COLOR ERROR DIFFUSION WITH GENERALIZED OPTIMUM NOISE SHAPING

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ABSTRACT

We optimize the noise shaping behavior of color error diffusion by designing an optimized error filter based on a proposed noise shaping model for color error diffusion and a generalized linear spatially-invariant model of the human visual system. Our approach allows the error filter to have matrix-valued coefficients and diffuse quantization error across channels in an opponent color representation. Thus, the noise is shaped into frequency regions of reduced human color sensitivity. To obtain the optimal filter, we derive a matrix version of the Yule-Walker equations which we solve by using a gradient descent algorithm.

1. INTRODUCTION

Color error diffusion is a high-quality method for color rendering of *continuous-tone* 24-bit digital color images on devices with limited color palettes such as low-cost displays and printers. The rendered images are commonly referred to as *halftones*. The high quality of error diffusion is due to the fact that it is a nonlinear feedback system. Quantization errors are filtered using an *error filter* and fed back to the input in order to shape the quantization noise into frequencies regions where humans are relatively less sensitive.

Kolpatzik and Bouman [1] and Akarun, Yardimci and Cetin [2] use error filters with matrix-valued coefficients to account for correlation among the color planes. The error filter by Kolpatzik and Bouman [1] filters each color error plane independently in an opponent color space [1]. Separate optimum scalar error filters are designed for the luminance and chrominance channels independently based on a separable model of the human visual system. However, no constraints are imposed on the error filter to ensure that all of the red-green-blue (RGB) quantization error is diffused. Akarun, Yardimci and Cetin [2] adapt the matrix-valued error filter coefficients using a least mean squares algorithm. This allows for cross-channel diffusion of color error. However, their method does not incorporate a human vision model.

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In this paper, we derive the optimum matrix-valued error filter using the matrix gain model [3] to model the noise shaping behavior of color error diffusion and a generalized linear spatially-invariant model (not necessarily separable) for the human color vision. We also incorporate the constraint that all of the RGB quantization error be diffused. We show that the optimum error filter may be obtained as a solution to a matrix version of the Yule-Walker equations. A gradient descent algorithm is proposed to solve the generalized Yule-Walker equations. In the special case that the constraints are removed, a separable color vision model used and the linear transformation into the opponent color space is unitary, our solution reduces to the solution derived by Kolpatzik and Bouman [1].

2. NOTATION

In this paper, boldface quantities written with a $\tilde{}$ represent matrices, whereas boldface quantities written without a $\tilde{}$ represent vectors. Capitalized quantities represent the frequency domain while lower case quantities represent the space domain. Scalar quantities are represented as usual as plain characters. The ith component of a vector $\mathbf{a}$ will be denoted by a_i whereas the (i,j)th element of a matrix $\tilde{\mathbf{A}}$ will be denoted by $a_{i,j}$. The vector with all of its elements equal to unity is denoted by $\mathbf{1}$.

Let $\mathbf{x}(m_1, m_2) \in [0, 255]^3$ represent the input RGB image to be halftoned. $\mathbf{X}(z_1, z_2)$ represents the z-transform of the RGB input image.

$$\mathbf{X}(z_1, z_2) = \sum_{m_1, m_2} \mathbf{x}(m_1, m_2) z^{-m_1} z^{-m_2} \tag{1}$$

We will use an index $\mathbf{m}$ to denote a 2-D spatial index (m_1, m_2) and $\mathbf{z}$ to denote the z-domain index (z_1, z_2).

3. VECTOR COLOR ERROR DIFFUSION

Fig. 1 shows the system block diagram for vector color error diffusion halftoning. The rendering scalar quantizer is defined by $\mathbf{Q} : \mathcal{R}^3 \mapsto \mathcal{U}$ where $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \mathcal{U}_3$ maps the modified input vector $\mathbf{u}(\mathbf{m})$ into a rendered output vector $\mathbf{b}(\mathbf{m})$. $\mathcal{U}_i, i = 1, 2, 3$ represents the alphabet used to represent the ith component of the rendered output. We

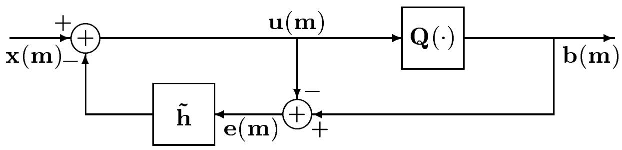


Figure 1: System block diagrams for block error diffusion halftoning where $\tilde{\mathbf{h}}$ represents a fixed 2-D nonseparable FIR error filter with matrix valued coefficients. The vector $\mathbf{m}$ represents the 2-D index (m_1, m_2).

assume that the output to be restricted to one bit per color plane with 255 representing the presence of a color component and 0 representing the absence of a color component, $\mathcal{U}_i = \{0, 255\}, \ \forall i$. The results of this paper are valid for any equal, uniform bit allocation among the RGB channel quantizers.

The quantization error vector $\mathbf{e}(\mathbf{m})$ is formed by subtracting the quantizer input from the output

$$\mathbf{e}(\mathbf{m}) = \mathbf{b}(\mathbf{m}) - \mathbf{u}(\mathbf{m}) \tag{2}$$

The error vector sequence is then filtered by an error filter $\tilde{\mathbf{h}}(\cdot)$ to produce the feedback signal. The error filter $\tilde{\mathbf{h}}(\cdot)$ is a filter with matrix-valued coefficients and will be denoted by the 3×3 matrix-valued sequence $\tilde{\mathbf{h}}(\cdot)$ with support set $\mathcal{S}$. $\tilde{\mathbf{H}}(\cdot)$ represents the z-transform of the matrix-valued multifilter defined by

$$\tilde{\mathbf{H}}(z_1, z_2) = \sum_{m_1, m_2} \tilde{\mathbf{h}}(m_1, m_2) z^{-m_1} z^{-m_2} \tag{3}$$

The filtering operation of a 2-D multifilter is defined by matrix-vector convolution given by

$$\left[\tilde{\mathbf{h}} \star \mathbf{e}\right](\mathbf{m}) = \sum_{\mathbf{k} \in \mathcal{S}} \tilde{\mathbf{h}}(\mathbf{k}) \mathbf{e}(\mathbf{m} - \mathbf{k}) \tag{4}$$

Here the error filter is assumed to have a causal support $\mathcal{S}$ with $(0,0) \notin \mathcal{S}$. We will assume a the standard 4-coefficient Floyd-Steinberg filter [4] support set. In the z domain the matrix-vector convolution becomes a linear transformation by an 3×3 transformation matrix given by

$$Z\left[\tilde{\mathbf{h}} \star \mathbf{e}\right](\mathbf{z}) = \tilde{\mathbf{H}}(\mathbf{z}) \mathbf{X}(\mathbf{z}) \tag{5}$$

The modified input is computed by subtracting the feedback signal from the input signal

$$\mathbf{u}(\mathbf{m}) = \mathbf{x}(\mathbf{m}) - \left[\tilde{\mathbf{h}} \star \mathbf{e}\right](\mathbf{m}) \tag{6}$$

4. FORMULATION OF THE DESIGN PROBLEM

We use the matrix gain model described in [3, 5] to predict the noise shaping behavior of the color error diffusion system. The matrix gain model linearizes the quantizer by considering the error diffusion system as two decoupled linear systems. The first linear system shapes the accounts for signal distortion and is called the *signal path* while the second linear system accounts for the noise shaping and is called the *noise path*. Validation methods for the matrix gain model are presented in [3, 5].

Based on the matrix gain model, we obtain the net noise component of the output as

$$\mathbf{b}_n(\mathbf{m}) = \left([\tilde{\mathbf{I}} - \tilde{\mathbf{h}}] \star \mathbf{n}\right)(\mathbf{m}) \tag{7}$$

Since signal shaping is typically desirable or in any case under user control [5] we only need to concentrate on the noise shaping. We define the objective function J as the average visually weighted noise energy in the output halftone. We use a linear spatially-invariant matrix-valued model for the human visual system denoted by the matrix valued filter function $\tilde{\mathbf{v}}(\cdot)$. We also define a constraint set $\mathcal{C}$ to ensure that all the quantization error (represented in a device independent RGB space) is diffused [5] .

Thus, the color error diffusion system $(\tilde{\mathbf{h}}(\cdot), \tilde{\mathbf{v}}(\cdot))$ for a given vision model $\tilde{\mathbf{v}}(\cdot)$ may be solved for an optimum filter $\tilde{\mathbf{h}}_{opt}(\cdot)$

$$\tilde{\mathbf{h}}_{opt}(\cdot) = \arg \min_{\tilde{\mathbf{h}}(\cdot) \in \mathcal{C}} J \tag{8}$$

where

$$J = E\left[\| \left(\tilde{\mathbf{v}} \star \left[\tilde{\mathbf{I}} - \tilde{\mathbf{h}}\right] \star \mathbf{n}\right)(\mathbf{m}) \|^2\right] \tag{9}$$

and

$$\mathcal{C} = \left\{ \tilde{\mathbf{h}}(\mathbf{i}), \ \mathbf{i} \in \mathcal{S} \mid \sum_{\mathbf{i}} \tilde{\mathbf{h}}(\mathbf{i}) \mathbf{1} = \mathbf{1} \right\} \tag{10}$$

5. OPTIMUM ERROR FILTER DESIGN

The objective function of (9) may be rewritten as

$$\begin{aligned} J \quad = \quad & E \, Tr \left[\tilde{\mathbf{a}}(\mathbf{m}) - \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \tilde{\mathbf{v}}(\mathbf{k}') \tilde{\mathbf{h}}(\mathbf{k}) \mathbf{n}(\mathbf{m} - \mathbf{k}' - \mathbf{k}) \right] \\ & \left[\tilde{\mathbf{a}}(\mathbf{m}) - \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \tilde{\mathbf{v}}(\mathbf{k}') \tilde{\mathbf{h}}(\mathbf{k}) \mathbf{n}(\mathbf{m} - \mathbf{k}' - \mathbf{k}) \right]^T \end{aligned} \tag{11}$$

where we have substituted $\mathbf{a}(\mathbf{m}) = (\tilde{\mathbf{v}} \star \mathbf{n})(\mathbf{m})$ and used the fact that for a vector $\mathbf{x}$, $\| \mathbf{x} \|^2 = Tr\left[\mathbf{x}\mathbf{x}^T\right]$, where Tr denotes the trace operation.

By taking the first partial derivatives of (9) with respect to $\tilde{\mathbf{h}}(\mathbf{i})$ for all $\mathbf{i} \in \mathcal{S}$ and setting them to zero, we obtain the first-order necessary conditions for an optimum solution. This requires that the trace be differentiated with respect to a matrix. To do this some results from linear algebra are required.

The following results are stated here without proof. For proofs of the following, please see [6]:

$$\begin{aligned} \frac{d}{d\tilde{\mathbf{X}}^T} f(\tilde{\mathbf{X}}) &= \left(\frac{d}{d\tilde{\mathbf{X}}} f(\tilde{\mathbf{X}})\right)^T \\ \frac{d}{d\tilde{\mathbf{X}}} Tr(\tilde{\mathbf{A}}\tilde{\mathbf{X}}) &= \tilde{\mathbf{A}}^T \\ \frac{d}{d\tilde{\mathbf{X}}} Tr(\tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}}) &= \tilde{\mathbf{A}}^T \tilde{\mathbf{B}}^T \\ \frac{d}{d\tilde{\mathbf{X}}} Tr(\tilde{\mathbf{X}}^T \tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}}) &= \tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}} + \tilde{\mathbf{A}}^T \tilde{\mathbf{X}}\tilde{\mathbf{B}}^T \\ Tr(\tilde{\mathbf{A}}\tilde{\mathbf{B}}) &= Tr(\tilde{\mathbf{B}}\tilde{\mathbf{A}}) \end{aligned} \tag{12}$$

Using the above results and the linearity of the trace operator we obtain after considerable algebra [5]

$$\sum_{\mathbf{k}} \tilde{\mathbf{v}}^T(\mathbf{k})\tilde{\mathbf{r}}_{\mathbf{an}}(-\mathbf{i}-\mathbf{k}) = \sum_{\mathbf{p}}\sum_{\mathbf{q}}\sum_{\mathbf{s}} \tilde{\mathbf{v}}^T(\mathbf{s})\tilde{\mathbf{v}}(\mathbf{q})\tilde{\mathbf{h}}(\mathbf{p}) \tilde{\mathbf{r}}_{\mathbf{nn}}(\mathbf{p}+\mathbf{q}-\mathbf{s}-\mathbf{i}) \qquad (13)$$

These equations may be regarded as a generalization of the Yule-Walker equations [6] from linear prediction theory to the matrix case, with a generalized linear spatially-invariant weighting. The above set of generalized Yule-Walker equations may be solved for the optimal filter subject to the constraints of (10) using the steepest descent algorithm [6].

We use a white noise image as an approximation to the uncorrelated noise image $\mathbf{n}(\mathbf{m})$. Thus, the required autocorrelation matrices are approximated as:

$$\tilde{\mathbf{r}}_{\mathbf{nn}}(\mathbf{k}) = E\left[\mathbf{n}(\mathbf{m})\mathbf{n}^T(\mathbf{m}+\mathbf{k})\right] \approx \delta(\mathbf{k}) \qquad (14)$$

$$\begin{aligned} \tilde{\mathbf{r}}_{\mathbf{an}}(\mathbf{k}) &= E\left(\sum_{\mathbf{t}} \tilde{\mathbf{v}}(\mathbf{t})\mathbf{n}(\mathbf{m}-\mathbf{t})\right)\mathbf{n}^T(\mathbf{m}+\mathbf{k}) \\ &\approx \sum_{\mathbf{t}} \tilde{\mathbf{v}}(\mathbf{t})\delta(\mathbf{k}+\mathbf{t}) = \tilde{\mathbf{v}}(-\mathbf{k}) \end{aligned} \qquad (15)$$

where $\delta(\mathbf{k})$ is the two-dimensional Kronecker delta function [6]. In the optimization the constraint is enforced by projection onto the convex constraint set. The convergence behavior of this algorithm is discussed in [7]. The algorithm is guaranteed to converge if the convergence parameter in the descent algorithm is chosen to be small enough [7].

The descent algorithm may be formulated as follows:

$$\begin{aligned} (\nabla J)(\tilde{\mathbf{h}}(\mathbf{i})) &\triangleq -\sum_{\mathbf{k}} \tilde{\mathbf{v}}^T(\mathbf{k})\tilde{\mathbf{v}}(\mathbf{i}+\mathbf{k}) \;+ \\ &\quad \sum_{\mathbf{p}}\sum_{\mathbf{q}}\sum_{\mathbf{s}} \tilde{\mathbf{v}}^T(\mathbf{s})\tilde{\mathbf{v}}(\mathbf{q})\tilde{\mathbf{h}}(\mathbf{p})\delta(\mathbf{p}+\mathbf{q}-\mathbf{s}-\mathbf{i}) \\ \tilde{\mathbf{f}}^{(\theta)}(\mathbf{i}) &\triangleq \tilde{\mathbf{h}}^{(\theta)}(\mathbf{i}) - \alpha(\nabla J)(\tilde{\mathbf{h}}^{(\theta)}(\mathbf{i})) \\ \tilde{\mathbf{h}}^{(\theta+1)}(\mathbf{i}) &= \mathcal{P}\left(\tilde{\mathbf{f}}^{(\theta)}(\mathbf{i})\right) \end{aligned} \qquad (16)$$

where θ refers to the iteration number, and $\mathcal{P}$ is the projection operator that projects the iterate into the constraint set $\mathcal{C}$, defined by (10). The convergence parameter $\alpha = 0.005$ in my simulations. The projection operator is defined as [8]

$$\mathcal{P}\left(\tilde{\mathbf{f}}^{(\theta)}(\mathbf{i})\right) \triangleq \tilde{\mathbf{f}}^{(\theta)}(\mathbf{i}) - \frac{1}{3\mid\mathcal{S}\mid}\left(\sum_{\mathbf{i}\in\mathcal{S}} \tilde{\mathbf{f}}^{(\theta)}(\mathbf{i}) - \tilde{\mathbf{I}}\right)\mathbf{1} \qquad (17)$$

Several random initial guesses were tried, and the descent algorithm was terminated when the changes in the objective function were below a predefined threshold. Using this method one may explore different minimizers (solutions that result in nearly the same objective function value). The uniformity in the dot distributions produced by different initial guesses was different. It has been shown [9] that frequency weighted mean squared error alone cannot guarantee optimum dot distributions. We chose a solution that had a reasonably uniform dot distribution.

6. SIMULATION RESULTS

The optimal filter that was obtained based on our calibrated color monitor and was tested on five standard color test images (*lena, peppers, pasta, fruits, hats*). Monitor calibration was performed by first measuring the monitor gamma, and then recording the CIE (X,Y,Z) co-ordinates of the monitor R, G, B guns respectively. First we undo the gamma correction on the RGB image to obtain device independent linear RGB tristimulus values, which are then transformed into XYZ space using the XYZ calibration matrix [10]. We used the pattern-separable model for the human visual system based on the work of Poirson and Wandell [11, 12] although our derivation of the optimum solution assumes the most general linear spatially-invariant color model. The Poirson and Wandell model consists of

1. A linear transformation $\tilde{\mathbf{T}}$ from XYZ into an opponent space and,
2. Separable spatial filtering on each channel using a different spatial filter on each channel. This operation may be regarded as a matrix convolution in the frequency domain by a filter with diagonal matrix-valued coefficients $\tilde{\mathbf{d}}(\cdot)$.

Thus, $\tilde{\mathbf{v}}(\mathbf{m})$ is computed as

$$\tilde{\mathbf{v}}(\mathbf{m}) = \tilde{\mathbf{d}}(\mathbf{m})\tilde{\mathbf{T}} \qquad (18)$$

The parameter spacifications for the model are given in [13].

To evaluate the noise shaping behavior, we produced undistorted halftones using the color signal distortion canceling method developed in [3]. In [3], it was shown that according to the matrix gain model, the quantization error image in the distortion canceling method is in fact the uncorrelated noise injection into the halftoning system. The effective noise shaping gain (in dB) of the optimal filter over the separable Floyd-Steinberg filter may be computed as

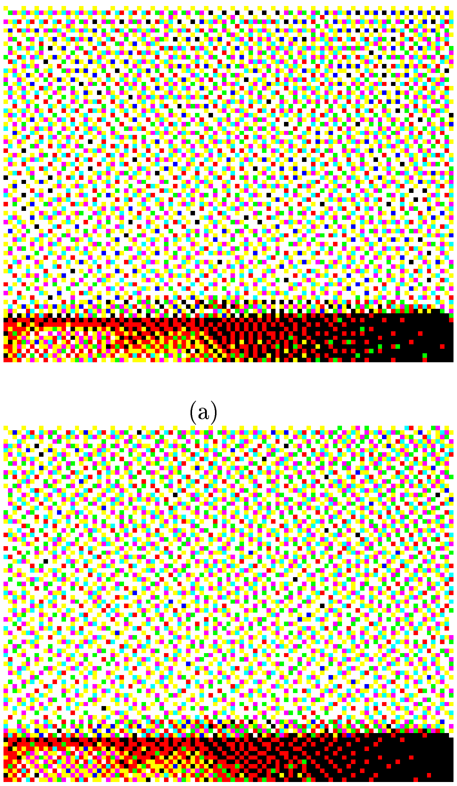
$$NG = 10\log_{10}\left(\frac{J_{fs}}{J_{opt}}\right) \qquad (19)$$

where the numerator and denominator in the argument of the log function are the objective functions computed using (9) for the optimal filter and the Floyd-Steinberg filter, respectively. Sample averages were used to estimate the expectations. Table 1 tabulates the noise gain of the optimal filter over using a separable Floyd-Steinberg error filter.

Fig. 2(a) shows a magnified view of the *pasta* image halftoned using a separable Floyd-Steinberg error filter on the R, G, and B color planes independently and 2(b) show the corresponding results for halftoning with the optimal error filter. The optimal filter results in less visible halftone noise. It significantly reduces color impulses when compared with scalar error diffusion using filters with scalar coefficients. The halftone noise patterns produced by conventional Floyd-Steinberg error scalar filter were significantly more visible when observed on the calibrated monitor as compared to the noise patterns produced by the optimal filter. It must be emphasized that since the optimal filter coefficients are dependent on a particular monitor configuration, the above design process must be applied on a case-by-case basis.

Image	*Noise gain* (dB)
lena	3.29
peppers	1.88
fruits	1.37
pasta	1.47
hats	1.05

Table 1: Noise gain of the optimal filter on standard test images.



(a)

(b)

Figure 2: Performance of the optimal filter. (a) Floyd-Steinberg error filter applied to the R, G, and B planes. (b) Optimum matrix-valued error filter performance. Note the decrease in visibility of color halftone noise achieved by using the optimum filter.

7. CONCLUSION

We have presented a method for optimum error filter design for RGB color error diffusion. The optimum filter minimizes the visibility (on average) of color halftone noise energy with respect to any given linear time-invariant human color vision model. The optimum filter has matrix-valued coefficients allowing for cross-channel diffusion of color errors and is obtained by solving a matrix extension of the Yule-Walker equations subject to diffusion constraints via a gradient descent algorithm.

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