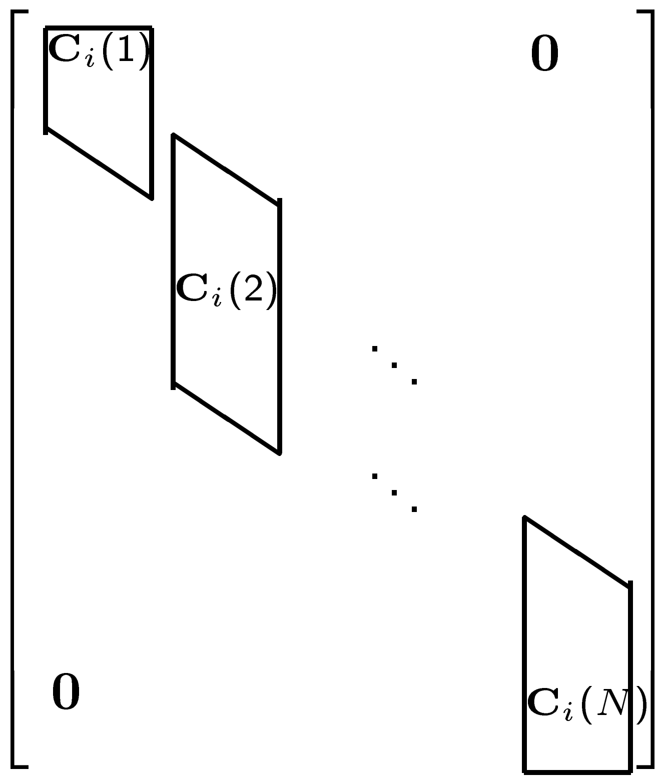
Definition of Channel and Kernel Matrices

Define the channel matrix

$$\mathcal{H}_i^m = \left[\begin{array}{ccc} h_1^m(L-1) & \cdots & h_1^m(0) \end{array} \right]$$

The kernel matrix $\mathcal{C}_i$ is defined as the shifted blocks of the PN spreading sequences

$$\mathcal{C}_i =$$



- Each block covers one symbol period and has Toeplitz structure
- Upper and lower blocks may be also partial because of the asynchronous operation of the uplink.

Definition of $\mathrm{C}_i(n)$ and Solution for S

A complete block of $\mathbf{C}_i(n)$ $n = 0, \ldots, N-1$ can be written as

$$\underbrace{\left.\begin{bmatrix} 0 & \cdots & 0 \\ \cdots & \ddots & \cdots \\ c_i(nL_c+1) & \ddots & 0 \\ \vdots & \ddots & c_i(nL_c+1) \\ c_i(nL_c+L_c) & \ddots & \vdots \\ 0 & \ddots & c_i(nL_c+L_c) \\ \vdots & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}\right\} 2L_c}_{L \text{ columns}}$$

If we use the equation $\mathcal{Y} = \mathcal{G}\mathbf{S}$ to solve for $\mathbf{S}$, then we get

$$\boxed{\mathbf{S} = \mathcal{G}^{\dagger}\mathcal{Y}}$$

Definition Of The Algorithm

- $\mathbf{H} = \mathbf{Y}\mathbf{W}^\dagger$ and $\mathbf{S} = \mathcal{G}^\dagger\mathcal{Y}$ allow us to use both frameworks to exploit the discrete-alphabet property of CDMA signals and knowledge of spreading codes.
- We adopt an Iterative Least Squares with Projection (ILSP) algorithm originally developed by Talwar, Viberg, and Paulraj for TDMA systems.
- We update $\mathbf{S}$ iteratively, which updates $\mathbf{W}$, and $\mathbf{H}$ under the constraint that the information symbols $\mathbf{S}$ are from finite alphabets.
- We continue to iterate until $\mathbf{S}$ or $\mathbf{H}$ converge.

Algorithm Outline

1. Randomly choose $\mathbf{S}_0$ and set $l = 0$
2. $l := l + 1$
   (a) $\begin{bmatrix} \mathbf{h}_{1,l} & \cdots & \mathbf{h}_{P,l} \end{bmatrix} = \mathbf{Y}\mathbf{W}_l^{\dagger}$ where

   $$\mathbf{W}_l = \begin{bmatrix} \mathbf{w}_{l,1}(NL_c) \\ \vdots \\ \mathbf{w}_{l,P}(NL_c) \end{bmatrix}$$

   (b) Construct $\mathcal{G}_l$ with the estimated channel parameters and PN sequences.
   (c) $\mathbf{S}_{l+1}$ is estimated through

   $$\mathbf{S}_{l+1} = \mathcal{G}_l^{\dagger}\mathcal{Y}.$$

   (d) Project $[s_{l,i}(k)]$ to closest discrete values.
3. Continue until $\mathbf{S}_{l+1} - \mathbf{S}_l = \mathbf{0}$.

Simulation Results

We simulate a single receiver CDMA system with $L_c = 16$, $P = 8$ and a multi-receiver system with L_c, $M = 2$, $P = 13$ and SNR $= 15$ dB.

- Pricipal Component Algorithm
- Proposed Method

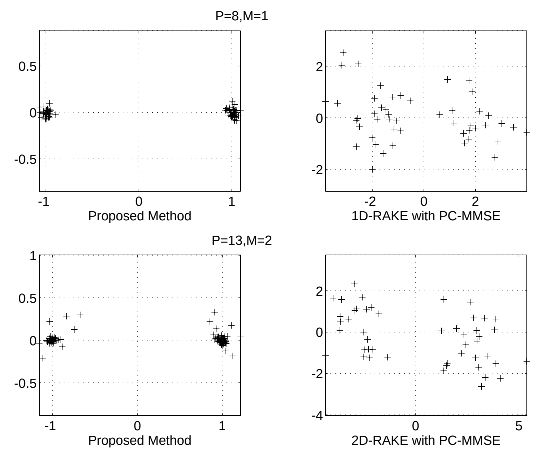


Figure 1: Signal constellations: 1-D RAKE, 2-D RAKE and our iterative method for the 1-D (M=1) and 2-D (M=2) cases.

Simulation Results

Compare the mean square errors of our channel estimation and teh principal component (PC) algorithm.

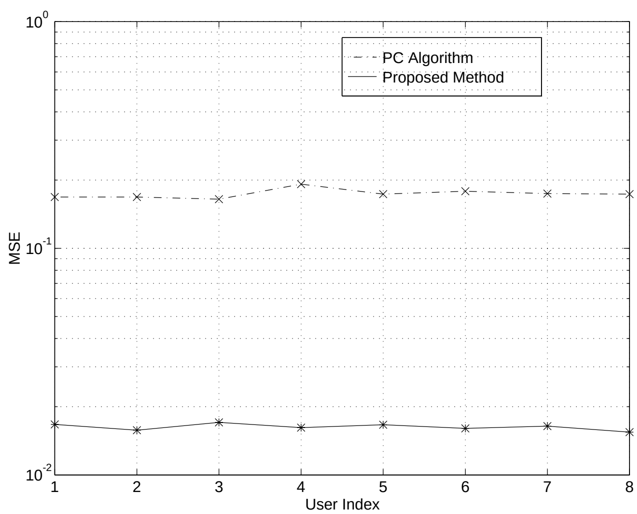


Figure 2: Comparison of two methods at SNR=15 dB

The proposed method offers better channel estimation.

Simulation Results

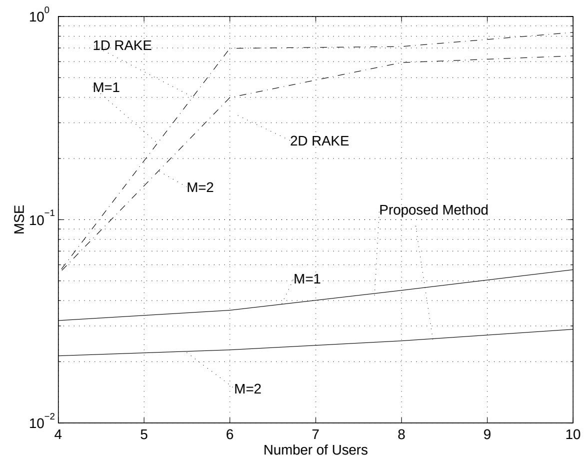


Figure 3: MSE vs. number of users using different receivers: 1-D RAKE, 2-D RAKE and our iterative method for the 1-D (M=1) and 2-D (M=2) cases.

# of Users	M=1	M=2
4	3.06	2.86
6	3.83	3.19
8	4.44	3.45
10	-	3.89
12	-	4.23
14	-	4.56
16	-	5.11

Table 1: Average Number of Iterations vs. Number of Antennas and Users

Conclusions

- We present a new approach for blind estimation of FIR channels in CDMA systems with aperiodic spreading sequences.
- We derive two frameworks to exploit the structure information of CDMA signals efficiently,
- We develop an iterative technique based on iterative least-squares and projection.
- We perform computer simulations to demonstrate the effectiveness of the purposed scheme over existing methods.
- Our future directions include proving a necessary and sufficient condition for identifiability.