Optimal Design of Real and Complex Minimum Phase Digital FIR Filters

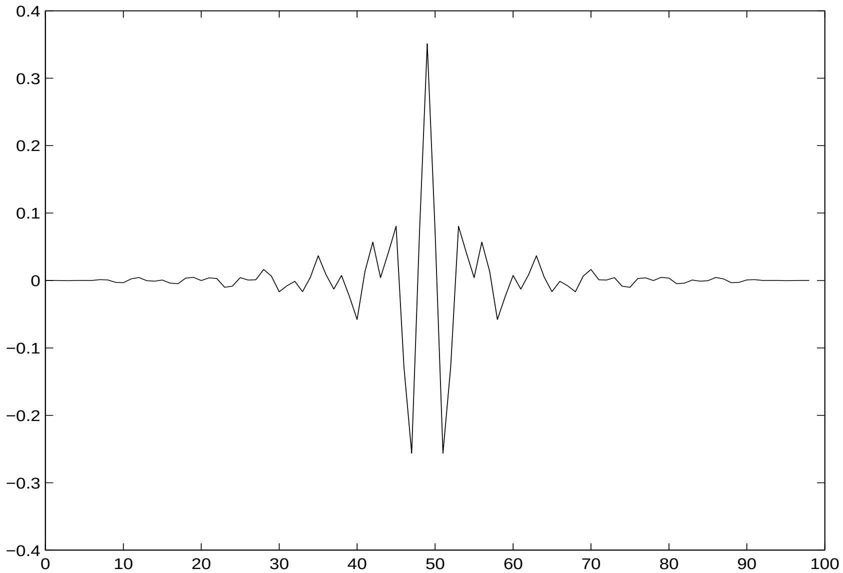
Niranjan Damera-Venkata and Brian L. Evans

Embedded Signal Processing Laboratory
Dept. of Electrical and Computer Engineering
The University of Texas at Austin
Austin, TX 78712-1084

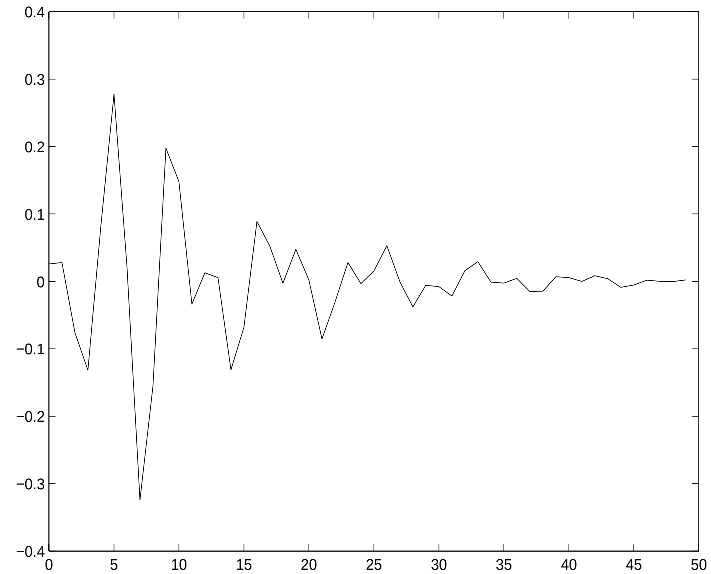
http://signal.ece.utexas.edu

Minimum Phase Digital FIR Filters

- **Properties**
  - **All zeros are on or inside the unit circle in the *z*-plane**
  - **Twice as many free parameters as linear phase filters of the same length**
  - **Minimum group delay**
  - **Minimum length to meet piecewise constant magnitude specifications**
- **Impulse Responses**



Linear Phase Filter Coefficients



Minimum Phase Filter Coefficients

Minimum Phase FIR Filter Design Algorithms

- **Spectral Factorization** [Chen & Parks 1986]
  1. Design linear phase filter for minimum phase power spectrum
  2. Factor polynomial transfer function
  3. Reconstruct minimum phase polynomial transfer function
- **Cepstral Deconvolution** [Boite & Leach 1981][Mian & Nainer 1982]
  1. Compute amplitude function for desired magnitude response
  2. Calculate *unique* minimum phase function using complex cepstrum
  3. Apply inverse fast Fourier transform (FFT) or solve nonlinear equations

Characteristic	Spectral Factorization	Cepstral Deconvolution	Proposed Algorithm
Computation	*Iterative*	*Non-Iterative*	*Non-Iterative*
Coefficient Accuracy	*High*	*Low*	*User Controlled*
Coefficient Data Type	*Real*	*Real*	*Real or Complex*
Magnitude Specification	*Piecewise Constant*	*Arbitrary*	*Arbitrary*
Dimensions	*1-D only*	*1-D*	*1-D or m-D*

Proposed Algorithm

1. Determine desired amplitude response $|X(e^{j\omega})|$

- ***Choice #1:*** **Piecewise constant magnitude response**
  - **Convert minimum phase filter design specifications to optimal linear phase filter specifications**
  - **Design optimal linear phase filter**
  - **Transform amplitude response to match that of desired optimal minimum phase filter using formulas by Chen and Parks**
- ***Choice #2:*** **Specify an arbitrary magnitude response**

Proposed Algorithm

2. Compute *unique* phase response from amplitude response

- **Use generalized Discrete Hilbert Transform relation for a causal minimum phase sequences**

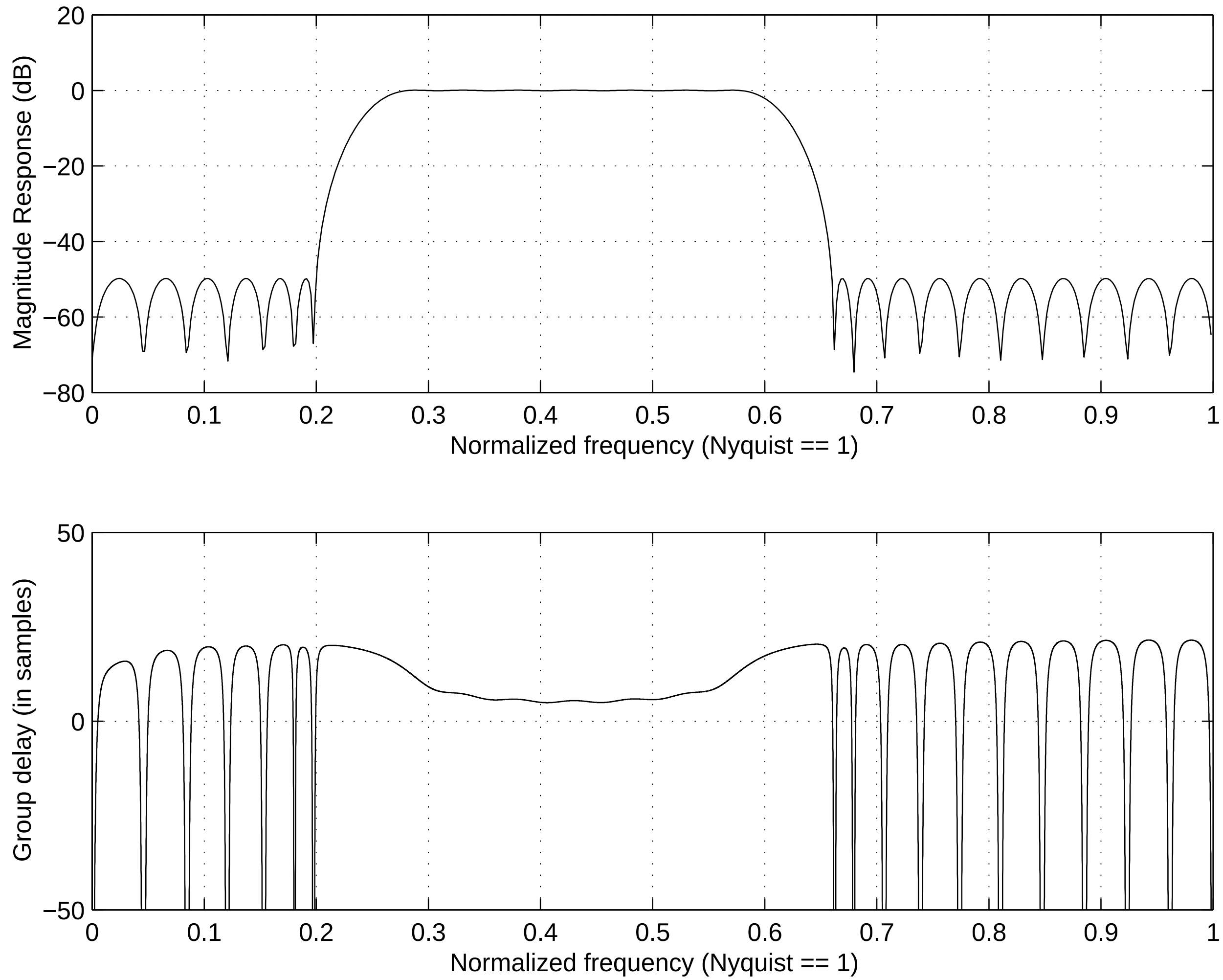
$$\arg X(e^{j\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} \log\left|X(e^{j\omega})\right| \cot\left(\frac{\omega-\theta}{2}\right) d\theta - K$$

  - **For real coefficients, $K = 0$ [Cizek 1970]**
  - **For complex case, we show that K is a constant, which disappears when taking the derivative of the phase to calculate group delay**

3. Compute the impulse response

- **Sample magnitude and phase response at M points**
- **Compute inverse FFT and truncate the result**

Real Minimum Phase Bandpass Filter Design



User Specifications

$f_{s1} = 0.20$

$f_{p1} = 0.28$

$f_{p2} = 0.58$

$f_{s2} = 0.66$

$\delta_p = 0.007844$

$\delta_s = 0.003255$

$\varepsilon = 0.001$

Computed Parameters

$\delta_p' = 0.007837$

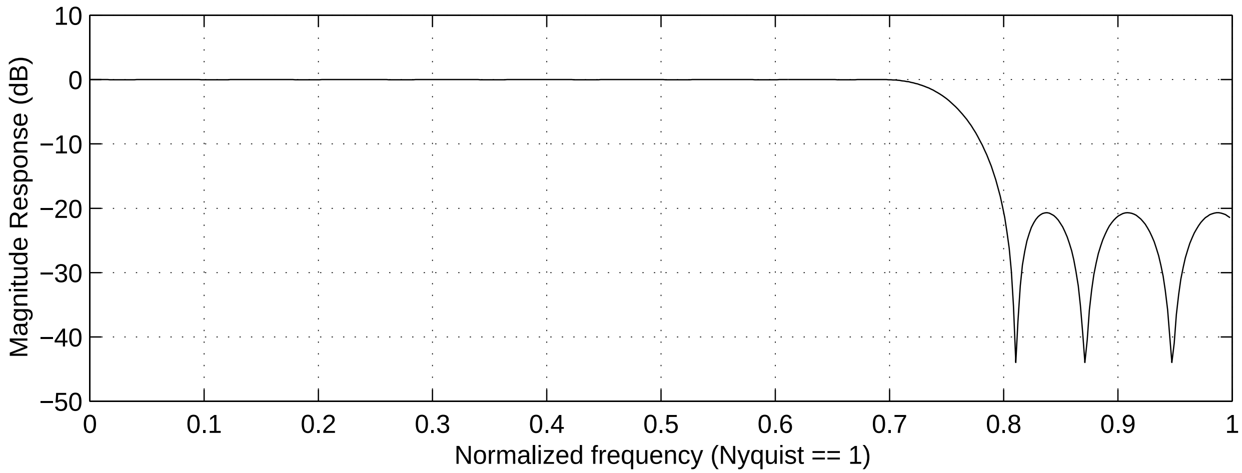
$\delta_s' = 0.003246$

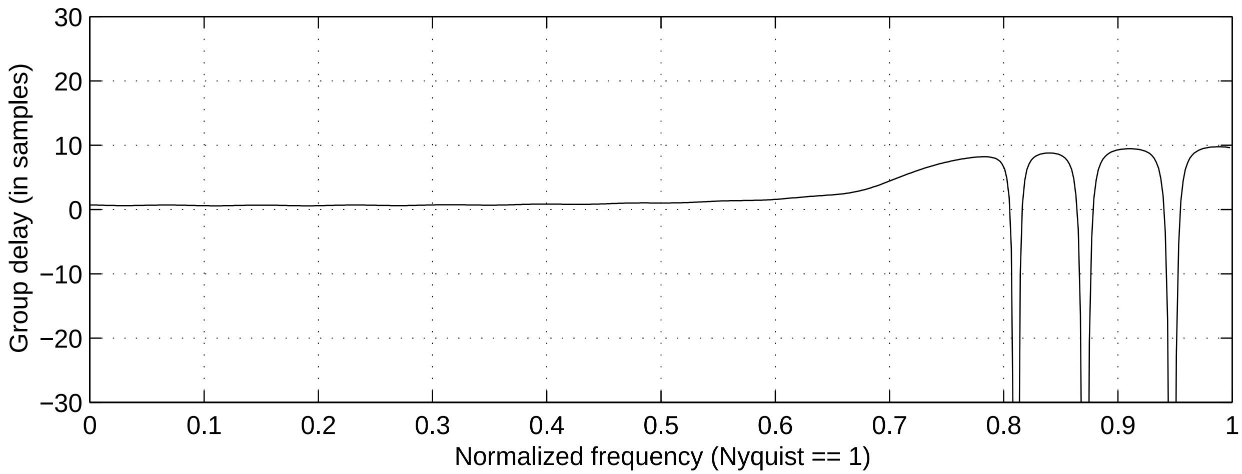
$M = 32768$

Proposed algorithm (near-linear phase passband): 50 taps

Parks-McClellan algorithm (linear phase): 99 taps

Complex Minimum Phase Lowpass Filter Design





User Specifications

$f_p = 0.7$

$f_s = 0.8$

$\delta_p = 0.002125$

$\delta_s = 0.092510$

$\varepsilon = 0.001$

Computed Parameters

$\delta_p' = 0.002125$

$\delta_s' = 0.092359$

$M = 32768$

Proposed algorithm (near-linear phase passband): **26 taps**

Karam-McClellan algorithm (linear phase passband): **51 taps**

Conclusion

- **Algorithm based on the Discrete Hilbert Transform (DHT)**
  - **Extension of the DHT relations to complex sequences**
  - **Optimal design algorithm for real *and* complex minimum phase FIR filters**
  - **Error bounds for real and complex filters**
  - **FFT length required to compute the DHT for a given coefficient accuracy**

Characteristic	Spectral Factorization	Cepstral Deconvolution	Proposed Algorithm
Computation	*Iterative*	*Non-Iterative*	*Non-Iterative*
Coefficient Accuracy	*High*	*Low*	*User Controlled*
Coefficient Data Type	*Real*	*Real*	*Real or Complex*
Magnitude Specification	*Piecewise Constant*	*Arbitrary*	*Arbitrary*
Dimensions	*1-D only*	*1-D*	*1-D or m-D*

Order of the FFT

- **Algorithm imposes causality on complex cepstral sequence**
  - **Complex cepstral sequence $x(n)$ has infinite duration**
  - **Truncation of $x(n)$ to M samples introduces error**
  - **User specifies coefficient accuracy as ε**
  - **Constraint on maximum error becomes** $\left| x\left(\frac{M}{2}\right) \right| < \varepsilon$
  - **Given ε, compute number of stopband zeros** N_s **and FFT length** $M = 2^m$
- **Complex filter case (loose upper bound is $M = 2\, N_s / \varepsilon$)**

$$m = \lceil 1 + \log_2 N_s - \log_2 \varepsilon \rceil$$

- **Tighter bound for real filter case (new result not in paper)**
  - **Stopband zeros are complex conjugate pairs spaced uniformly on unit circle**

$$m = \left\lceil 2 + \log_2 \left| \sum_{i=0}^{l} \cos\left(2\pi\left(f_s + i\frac{1 - 2f_s}{N_s - 1}\right)\right) \right| - \log_2 \varepsilon \right\rceil$$

  - f_s **is the stopband frequency and** $l = \left\lfloor \frac{(N_s - 1)}{2} \right\rfloor$