Optimum Channel Shortening for Discrete Multitone Transceivers

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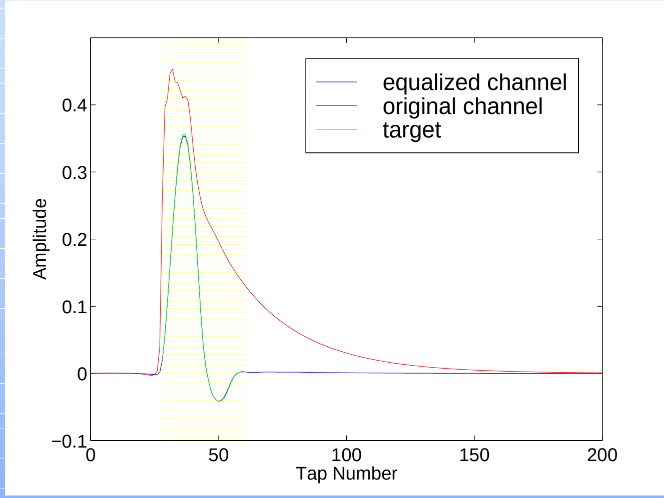
- Problem
  - Design an FIR equalizer that optimizes channel capacity
- Motivation
  - Distribution of ISI power in frequency affects capacity
- Solution
  - Derive cost function for ISI power as function of equalizer taps
  - Solve constrained quadratic optimization problem
- Contributions
  - Decompose equalized channel into signal, noise, and ISI paths
  - Model subchannel SNR based on this decomposition
  - Develop a design method to maximize subchannel SNR
- Results
  - Generalize maximum shortening SNR with frequency weighting
  - Outperforms best methods by up to 18% for ADSL channels

Introduction

- Wireline discrete multitone modulation
  - Multicarrier modulation by using FFT
  - Add cyclic prefix between symbols to fight intersymbol interference (ISI)
  - FIR time-domain equalizer (TEQ) shortens channel length to be less than cyclic prefix length



- Problem:
  - Design a TEQ that optimizes channel capacity
- Solution
  - Derive cost function for ISI power as function of TEQ taps
  - Solve constrained quadratic optimization problem

Channel Capacity

- Multicarrier channel capacity

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{\text{SNR}_i}{\Gamma}\right) = N \log_2\left(1 + \frac{\text{SNR}_{geom}}{\Gamma}\right)$$

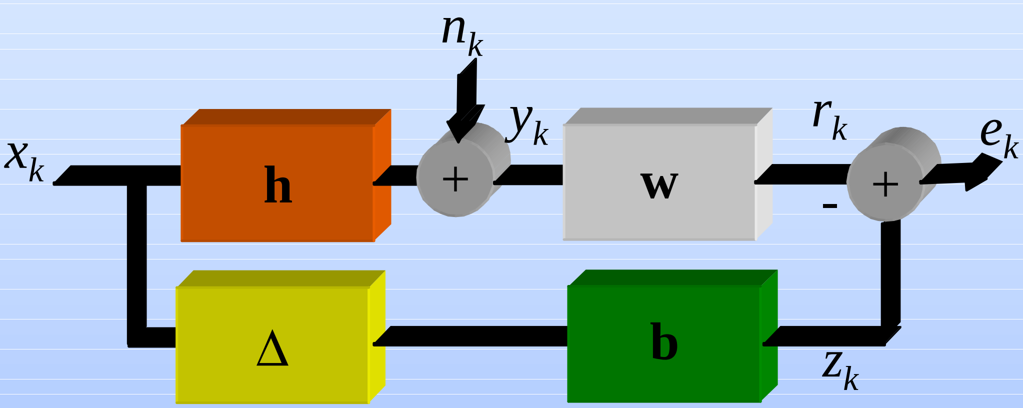
N: # subchannels SNR_i: SNR in subchannel i Γ: SNR gap

$$\text{SNR}_{geom} = \Gamma\left(\left[\prod_{i=1}^{N}\left(1 + \frac{\text{SNR}_i}{\Gamma}\right)\right]^{1/N} - 1\right)$$

- Previous methods *do not* maximize channel capacity
  - Minimum mean squared error method [Chow & Cioffi 1992]
  - Maximum shortening SNR method [Melsa, Younce, Rohrs 1996]
  - Maximum geometric SNR method [Al-Dhahir & Cioffi, 1996]

Minimum Mean Squared Error (MMSE) Method

- Chose length of b_k to shorten length of $h_k * w_k$
- Minimize MSE so that $h_k * w_k \cong b_{k-\Delta}$
- Zeros low SNR bands

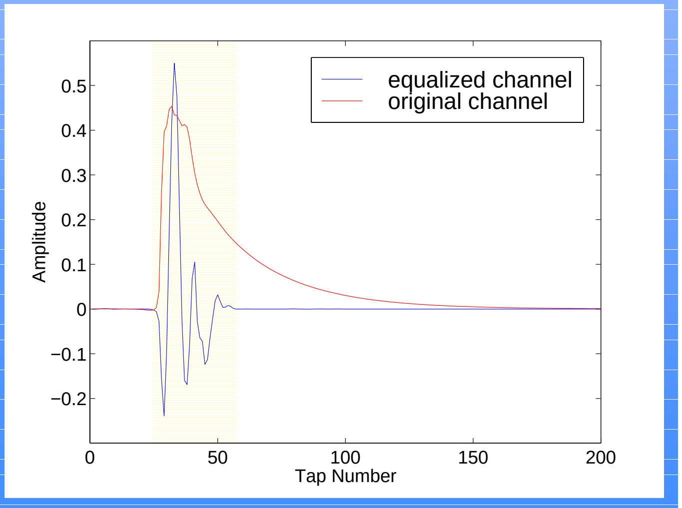


Maximum Shortening SNR (SSNR) Method

- For each possible window,

$$\max_{\mathbf{w}}(\text{SSNR}) = \max_{\mathbf{w}} 10 \log_{10}\left(\frac{\text{energy inside window}}{\text{energy outside window}}\right)$$

- Closed-form solutions exist
- Assume known channel impulse response



Maximum Geometric SNR Method

- Maximize approximate geometric SNR
- Keep MSE smaller than MSE_{max}
- Disadvantages
  - Subchannel SNR definition ignores ISI
  - Objective function ignores interdependence of **b** and **w**
  - Requires solution of nonlinear constrained optimization problem
  - Has the same drawbacks as the MMSE method
  - MSE_{max} has to be tuned for different channels

$$\mathrm{SNR}_{geom} \approx \left[\prod_{i=1}^{N} \mathrm{SNR}_i \right]^{1/N}$$

$$\mathrm{SNR}_\mathrm{i} = \frac{S_x |B_i|^2}{S_{n,i} |W_i|^2}$$

$$|B_i| \approx |H_i||W_i|$$

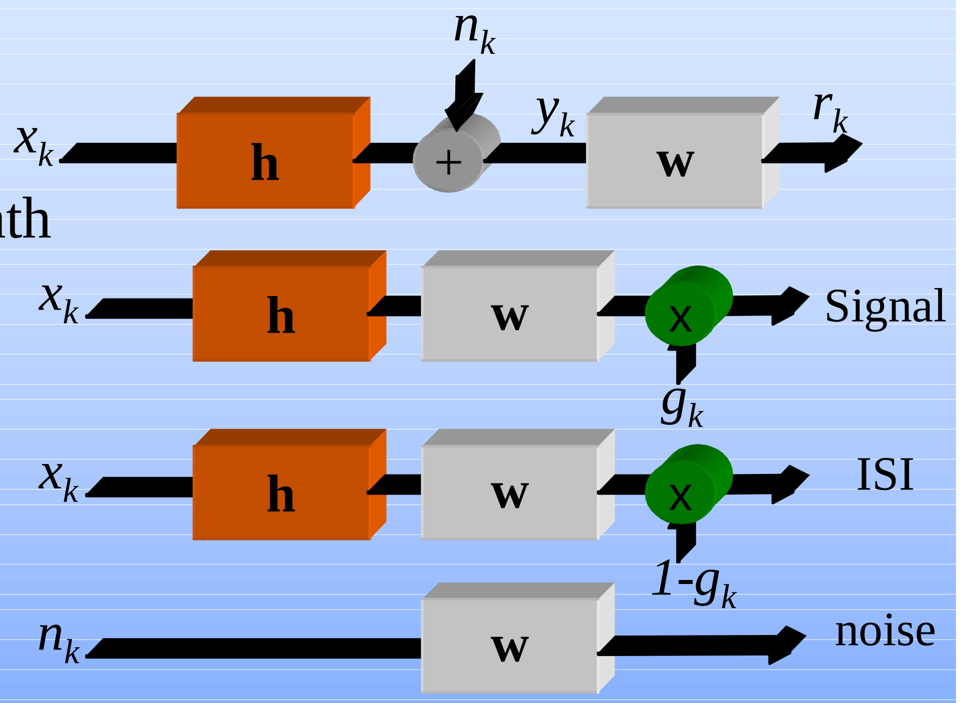
$|\mathrm{H_i}|$: Channel gain in $\mathrm{i^{th}}$ subchannel

$|\mathrm{W_i}|$: Equalizer gain in $\mathrm{i^{th}}$ subchannel

$|\mathrm{B_i}|$: Target gain in $\mathrm{i^{th}}$ subchannel

Proposed Subchannel SNR Model

- Partition equalized channel
  - signal path, ISI path, noise path
- Subchannel SNR is defined as



$$\mathrm{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}}$$

Observation: Equalizer affects SNR only by changing the ISI term

$$\text{signal power} = S_{x,i}\left|H_{signal,i}\right|^2$$
$$\text{noise power} = S_{n,i}\left|H_{noise,i}\right|^2$$
$$\text{ISI power} = S_{x,i}\left|H_{ISI,i}\right|^2$$

$$g_k = \begin{cases} 1 & d \le k < d + \upsilon \\ 0 & otherwise \end{cases}$$

Proposed Design Method

- Minimize ISI to maximize SNRs in subchannels

$$\sum_{i=1}^{N/2} S_{x,i} \left| H_{ISI,i} \right|^2 = \sum_{i=1}^{N/2} S_{x,i} \left| \mathbf{q}_i \mathbf{D} \mathbf{H} \mathbf{w} \right|^2 = \mathbf{w}^T \mathbf{A} \mathbf{w}$$

D: diagonal matrix of 1-g_k

H: channel convolution matrix

$\mathbf{q}_i$: FFT vector

- Constrain energy of the equalized channel impulse response

$$\left\| \mathbf{H} \mathbf{w} \right\|^2 = \mathbf{w}^T \mathbf{B} \mathbf{w} = 1$$

- Optimum in the sense of minimizing total ISI power
  - Equivalent to maximizing subchannel SNRs
  - Equivalent to maximizing channel capacity
- Generalization of the maximum SSNR method
  - Equivalent if $S_{x,i}$=1

Simulation Results

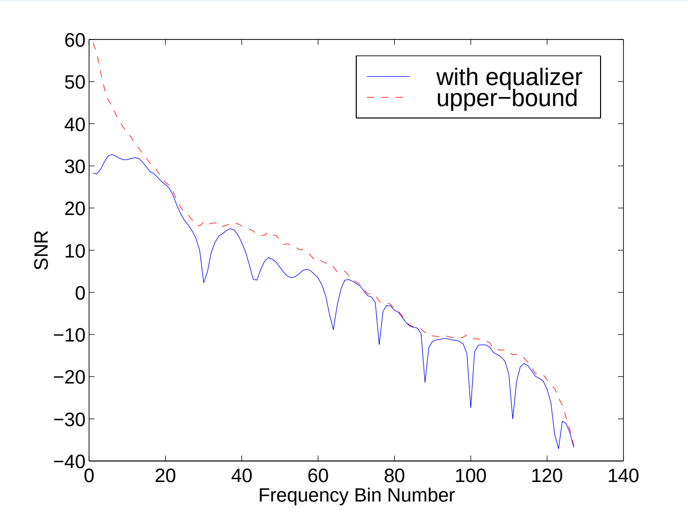
- Proposed method improves bit rate by up to 18%
- Max SSNR method outperforms max geometric SNR method
- Similar SNR distribution for MMSE and max geometric SNR
  - MSE is constrained in max geometric SNR method

Method	*Geometric SNR (dB)*	*Bits/ symbol*
Proposed	15.14	242
Max SSNR	13.90	205
Max geo SNR	12.44	166
MMSE	12.30	162

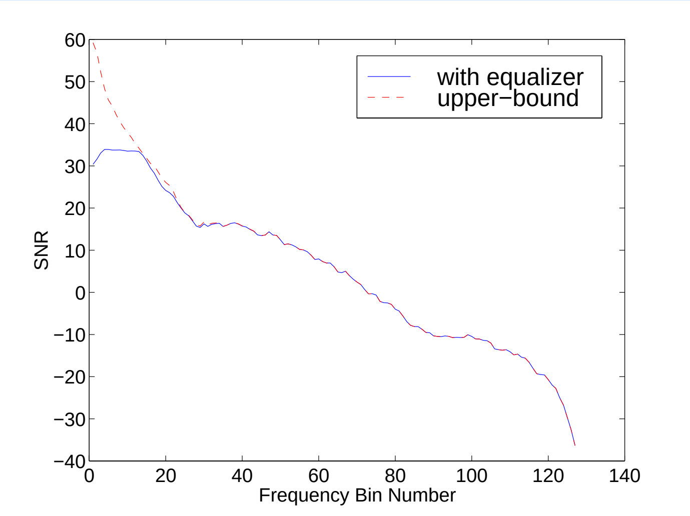
Simulation Parameters	
Channel	*CSA loop 1*
Sampling Rate	2.208 Mhz
Channel length	512 samples
AWGN power	-110 dBm/Hz
NEXT noise mask	$10^{-13} f^{3/2}$
MFB	25 dB
Cyclic prefix	16
Number of taps	21
FFT size	128
Margin	6 dB
Coding Gain	5 dB

Simulation Results

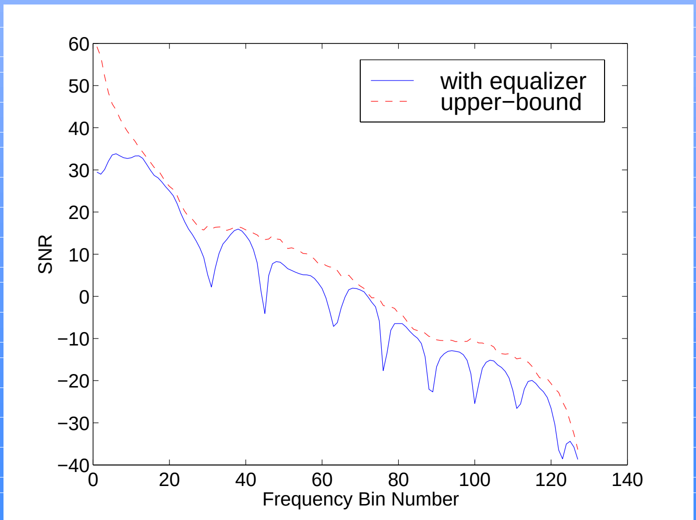
Minimum MSE



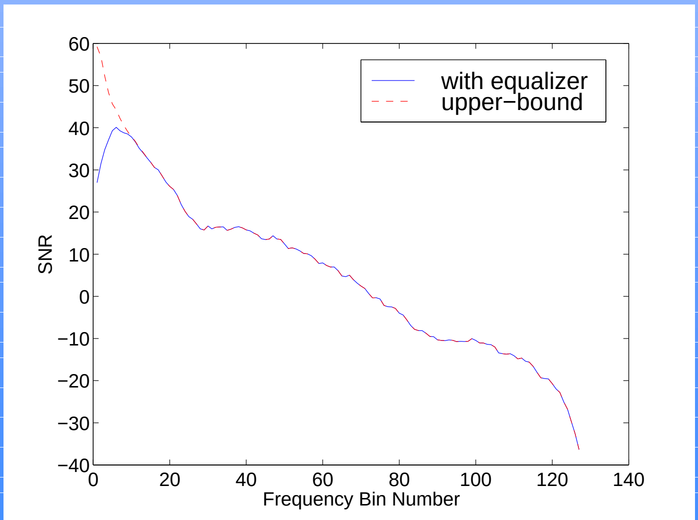
Maximum SSNR



Maximum geometric SNR



Proposed method



Conclusion

- Distribution of ISI power in frequency affects capacity
  - ISI in subchannels with low noise decreases SNR dramatically
  - High noise power dominates ISI but keeps SNR unaffected
- Contributions
  - Decompose equalized channel into signal, noise, and ISI paths
  - Model subchannel SNR based on this decomposition
  - Develop FIR TEQ design method to maximize subchannel SNR
- Proposed FIR TEQ design method
  - Generalize max SSNR method by frequency weighting of ISI
  - Outperforms best methods by up to 18% for ADSL channels