COCHANNEL SIGNAL SEPARATION IN FADING CHANNELS USING A MODIFIED CONSTANT MODULUS ARRAY

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ABSTRACT

We convert a constant modulus (CM) array into a robust smart antenna by modifying the error criterion to be a weighted sum of conventional CM array error and decision-directed equalization error. The new error criterion enables the CM array to (1) separate digital cochannel signals with multipath and inter-symbol interference and (2) track fading signals. The key contribution is that the modified error criterion adds phase sensitivity to the otherwise phase insensitive CM error criterion. We present computer simulations to show the signal tracking properties of the CM array using the modified error criterion in a fading environment.

1. INTRODUCTION

In mobile communication systems, the transmitted signal may be unintentionally reflected, refracted, or scattered on its way to the receiver. The received signal is a linear combination of delayed, scaled, and attenuated versions of the transmitted signal. Other users transmitting at the same frequency cause cochannel interference. Vehicle motion affects the received frequency because of Doppler shift and creates standing waves. Standing waves produce regions of high and low amplitudes, which is known as fading. Fading causes both attenuation and phase shift, which in turn cause errors at the receiver. Attenuation decreases SNR and phase shifts rotate the signal constellation which cause errors at the receiver.

One way to reduce errors at the receiver is to use a smart antenna system. Smart antenna systems improve signal recovery in severe cochannel signal environments. One smart antenna system, the multistage constant modulus (CM) array [1], is capable of separating cochannel signals. Fig 1 shows the stages in a CM array. Each stage consists of two components: (i) a weight-and-sum beamformer adapted by the constant modulus algorithm (CMA) [2] that captures one source, and (ii) a signal canceler adapted by the least-mean-squares (LMS) algorithm [3] that removes the captured source from the array input.

The CMA is a blind equalization algorithm (i.e., does not require a training or pilot signal) that makes use of the property that the transmitted signals are of constant amplitude. The CMA in [2] is insensitive to the phase of received signals because the error which is used to update the weights comprises of only amplitude differences. In this paper, we propose a modification to the CM array which makes the error dependent on both phase and amplitude of the output. In digital communications, the phase of the received signal is important because a shift in the phase would rotate the constellation of the received signal thus causing decision errors at the decoder.

Section 2 reviews the operation of a CM array and the CM algorithm in [2]. Section 3 describes our channel model. Section 4 discusses decision-directed equalization and our modification to the CMA error criterion. Section 5 presents computer simulations to show the robustness of signal tracking using the modified error criterion in a fading environment. Section 6 concludes the paper.

2. BACKGROUND

Each stage of the constant modulus (CM) array consists of a constant modulus beamformer and an adaptive signal canceler. The purpose of the beamformer is to lock onto and track a particular user. Section 2.1 describes the weight-and-sum beamformer which is adapted by the constant modulus algorithm (CMA). Section 2.2 describes the adaptive signal canceler which uses a least-mean-squares (LMS) algorithm [3] to remove the captured source from the array input.

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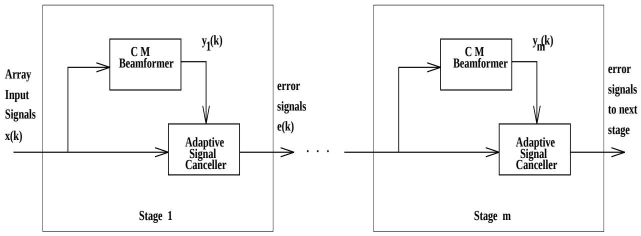


Figure 1: Multistage constant modulus array. Signals $x(k)$, $y_m(k)$, and $e(k)$ are complex-valued.

2.1. Constant Modulus Beamformer

The CM beamformer with complex input $\mathbf{x}(k)$ and output $y(k) = \mathbf{w}^H(k)\ \mathbf{x}(k)$ is shown in Fig 1. The complex weight vector $\mathbf{w}(k) = [w_1(k), w_2(k), \cdots, w_N(k)]^T$ is updated using the CM algorithm according to

$$\mathbf{w}(k+1) = \mathbf{w}(k) + 2\ \mu_{cma}\ \mathbf{x}(k)\ \epsilon^*_{cma}(k) \tag{1}$$

where $\mu_{cma} > 0$ is the step size and $\epsilon_{cma}(k)$ is the CMA error

$$\epsilon_{cma}(k) = \frac{y(k)}{\mid y(k) \mid} - y(k) = y(k)\left(\frac{1}{\mid y(k) \mid} - 1\right) \tag{2}$$

where $\mid y(k) \mid\ = \sqrt{y(k)\ \ y^*(k)}$. The term $\dfrac{y(k)}{\mid y(k) \mid}$ is called the instantaneous modulus (amplitude) of the received signal. The error criterion in (2) does not contain any phase information, thus the update in (1) is phase insensitive.

The update of the weights in (1) is similar to the update used in the LMS algorithm [3] except that the instantaneous modulus $\dfrac{y(k)}{\mid y(k) \mid}$ acts as the "desired response" signal. The CMA tries to make the instantaneous modulus constant. Thus, it can be used only with signals of constant modulus (amplitude) such as FSK and QPSK [4].

The value of step size μ_{cma} has to be chosen appropriately (as is the case with most of the adaptive algorithms) so that the algorithm converges fast. If the value is too small, the algorithm takes a long time to converge; if it is too large, then the algorithm may diverge. It has been proved that if $0 < \mu_{cma} < \dfrac{2}{\lambda_{max}}$ then the algorithm converges [5]. λ_{max} is the maximum eigenvalue of input autocorrelation matrix.

2.2. Adaptive Signal Canceler

Every stage of a CM array contains an adaptive signal canceler. The output $y(k)$ is weighted by the canceler weights $\mathbf{u}(k) = [u_1(k)\ \ u_2(k) \cdots u_N(k)]^T$ which is subtracted from $\mathbf{x}(k)$ to generate the error vector $\mathbf{e}(k)$. This error vector serves as the input to the next stage and is also used to update the canceler weights as follows

$$\mathbf{u}(k+1) = \mathbf{u}(k) + 2\ \mu_{lms}\ y^*(k)\ e(k) \tag{3}$$

where $\mu_{lms} > 0$ is the step size. The weights of canceler estimate the columns of the array response matrix which in turn gives an estimate of the directions of arrival of various signals. We define the array response matrix in the next section.

3. CHANNEL MODEL FOR DIGITAL SIGNALS

We assume that the transmitted signals are narrowband and that the receiver antenna array is in the far field of the transmitter. The baseband analog waveform transmitted by the l^{th} source is

$$s_l(t) = \sum_{n=0}^{\infty} d_l(n)\ \ g(t - nT), \qquad l = 1, \ldots L \tag{4}$$

where $d_l(n)$ are the digital symbols, T is the symbol period, and $g(t)$ is the pulse shape. The $d_l(n)$ terms are symbols of constant modulus (amplitude) such as BPSK and QPSK so that all of the $d_l(n)$ terms lie on a circle of the same radius. We assume that the l^{th} source propagates along M_l paths, each with a different attenuation and propagation delay [6].

The received baseband signal at the m^{th} antenna element of a uniform linear array is

$$x_m(t) = \sum_{l=1}^{L}\sum_{i=1}^{M_l} \alpha_{li}(k)\ e^{j2\pi f_c \tau_{li}}\ e^{j\phi_{mli}}\ s_l(t - \tau_{li}) + n_m(t) \tag{5}$$

where f_c is the carrier frequency, τ_{li} is the propagation delay of the i^{th} multipath of the l^{th} signal, $\alpha_{li}(k)$ is the corresponding attenuation, $n_m(t)$ is white Gaussian noise and

$$\phi_{mli} = 2\pi\frac{d}{\lambda}(m-1)\sin(\theta_{li})$$

where d is the inter-sensor spacing, λ is the carrier wavelength, and θ_{li} is the signal angle of arrival (AOA) for path i of the l^{th} source. The array input can be compactly written as

$$\mathbf{x}(k) = \mathbf{A}(k)\ \mathbf{s}(k) + \mathbf{n}(k) \tag{6}$$

where $\mathbf{A}$ is called the array response matrix. We assume that $\mathbf{s}(k)$ and $\mathbf{n}(k)$ have zero mean and are uncorrelated with each other. We model the fading of attenuation coefficients using a Rayleigh fading channel [7] with Doppler shift. The attenuation terms are given by

$$\alpha_{li}(k) = \frac{1}{\sqrt{M}}\sum_{m=1}^{M} e^{j(\Omega_i k \cos(\gamma_m) + \phi_m)} \tag{7}$$

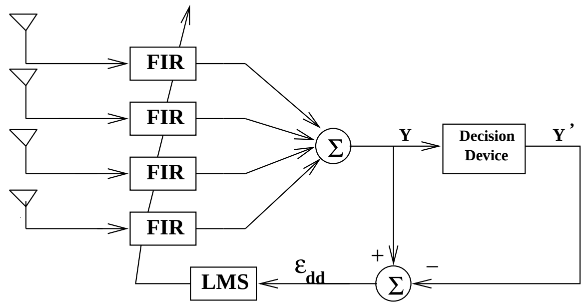


Figure 2: A decision directed beamformer shown for an array of four sensors.

where $\Omega_i = 2\pi \dfrac{f_i}{f_s}$, with f_i as the Doppler shift of the i^{th} source and f_s as the symbol rate, M is the number of scattering paths received, $\gamma_m = \dfrac{2\pi m}{M}$ which is based on the scatterers being uniformly distributed, and ϕ_m is a random variable uniformly distributed on $[-\pi, \pi]$ representing the initial phase of the mth scattering path.

4. MODIFIED ERROR CRITERION

In this section, we convert a constant modulus (CM) array into a robust smart antenna by modifying the error criterion to be a weighted sum of the conventional CM array error and decision-directed equalization error. The modified error criterion adds phase sensitivity to the otherwise phase insensitive CM error criterion which enables the CM array to (1) separate digital cochannel signals with multipath and inter-symbol interference and (2) track fading signals. In addition, phase is important for constellations which are based on phase like BPSK and QPSK. Section 4.1 explains decision directed equalization and shows the possibility of error propagation if the decisions are in error. Section 4.2 derives the proposed modified error criterion.

4.1. Decision Directed Beamforming

If the output of a communication channel were the correct transmitted sequence, then the output may be used as the "desired" response for the purpose of adaptive equalization. This method of equalization, called *Decision Directed Equalization* [5], can only be used if the output is free of errors; otherwise, an error in output will propagate through the receiver. The error ϵ_{dd} for updating the weights using decision directed equalization is

$$\epsilon_{dd} = \mathbf{Y} - \mathbf{Y}' \tag{8}$$

where $\mathbf{Y}$ is the output of the beamformer and $\mathbf{Y}'$ is the output of the decision device.

The primary disadvantage of using decision directed equalization is the propagation of errors when wrong decisions are made. In a real-time application, pure decision directed equalization cannot be performed. When we start the receiver, we get errors as the weights of the receiver filter are not set correctly. Since the decisions are in error, we cannot use decision directed equalization to update the weights of the receiver filter; thus, an initial training signal is needed.

4.2. The Modified Error Criterion

The primary advantages of CM beamformers are that they do not require a reference signal (because it performs blind equalization) and they can be implemented in real time (even on fixed-point processors). Its close resemblance to the LMS algorithm means that a hardware or software subsystem configured to use the complex LMS algorithm could be used directly for the CM algorithm.

From (2), ϵ_{cma} does not contain any phase information in it; i.e., the update in (1) is insensitive to phase shifts. Therefore, a conventional CM beamformer would not be able to give the desired response for a wireless communication system with fading effects. For signals such as QPSK and FSK, the decision depends on the phase of the output wave and not on the amplitude. Thus, a phase shift in the output could result in many wrong decisions. In addition, the CM array captures the source having the maximum power. When there is deep fading, the CM beamformer captures the interfering source, thus causing erroneous decisions. Thus, the need arises for a modified error criterion that would keep the advantages of CM array while removing its disadvantages.

The key advantage in decision directed equalization is its sensitivity to phase in updating the weights, which prevents an error if we have a phase offset. Decision directed equalization also has the ability to track small frequency offsets because it would adjust its weights to track the change in frequency. For a conventional CM array, however, a small frequency offset would result in a phase offset that cannot be corrected by CMA due to its insensitivity to phase. The decision directed equalization would also latch onto to a captured source even in case of deep fading. All the above predictions are based on the assumption that the decisions made are correct, and this assumption may not be valid in practice.

To overcome the drawbacks of CM array, we propose a new error criterion which is a weighted mean of the CM error ϵ_{cma} and the decision directed error ϵ_{dd}

$$\epsilon(k) = \alpha_{cma}\, \epsilon_{cma}(k) + \alpha_{dd}\, \epsilon_{dd}(k) \tag{9}$$

where α_{cma} is the weight of CM error and α_{dd} is the weight of decision directed error and

$$\alpha_{cma} + \alpha_{dd} = 1 \tag{10}$$

During the initial stage when the weights of the beamformer have not converged, i.e., when the decisions may not be correct, we use only the CM error by setting $\alpha_{cma} = 1$ and $\alpha_{dd} = 0$. As the weights converge and the output becomes stable and decisions become correct, we move from pure CM error to the modified error given in (9) with both α_{cma} and α_{dd} being nonzero. We determine this transition based on the absolute value of the CM error. When the error ϵ_{cma} becomes less than a threshold (1% of the maximum CM error in our case), then we switch from a CM error criterion to modified error criterion. The modified error builds phase sensitivity into the update equation and improves the performance of the CM array in the following ways:

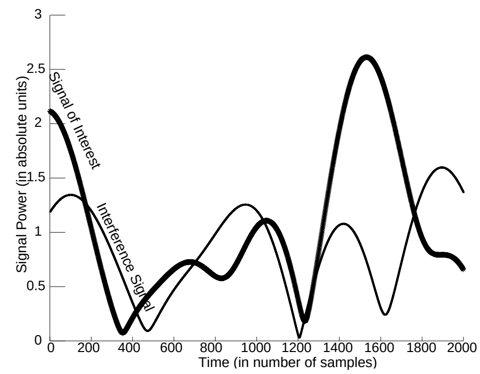
- it latches onto a captured signal irrespective of the power of the signal,
- it overcomes small frequency offsets, and
- it reduces phase offsets in the CM array.

The first property implies that the modified error criterion gives the correct output even when the captured signal has destructive fading, which occurs when the power of the captured signal is less than that of the interference. The new error criterion would perform better than the CM error by itself and make decisions more reliable.

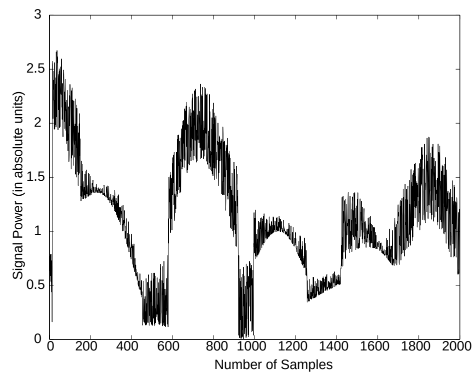
5. COMPUTER SIMULATIONS

We present the results of a computer simulation for the case where the modified error could separate two cochannel sources undergoing fading but the traditional CM array fails to latch onto one signal. The simulation uses $L = 2$ users, $N = 4$ antenna elements, and $M_1 = M_2 = 12$ multipaths. The symbol rate f_s is 24,300 baud and the fading frequency f_i is 72 Hz to correspond to a vehicle traveling at approximately 45 MPH with a transmitting frequency of 1.8 GHz. For the pulse shape $g(t)$ we used a square-root raised-cosine spectrum [8] with a roll-off parameter of $\beta = 0.35$. We transmit QPSK signals.

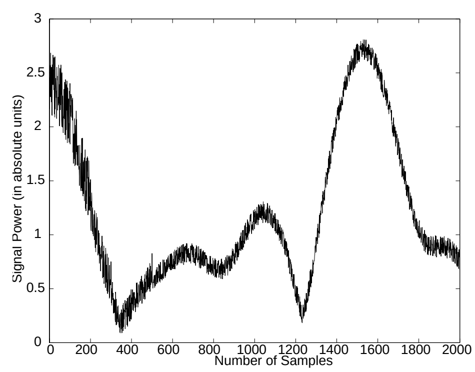
Fig 3(a) shows the amplitude of the received signal in a fading channel. Initially, the amplitude of the first signal is greater than that of the second. The first signal fades with time, and eventually, the power of the second signal becomes greater than that of the first. We refer to the crossover point as the critical point. A traditional CM array would capture the first signal until the critical point. After the critical point, the CM



(a)



(b)



(c)

Figure 3: Performance of constant modulus array for two different error criteria: **(a)** power of fading signals, **(b)** output power of first beamformer using traditional CMA, and **(c)** output power of first beamformer using modified error criterion. The modified error criterion allows for signals to be tracked in in a fading environment.

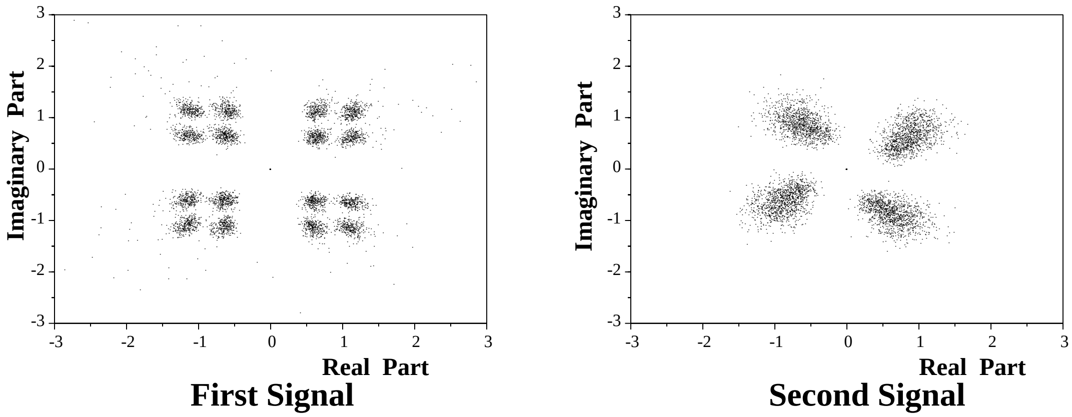


Figure 4: Cochannel Signals (SNR = 10 dB) separated by a CM array using the modified error criterion

array would be forced to capture the second signal, thereby causing errors at the decision device because the decision device thinks that it is still receiving the first signal. Fig 3(b) plots the output signal power using the traditional CMA. The CM beamformer clearly captures the interference signal when interference becomes stronger than the signal of interest.

The problem of capturing different sources is overcome by using the modified error criterion. Fig 3(c) shows the output signal power using the modified error criterion. Using the modified error criterion, the CM array latches onto one signal even during deep fades. Fig 4 displays the received constellation of the separated signals using the modified error criterion. The signals have been clearly separated without any errors after we switch from pure CM error to the modified error.

6. CONCLUSION

We present a new error criterion, which is a weighted mean of the Constant Modulus and Decision Directed Error criteria, to improve the performance of CM beamformer. The new error criterion allows the beamformer to latch onto a signal even during deep fades because it adds phase sensitivity to the beamformer. The use of a modified CM error criterion requires an additional comparison and two additions, which is insignificant with respect to the complexity of the CM array. The cochannel signal separator has been shown to work in simulations of severe signal conditions such as when the cochannel interference is stronger than the signal of interest. Additional advantages of using the CM cochannel signal separator are that (a) its canceler weights can be used to estimate the angles of arrival, and (b) it can be used to estimate the number of cochannel signals.

The step size μ_{cma} for the beamformer and μ_{lms} for the canceler could change with the situation. They should be appropriately chosen so that both the algorithms converge in a few iterations. We use 0.01 for both μ_{cma} and μ_{lms}. Another decision that the CM array has to make is the switch from CM mode to modified error mode. We perform the shift when the error from the CM mode falls below a threshold of 1% to the maximum CM error. This shift should be performed once the user is sure that the decisions on the output are error free. It could be made either empirically or when the mean CM error reaches a constant value.

7. REFERENCES

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