OPTIMUM CHANNEL SHORTENING FOR DISCRETE MULTITONE TRANSCEIVERS

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ABSTRACT

We propose an optimum channel shortening method for discrete multitone (DMT) transceivers. The proposed method shortens a given channel to a desired length while maximizing the number of bits transmitted on a DMT symbol. The key to the optimum solution is the definition of the SNR in a subchannel using the equivalent signal, noise, and ISI paths in the system. Our simulation results show that the proposed method outperforms the best existing method with a 18% increase in the bit rate. We show that the maximum shortening SNR method is a special case of the proposed method and both methods are nearly equivalent when the input energy distribution is constant over all subchannels.

1. INTRODUCTION

Discrete-multitone (DMT) modulation is a popular method for high-speed data transmission over spectrally shaped channels, e.g. in the asymmetric digital subscriber line (ADSL) standard [1]. In DMT, a channel is partitioned into a large number of independent subchannels using the inverse fast Fourier transform (FFT). The total number of bits transmitted is the sum of the number of bits transmitted in each subchannel. For an ideal channel, the subchannels are orthogonal, which enables recovery of the information at receiver by using an FFT operation. A spectrally shaped channel, however, causes inter-symbol interference (ISI) and destroys orthogonality between subchannels so that they cannot be separated at the receiver [2].

For $N/2$ subchannels, a DMT symbol has N samples. Prepending a guard period of ν samples to each DMT symbol eliminates ISI when $\nu \geq L-1$ [3], where L is the channel impulse response length. The guard period samples are typically the last ν samples of the DMT symbol, a.k.a. a cyclic prefix. The channel throughput is reduced by a factor of $N/(N+\nu)$. When ν gets large relative to N this factor decreases and the performance loss can be prohibitive. Hence, a channel shortening equalizer is required to shorten the effective length of the channel to the length of the cyclic prefix which is chosen to be relatively small.

Section 2 reviews three channel shortening methods. Section 3 proposes a channel shortening method to optimize maximum bit rate. Section 4 gives simulation results.

This work was supported by a US NSF Career Award.

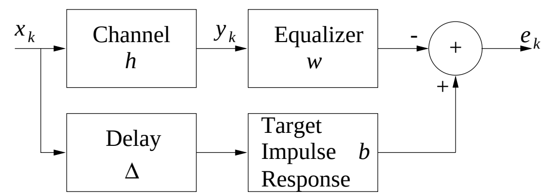


Figure 1: MMSE equalizer (a.k.a. time domain equalizer)

2. CHANNEL SHORTENING METHODS

Channel shortening methods shorten the effective channel impulse response to be less than or equal to ν.

2.1. Minimum Mean-Squared Error Method

The minimum Mean-Squared Error (MSE) method (a.k.a. time domain equalizer or TEQ) [4] uses the structure shown in Fig. 1 to calculate the equalizer coefficients. If the error in Fig. 1 could be forced to be zero, then the equalized impulse response would be equal to the target impulse response (TIR) with a time delay difference. By controlling the TIR length, we control the length of the equalized channel. Given the length of the TIR, the goal is to find the best TIR and an equalizer which minimize the MSE. Note that the lower path in Fig. 1 is not physically implemented.

For minimum MSE, the TIR and equalizer must satisfy

$$\mathbf{b}^T \mathbf{R_{xy}} = \mathbf{w}^T \mathbf{R_{yy}} \tag{1}$$

where $\mathbf{R_{xy}}$ and $\mathbf{R_{yy}}$ are the input-output cross correlation and output autocorrelation matrices, respectively [4]. Then

$$\text{MSE} = \mathbf{b}^T \left[\mathbf{R_{xx}} - \mathbf{R_{xy}} \mathbf{R_{yy}^{-1}} \mathbf{R_{yx}}\right] \mathbf{b} = \mathbf{b}^T \mathbf{R_{x|y}} \mathbf{b} \tag{2}$$

Adding a constraint on $\mathbf{b}$ to prevent the trivial solution $\mathbf{b} = \mathbf{0}$, the equalizer design problem becomes

$$\min_{\mathbf{b}} \mathbf{b}^T \mathbf{R_{x|y}} \mathbf{b} \;\; \text{s.t.} \;\; \|\mathbf{b}\| = 1 \tag{3}$$

The solution for $\mathbf{b}$ is the eigenvector of $\mathbf{R_{x|y}}$ corresponding to the minimum eigenvalue. The corresponding equalizer settings can be calculated from (1). This channel shortening method does not maximize bit rate because minimizing the MSE does not necessarily increase the bit rate [5].

2.2. Maximum Shortening SNR Method

This method minimizes the energy of the effective channel impulse response outside a window with the desired length ν while keeping the energy within the window constant [6]. No TIR is used, so only the upper path in Fig. 1 is used. The equalized impulse response can be written in matrix form as $\mathbf{h}_{\text{eff}} = \mathbf{H}\mathbf{w}$ where $\mathbf{H}$ is the convolution matrix of the channel impulse response. Construct the vector $\mathbf{h}_{\text{win}}$ from the samples of $\mathbf{h}_{\text{eff}}$ which are in a given window of size ν and $\mathbf{h}_{\text{wall}}$ from the samples outside the window. The channel shortening problem is defined as minimizing the energy of $\mathbf{h}_{\text{wall}}$ while satisfying the constraint $\|\mathbf{h}_{\text{win}}\|^2 = 1$ to prevent the trivial solution. The energy outside and inside the window, respectively, can be expressed as

$$\mathbf{h}_{\text{wall}}^T\mathbf{h}_{\text{wall}} = \mathbf{w}^T\mathbf{H}_{\text{wall}}^T\mathbf{H}_{\text{wall}}\mathbf{w} = \mathbf{w}^T\mathbf{A}\mathbf{w} \tag{4}$$

$$\mathbf{h}_{\text{win}}^T\mathbf{h}_{\text{win}} = \mathbf{w}^T\mathbf{H}_{\text{win}}^T\mathbf{H}_{\text{win}}\mathbf{w} = \mathbf{w}^T\mathbf{B}\mathbf{w} \tag{5}$$

Thus, the aim is to minimize $\mathbf{w}^T\mathbf{A}\mathbf{w}$ while satisfying the constraint $\mathbf{w}^T\mathbf{B}\mathbf{w} = 1$. The solution is given as [6]

$$\mathbf{w}_{\text{opt}} = (\sqrt{\mathbf{B}}^T)^{-1}\mathbf{q}_{\min} \tag{6}$$

where $\mathbf{q}_{\min}$ is the eigenvector corresponding to the minimum eigenvalue $\lambda_{\min}$of the matrix

$$\mathbf{C} = \left(\mathbf{Q}\sqrt{\mathbf{\Lambda}}\right)^{-1}\mathbf{A}\left(\sqrt{\mathbf{\Lambda}}\mathbf{Q}^T\right)^{-1} \tag{7}$$

The columns of $\mathbf{Q}$ consist of the orthonormal eigenvectors of $\mathbf{B}$, and $\mathbf{\Lambda}$ is a diagonal matrix with the entries of the eigenvalues. This approach is optimum in the sense of maximum shortening SNR which is defined as

$$\mathrm{SSNR}_{\text{opt}} = 10\log_{10}\left(\frac{\mathbf{w}_{\text{opt}}^T\mathbf{B}\mathbf{w}_{\text{opt}}}{\mathbf{w}_{\text{opt}}^T\mathbf{A}\mathbf{w}_{\text{opt}}}\right) = 10\log_{10}\left(\frac{1}{\lambda_{\min}}\right) \tag{8}$$

No previous study has been made on the optimality of this algorithm in the maximum bit rate sense. Our simulation results show that this approach is in fact near optimum.

2.3. Geometric TEQ Method

This is the first method to optimize the equalizer for maximum bit rate [5]. First, a TIR is calculated to maximize the number of bits transmitted in one DMT symbol (b_{DMT}). Then, the equalizer is calculated by using (1). Its structure is equivalent to the MMSE method in Fig. 1.

Assuming that the subchannels can be modeled as additive white Gaussian noise channels,

$$b_{DMT} = \sum_{i=1}^{N/2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{,_i}\right) \tag{9}$$

where i is the subchannel index, SNR_i is the SNR of the i^{th} subchannel, and $,_i$ is the SNR gap for achieving channel capacity in subchannel i [7]. Assuming that the SNR gap is constant in each subchannel, i.e. $,_i = ,$ $\forall i$,

$$\mathrm{SNR}_{geom} = ,\left(\left[\prod_{i=1}^{N/2}\left(1 + \frac{SNR_i}{,}\right)\right]^{2/N} - 1\right) \tag{10}$$

so that (9) can be rewritten as

$$b_{\text{DMT}} = N\log_2\left(1 + \frac{\mathrm{SNR}_{geom}}{,}\right) \tag{11}$$

So, maximizing geometric SNR maximizes b_{DMT}.

The method is based on the following approximation of the geometric SNR which is obtained by ignoring the 1 and -1 terms in (10):

$$\mathrm{SNR}_{geom} \approx \left[\prod_{i=1}^{N/2} SNR_i\right]^{2/N} \tag{12}$$

This approximation is valid with the assumptions that the input SNR is high enough to ignore the dependency of the geometric SNR to , and that the entire available bandwidth is used. SNR_i is defined as follows by ignoring the ISI term in the denominator and assume that $|B_i| \approx |H_i||W_i|$

$$\mathrm{SNR}_i = \frac{S_x|B_i|^2}{S_{n,i}|W_i|^2} \tag{13}$$

where S_x is signal power across the entire bandwidth, $S_{n,i}$ is the noise power in the i^{th} subchannel and B_i, W_i are the gain of $\mathbf{b}$ and $\mathbf{w}$ in the i^{th} subchannel, respectively.

In this case, the problem is converted to the maximization of the objective function

$$L(b) = \frac{2}{N}\sum_{i=1}^{N/2}\ln|B_i|^2 \tag{14}$$

which also assumes that the noise at the output of the equalizer is independent of $\mathbf{b}$. A unit-energy constraint on $\mathbf{b}$ is used to prevent an infinite gain equalizer. But using only this constraint maximizes the cost function for $|B_i|^2 = 1$ $\forall i$, which implies full equalization (zero forcing equalization) of the channel. Therefore an additional constraint is required to keep the MSE error relatively small. Thus, the problem of finding optimum settings for the TIR can be stated as

$$\max_{\mathbf{b}}\sum_{i=1}^{N/2}\ln|B_i|^2 \text{ s.t. } \|\mathbf{b}\|^2 = 1 \text{ and } \mathbf{b}^T\mathbf{R}_{\mathbf{\Delta}}\mathbf{b} \le \mathrm{MSE}_{\max}$$

This nonlinear constrained optimization problem does not have a closed-form solution, but may be solved by numerical methods. Although this method incorporates the geometric SNR, it is not optimum because of the approximations. For example, the definition in (13) does not include the effect of ISI on the SNR, and the assumption $|B_i| \approx |H_i||W_i|$ holds only after the channel has been equalized.

3. OPTIMUM CHANNEL SHORTENING

We propose a new method to compute the equalizer settings that maximize b_{DMT}. Unlike the geometric TEQ approach, we define SNR_i with the ISI term as

$$\mathrm{SNR}_i = \frac{S_{x,i}|H_{signal,i}|^2}{S_{n,i}|H_{noise,i}|^2 + S_{x,i}|H_{ISI,i}|^2} \tag{15}$$

where $S_{x,i}$, $S_{n,i}$, $H_{signal,i}$, $H_{noise,i}$, and $H_{ISI,i}$ are the signal power, noise power, signal path gain, noise path gain,

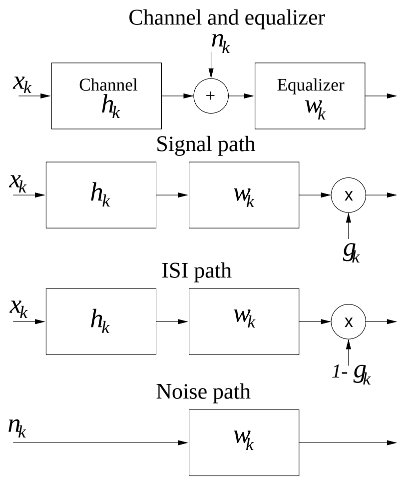


Figure 2: Equalized channel and the corresponding signal, noise, and ISI paths.

and ISI path gain in the i^{th} subchannel, respectively. Ideally, the equalizer shortens the channel so that it would fit in a window of size ν. The part of the equalized impulse response which is inside the window is the signal path, and the part outside the window is the ISI path:

$$\begin{aligned} h_{signal} &= (h_k * w_k) g_k \\ h_{ISI} &= (h_k * w_k)(1 - g_k) \\ h_{noise} &= w_k \end{aligned} \tag{16}$$

Here, '*' denotes convolution, h_k and w_k are the channel and equalizer impulse responses, respectively, and

$$g_k = \begin{cases} 1 & d \le k < d + \nu \\ 0 & \text{otherwise} \end{cases}$$

The noise path consists only of the equalizer because the noise process passes only through the equalizer. All three paths are shown in Fig. 2.

Since most energy of the equalized impulse response is in the window, the window on the signal path does not contribute to the signal path considerably and can be ignored:

$$\mathrm{SNR}_i = \frac{S_{x,i}|H_i|^2|W_i|^2}{S_{n,i}|W_i|^2 + S_{x,i}|H_{ISI,i}|^2} \tag{17}$$

If the ISI term could be forced to be zero, then W_i would drop out and SNR_i would not depend on the equalizer:

$$\mathrm{SNR}_i = \frac{S_{x,i}|H_i|^2}{S_{n,i}} \tag{18}$$

The SNR in (18) is the highest achievable SNR (matched filter bound). Rewriting (16) in matrix form,

$$\begin{aligned} \mathbf{h}_{signal} &= diag(\mathbf{g})\mathbf{H}\mathbf{w} \\ \mathbf{h}_{ISI} &= ((\mathbf{I} - diag(\mathbf{g}))\,\mathbf{H}\mathbf{w} = \mathbf{D}\mathbf{H}\mathbf{w} \\ \mathbf{h}_{noise} &= \mathbf{w} \end{aligned} \tag{19}$$

where bold lower case letters represent their corresponding signals in vector form, $\mathbf{H}$ represents the $N \times N_w$ size convolution matrix of h_k (N_w is the length of $\mathbf{w}$), and $diag(\cdot)$ forms a diagonal matrix from vector argument. The inner product of

$$\mathbf{q}_i = \begin{bmatrix} 1 & e^{j2\pi i/N} & e^{j2\pi 2i/N} & \cdots & e^{j2\pi(N-1)i/N} \end{bmatrix}$$

with a vector returns the i^{th} DFT coefficient for that vector. We minimize the ISI part of (17) by minimizing

$$\begin{aligned} L(\mathbf{w}) &= \sum_{i=1}^{N/2} S_{x,i}|H_{ISI,i}|^2 = \sum_{i=1}^{N/2} S_{x,i} \mid \mathbf{q}_i\mathbf{D}\mathbf{H}\mathbf{w} \mid^2 \\ &= \mathbf{w}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{D}^{\mathrm{H}} \left(\sum_{i=1}^{N/2} \mathbf{q}_i^{\mathrm{H}} S_{x,i}\mathbf{q}_i \right) \mathbf{D}\mathbf{H}\mathbf{w} \\ &= \mathbf{w}^{\mathrm{H}}\mathbf{A}\mathbf{w} \end{aligned} \tag{20}$$

subject to a constraint on $\mathbf{w}$ to prevent the trivial solution. We propose to constrain the energy of the equalized channel impulse response to unity:

$$\|\mathbf{H}\mathbf{w}\|^2 = \mathbf{w}^T\mathbf{H}^T\mathbf{H}\mathbf{w} = \mathbf{w}^T\mathbf{B}\mathbf{w} = 1 \tag{21}$$

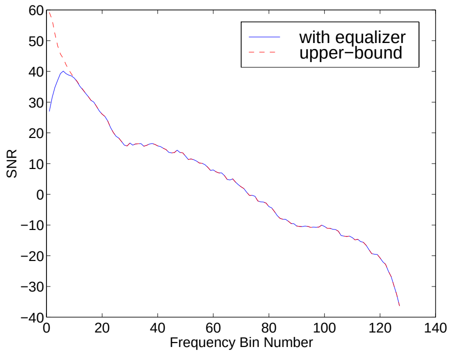
We have converted the shortening problem to a constrained minimization problem. Its solution is equivalent to that of the maximum shortening SNR method in (6) and (7). The maximum shortening SNR method minimizes the energy of the part of the equalized impulse response that causes ISI. We have shown that minimizing a frequency weighted sum of the energy is optimum, and the optimum weights are the input energy distribution.

The maximum shortening SNR method and proposed method are equivalent when the input signal power distribution is constant over frequency, as seen from (20). If $S_{x,i} = 1,\ \forall i$, then $L(\mathbf{w})$ is equal to the energy of the part of the equalized impulse response outside of the window, i.e. $\mathbf{h}_{\mathrm{wall}}$ in (4). The proposed method constrains the energy of the entire equalized impulse response, whereas the maximum shortening SNR method constrains only the part within the window to be unity.

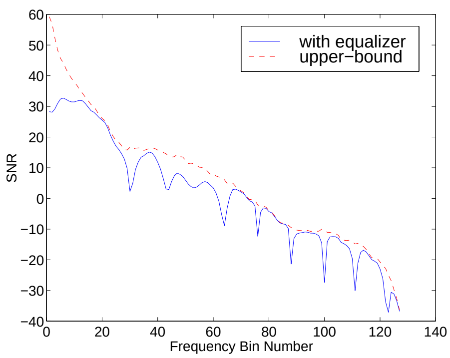
4. SIMULATION RESULTS

Simulations use the carrier-serving-area loop number one, and process 512 samples sampled at 2.208 MHz. The two-sided white Gaussian noise power is -110 dBm/Hz. Crosstalk noise is modeled as $|H_{NEXT}(f)|^2 = 10^{-13}f^{3/2}$ [3]. Input power is set so that the matched filter bound of the SNR is 25 dB. We use a 21-tap equalizer, a cyclic prefix length ν of 16, and an FFT size of $N = 128$. A coding gain of 5 dB and a margin of 6 dB are assumed when calculating b_{DMT}. In the maximum geometric SNR method, the MSE threshold (MSE_{max}) is chosen to be 0.025 [5].

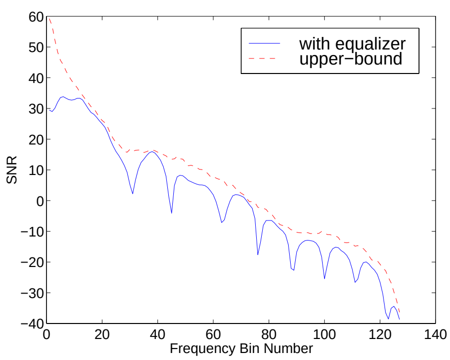
As shown in Table 1, the proposed method improves the effective bits per symbol by 18% over the best previously reported method of maximum shortening SNR, and has the highest geometric SNR. The MMSE method gives the lowest performance in geometric SNR and effective bits per symbol. Although not designed to optimize the geometric



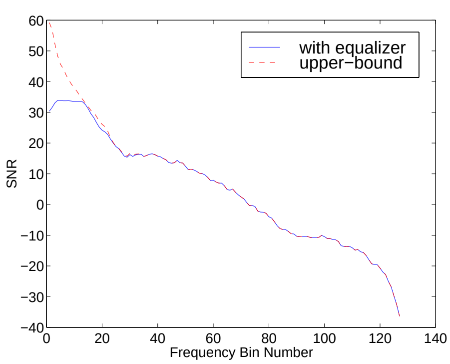
(a) Proposed method



(b) MMSE-UEC method



(c) Geometric TEQ method



(d) Maximum SSNR method

Figure 3: SNR distribution over frequency

Method	Geometric SNR (dB)	Bits per symbol
Proposed	15.14	242
MMSE	12.30	162
geometric TEQ	12.44	166
max. short. SNR	13.90	205

Table 1: Simulation results for $N = 128$, $\nu = 16$, 21-tap equalizer, 5 dB coding gain, 6 dB subchannel margin, 512 channel length, and 2.208 MHz sampling rate.

SNR, the maximum shortening SNR method gives better geometric SNR results than the geometric SNR method.

Fig. 3 (a)-(d) show the frequency distribution of the SNR after equalization, and the upper bound of the SNR given by (18). SNR distributions for the proposed and maximum shortening SNR methods exactly follow the upper bound at high frequencies. At lower frequencies, the maximum shortening SNR solution diverges from the upper bound before the proposed method does. Although the divergence is very small, the effect on the geometric SNR is dramatic. The SNR distributions for MMSE and geometric TEQ methods appear similar. The geometric TEQ approach is constrained with a maximum MSE, which roughly tracks MMSE. The only improvement the geometric TEQ solution has over the MMSE solution that the notches smaller. The MMSE solution may appear better than the geometric TEQ solution because the MMSE solution follows the upper bound more tightly. Deeper notches in the MMSE solution, however, decrease the geometric mean of the SNR more than the evenly distributed offset in the geometric TEQ solution.

5. REFERENCES

[1] ANSI T1.413-1995, "Network and customer installation interfaces: ADSL metallic interface."

[2] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Communications Magazine*, vol. 28, pp. 5–14, May 1990.

[3] T. Starr, J. M. Cioffi, and P. J. Silverman, *Understanding Digital Subscriber Line Technology.* Prentice-Hall, 1999.

[4] N. Al-Dhahir and J. M. Cioffi, "Efficiently computed reduced-parameter input-aided MMSE equalizers for ML detection: A unified approach," *IEEE Trans. on Info. Theory*, vol. 42, pp. 903–915, May 1996.

[5] N. Al-Dhahir and J. M. Cioffi, "Optimum finite-length equalization for multicarrier transcievers," *IEEE Trans. on Comm.*, vol. 44, pp. 56–63, Jan. 1996.

[6] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse response shortening for discrete multitone transceivers," *IEEE Trans. on Comm.*, vol. 44, pp. 1662–1672, Dec. 1996.

[7] J. M. Cioffi, *A Multicarrier Primer.* Amati Comm. Corp. and Stanford University, T1E1.4/91-157, Nov. 1991.