OBJECTIVE

- Develop a formal mathematical framework for analysis and design of error diffusion algorithms for digital image halftoning
  - Model halftoning as two-dimensional delta-sigma modulation
  - Derive objective measures for subjective quality of edge sharpening and noise in halftoned images
- Applications:
  - Design of optimal error diffusion filters with respect to subjective quality
  - Optimize quality of halftoned oversampled images

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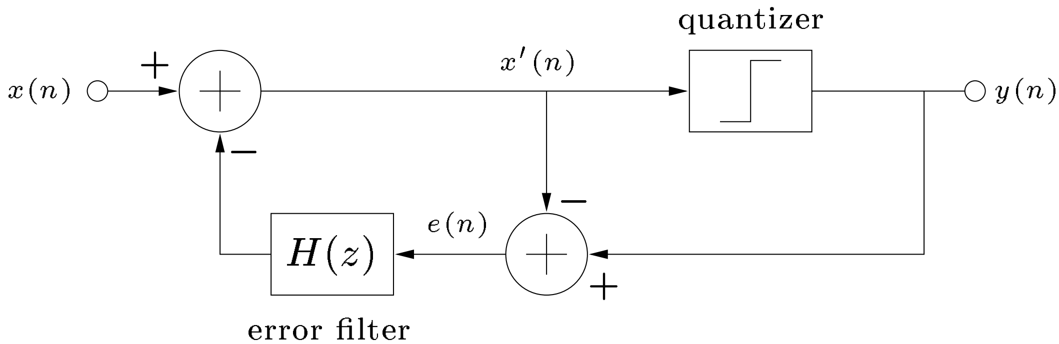
LINEAR ANALYSIS



Equivalent circuit of error diffusion

- Assume quantizer adds white noise uncorrelated with input
- Output given by

$$Y(z) = X(z) + N(z)(1 - H(z))$$

- Signal transfer function (STF) is flat, noise transfer function (NTF) is high-pass
- Circuit is equivalent in form to a noise-shaping feedback coder

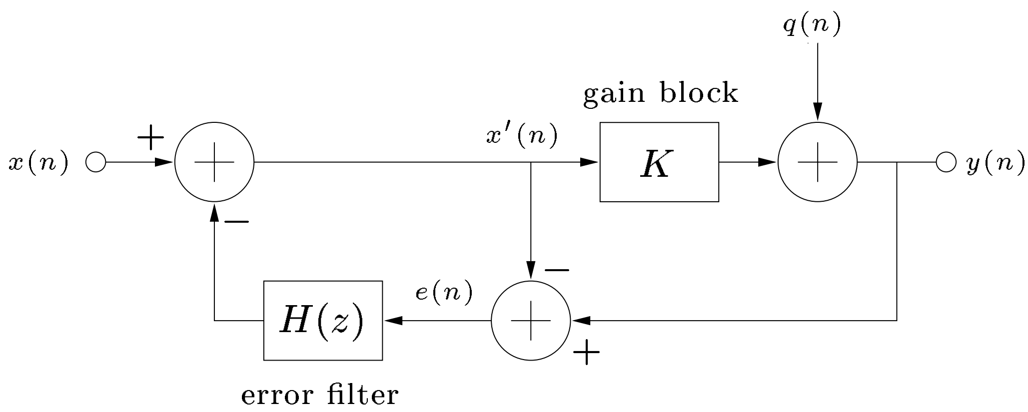
ERROR IMAGE

Original image

Error image

- Error image is highly correlated with input (Knox, 1992)
- Correlation is higher for larger error filters
- Degree of image sharpening increases with correlation
- Suggests *linear gain model* for quantizer
- Signal and noise paths modeled separately (Ardalan and Paulos, 1987)

LINEAR GAIN MODEL



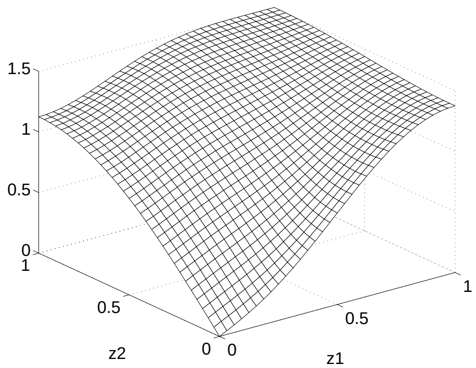
Equivalent circuit with linearized quantizer

- Output given by

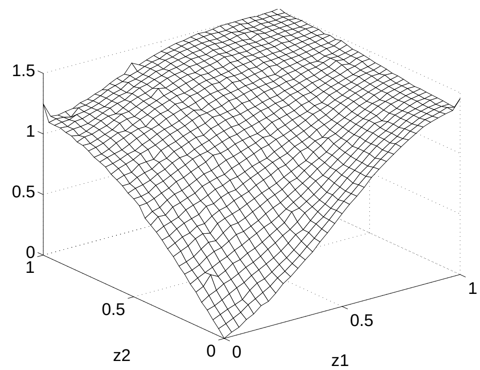
$$Y(z) = \underbrace{\frac{K}{1+(K-1)H(z)}}_{\text{STF}} X(z) + \underbrace{\frac{1-H(z)}{1+(K-1)H(z)}}_{\text{NTF}} N(z)$$

- K is measured empirically; varies with image and error filter
- Accounts for image sharpening
- Noise treated separately ($K = 1$)

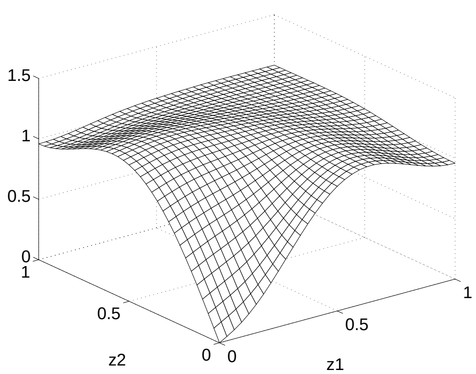
NOISE TRANSFER FUNCTION



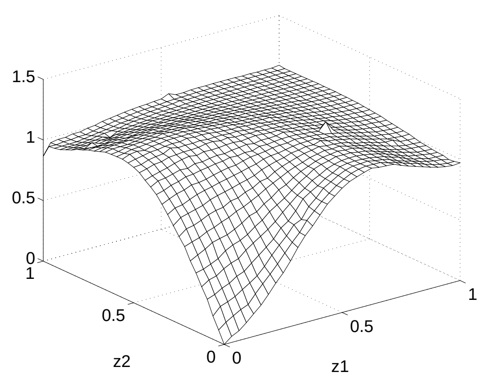
Predicted



Measured



Predicted

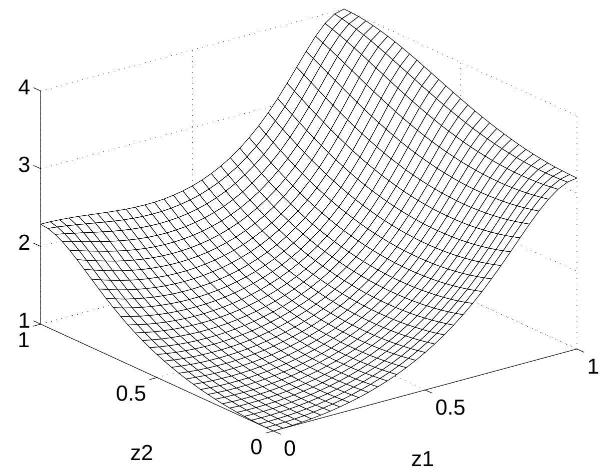


Measured

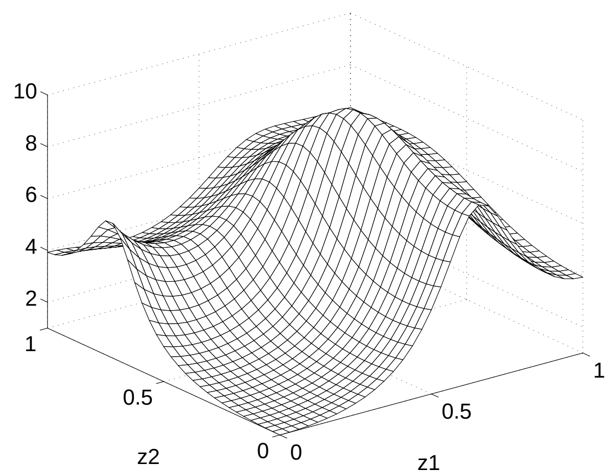
Top: Floyd-Steinberg. Bottom: Jarvis *et al*.

- Similar results for 1-D delta-sigma modulators

SIGNAL TRANSFER FUNCTION



Floyd-Steinberg STF, $K = 2.0$

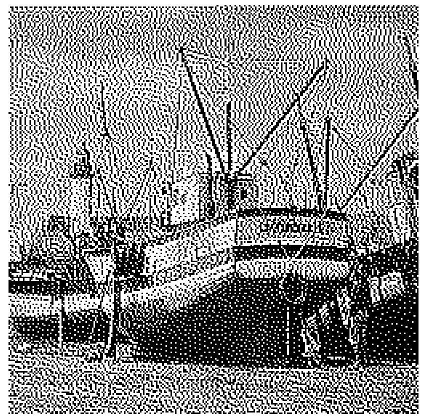


Jarvis *et al*. STF, $K = 4.5$

- Linear gain model accounts for sharpening seen with large error filters

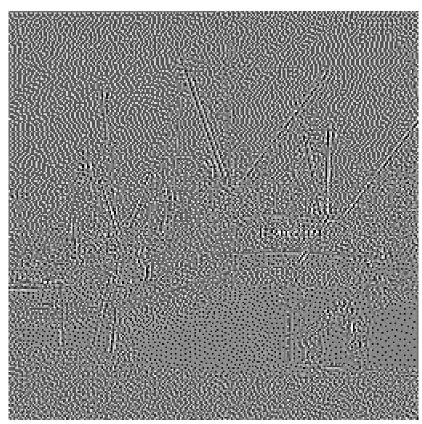
RESULTS OF LINEAR MODEL

Original image



Jarvis halftoned

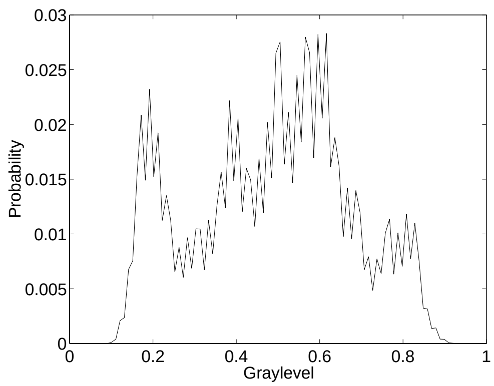
Gain model output



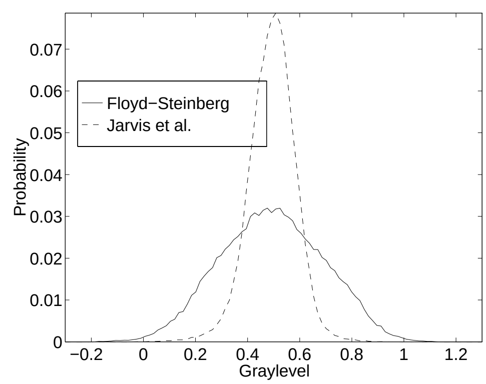
Difference

- Sharpening is decoupled from noise
- Effect of noise shaping can be quantified

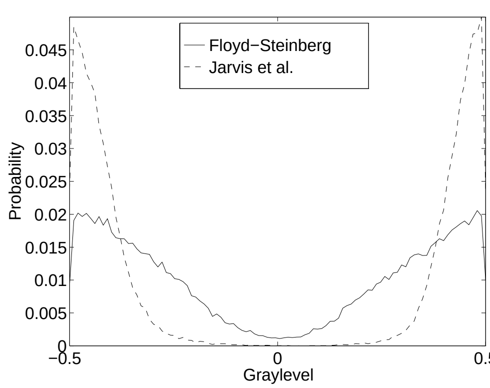
HISTOGRAMS



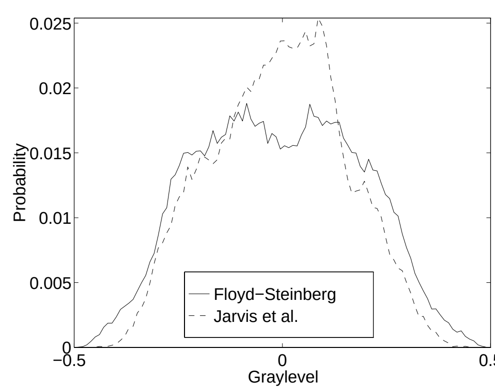
Original image



Quantizer input



Quantizer error



Filter output

- Narrow histogram at quantizer input leads to higher effective quantizer gain, K
- Quantizer error bounded by ± 0.5

SMALL ERROR FILTERS I

- Can small error filters be designed to sharpen as much as large filters?
  - Design large sharpening filter
  - Construct smaller filter whose frequency response is closest to the large filter in a mean square sense (Wong, 1996):

$$g_n = h_n + \alpha$$

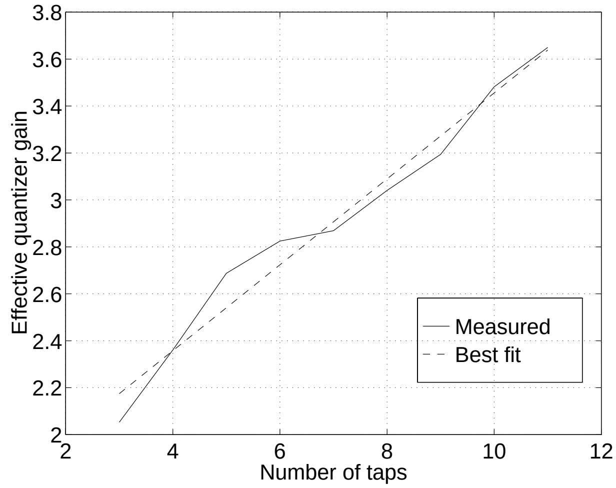
  where:

  h_n, g_n are the coefficients of the large and small error filters, respectively

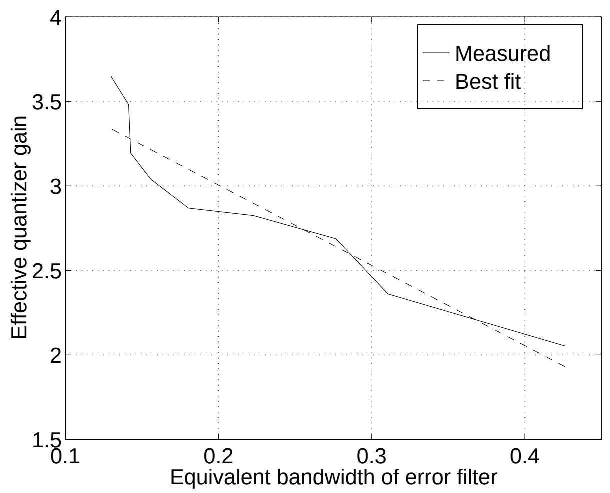
  α is a constant chosen to satisfy the gain constraint at DC

- Result: sharpening ability falls off linearly as number of filter taps decreases
- Degree of sharpening related to bandwidth of error filter

SMALL ERROR FILTERS II



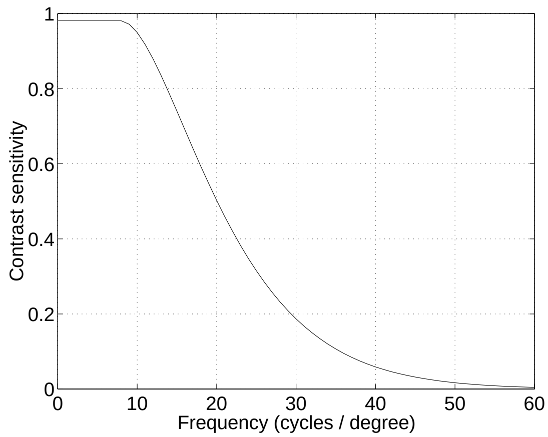
Variation of quantizer gain with filter size

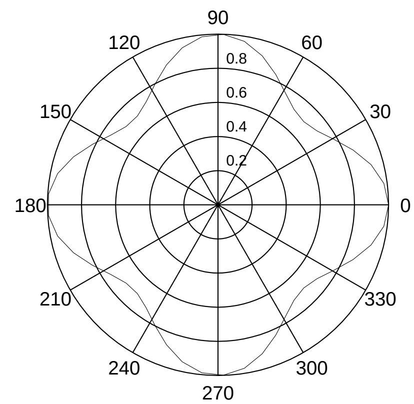


Variation of quantizer gain with filter bandwidth

- Sharpening correlated with bandwidth

VISUAL SYSTEM MODEL





Radial Angular

Sensitivity of human visual system

- Noise isolated by subtracting sharpened, noiseless image from halftoned image
- Weighted noise figure computed using visual system model

CONCLUSION

- Summary
  - Error diffusion can be modeled as a noise-shaping feedback coder, a form of two-dimensional delta-sigma modulation
  - Quantizer can be modeled by a gain block plus additive noise
  - Objective measures of subjective quality by decoupling edge sharpening and noise effects:
    - Edge sharpening proportional to gain
    - Weight noise by perceptual SNR measure
- Future work
  - Design of optimal error diffusion filters with respect to subjective quality using constrained nonlinear optimization
  - Optimize algorithm complexity and subjective quality for halftoned oversampled images
    - Combine error diffusion (sharpening) with interpolation (smoothing)