Color Error Diffusion with Generalized Optimum Noise Shaping

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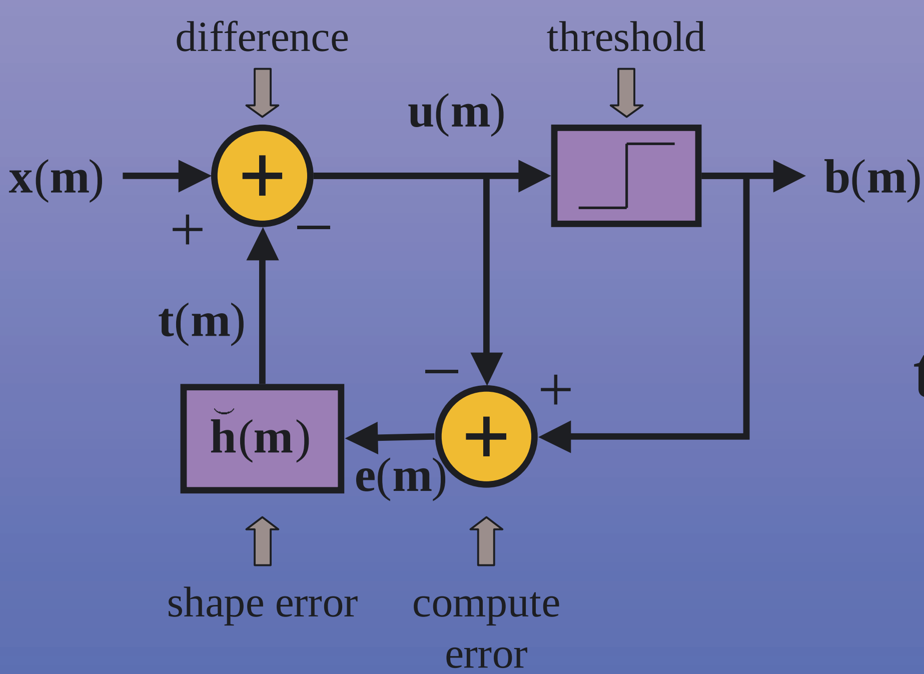
Optimum color noise shaping

- **Vector color error diffusion halftone model**
  - We use the matrix gain model [*Damera-Venkata and Evans, 2001*]
  - Predicts signal frequency distortion
  - Predicts shaped color halftone noise
- **Visibility of halftone noise depends on**
  - Model predicted noise shaping
  - Human visual system model (assumed as an LSI system)
- **Formulation of design problem**
  - Given HVS model and matrix gain model find the color error filter that minimizes average visible noise power subject to certain diffusion constraints
  - Solution leads to a filter with matrix valued coefficients

Vector Color Error Diffusion

- **Error filter has matrix-valued coefficients**
- **Algorithm for adapting matrix coefficients**

[Akarun, Yardimci, Cetin 1997]



$$\mathbf{t}(\mathbf{m}) = \sum_{\mathbf{k}\in\wp} \underbrace{\mathbf{h}(\mathbf{k})}_{\text{matrix}} \; \underbrace{\underline{\mathbf{e}}(\underline{\mathbf{m}} - \mathbf{k})}_{\text{vector}}$$

The Matrix Gain Model

- **Replace scalar gain with a matrix**

$$\breve{\mathbf{K}}_s = \arg\min_{\breve{\mathbf{A}}} E \left\| \mathbf{b}(\mathbf{m}) - \breve{\mathbf{A}}\,\mathbf{u}(\mathbf{m}) \right\|^2 \Big| = \breve{\mathbf{C}}_{\mathbf{bu}} \breve{\mathbf{C}}_{\mathbf{uu}}^{-1}$$

$$\breve{\mathbf{K}}_n = \breve{\mathbf{I}}$$

$\mathbf{u}(\mathbf{m})$ quantizer input
$\mathbf{b}(\mathbf{m})$ quantizer output

  - Noise is uncorrelated with signal component of quantizer input
  - Convolution becomes matrix–vector multiplication in frequency domain

$$\mathbf{B}_n(\mathbf{z}) = \left(\breve{\mathbf{I}} - \breve{\mathbf{H}}(\mathbf{z})\right) \mathbf{N}(\mathbf{z})$$

Noise component of output

$$\mathbf{B}_s(\mathbf{z}) = \breve{\mathbf{K}}\left(\breve{\mathbf{I}} + \breve{\mathbf{H}}(\mathbf{z})\left(\breve{\mathbf{K}} - \breve{\mathbf{I}}\right)\right)^{-1} \mathbf{X}(\mathbf{z})$$

Signal component of output

Designing of the Error Filter

- **Eliminate linear distortion filtering before error diffusion**
- **Optimize error filter h(m) for noise shaping**

$$\min E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right] = E\left\|\breve{\mathbf{v}}(\mathbf{m}) * \left(\breve{\mathbf{I}} - \breve{\mathbf{h}}(\mathbf{m})\right) * \mathbf{n}(\mathbf{m})\right\|^2 \Big|$$

Subject to diffusion constraints

$$\sum_{\mathbf{m}} \breve{\mathbf{h}}(\mathbf{m}) \,|\, \mathbf{1} = \mathbf{1}$$

where

$\breve{\mathbf{v}}(\mathbf{m})$ linear model of human visual system

$*$ matrix-valued convolution

Generalized Optimum Solution

- **Differentiate scalar objective function for visual noise shaping with respect to matrix-valued coefficients**

$$\frac{d\left\{E\left[\left\|\mathbf{b}_n(\mathbf{m})\right\|^2\right]\right\}}{d\mathbf{h}(\mathbf{i})} = \mathbf{0} \quad \forall \mathbf{i} \in \wp$$

- **Write the norm as a trace and then differentiate the trace using identities from linear algebra**

$$\left\|\mathbf{x}\right\| = Tr\left(\mathbf{x}\mathbf{x}'\right)$$

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}'$$

$$\frac{d\left\{Tr\left(\breve{\mathbf{X}}'\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}} + \breve{\mathbf{A}}'\breve{\mathbf{X}}\breve{\mathbf{B}}'$$

$$\frac{d\left\{Tr\left(\breve{\mathbf{A}}\breve{\mathbf{X}}\breve{\mathbf{B}}\right)\right\}}{d\breve{\mathbf{X}}} = \breve{\mathbf{A}}'\breve{\mathbf{B}}'$$

$$Tr\left(\breve{\mathbf{A}}\breve{\mathbf{B}}\right) = Tr\left(\breve{\mathbf{B}}\breve{\mathbf{A}}\right)$$

Generalized Optimum Solution (cont.)

- **Differentiating and using linearity of expectation operator give a generalization of the Yule-Walker equations**

$$\sum_{\mathbf{k}} \breve{\mathbf{v}}'(\mathbf{k})\breve{\mathbf{r}}_{\mathbf{an}}(-\mathbf{i}-\mathbf{k}) = \sum_{\mathbf{p}}\sum_{\mathbf{q}}\sum_{\mathbf{s}} \breve{\mathbf{v}}'(\mathbf{s})\breve{\mathbf{v}}(\mathbf{q})\breve{\mathbf{h}}(\mathbf{p})\breve{\mathbf{r}}_{\mathbf{nn}}(-\mathbf{i}-\mathbf{s}+\mathbf{p}+\mathbf{q})$$

where

$$\mathbf{a}(\mathbf{m}) = \breve{\mathbf{v}}(\mathbf{m}) * \mathbf{n}(\mathbf{m})$$

- **Assuming white noise injection**

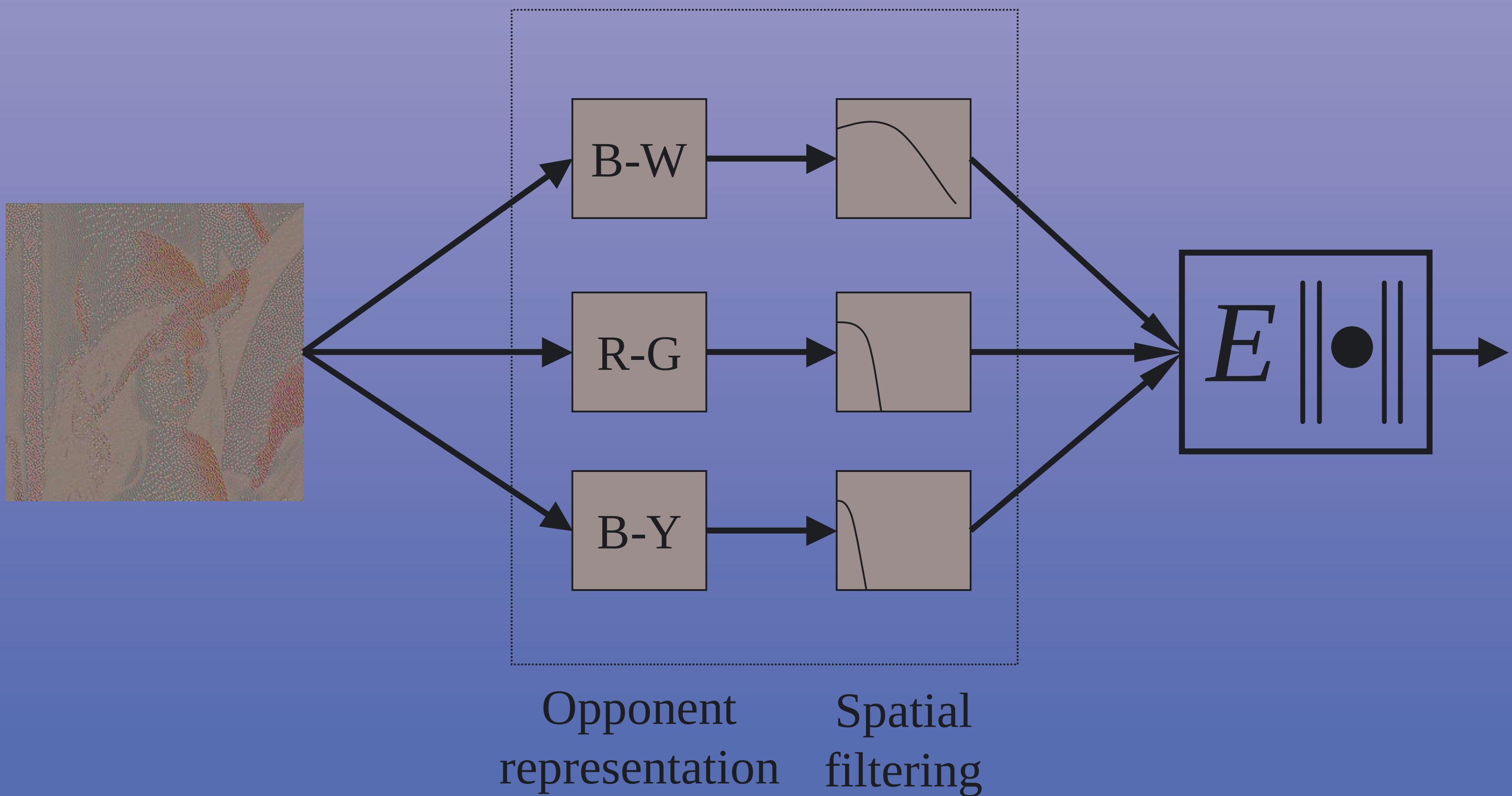
$$\mathbf{r}_{\mathbf{nn}}(\mathbf{k}) = E\left[\mathbf{n}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})\right] \approx \delta(\mathbf{k})$$

$$\mathbf{r}_{\mathbf{an}}(\mathbf{k}) = E\left[\mathbf{a}(\mathbf{m})\,\mathbf{n}'(\mathbf{m}+\mathbf{k})\right] \approx \breve{\mathbf{v}}(-\mathbf{k})$$

- **Solve using gradient descent with projection onto constraint set**

Linear Color Vision Model

- **Pattern-Color separable model [Poirson and Wandell, 1993]**
  - Forms the basis for S-CIELab [Zhang and Wandell, 1996]
  - Pixel-based color transformation



Linear Color Vision Model

- **Undo gamma correction on RGB image**
- **Color separation**
  - Measure power spectral distribution of RGB phosphor excitations
  - Measure absorption rates of long, medium, short (LMS) cones
  - Device dependent transformation ***C*** from RGB to LMS space
  - Transform LMS to opponent representation using ***O***
  - Color separation may be expressed as ***T = OC***
- **Spatial filtering is incorporated using matrix filter** $\breve{\mathbf{d}}(\mathbf{m})$
- **Linear color vision model**

$\breve{\mathbf{v}}(\mathbf{m}) = \breve{\mathbf{d}}(\mathbf{m})\breve{\mathbf{T}}$ where $\breve{\mathbf{d}}(\mathbf{m})$ is a diagonal matrix

Original Image

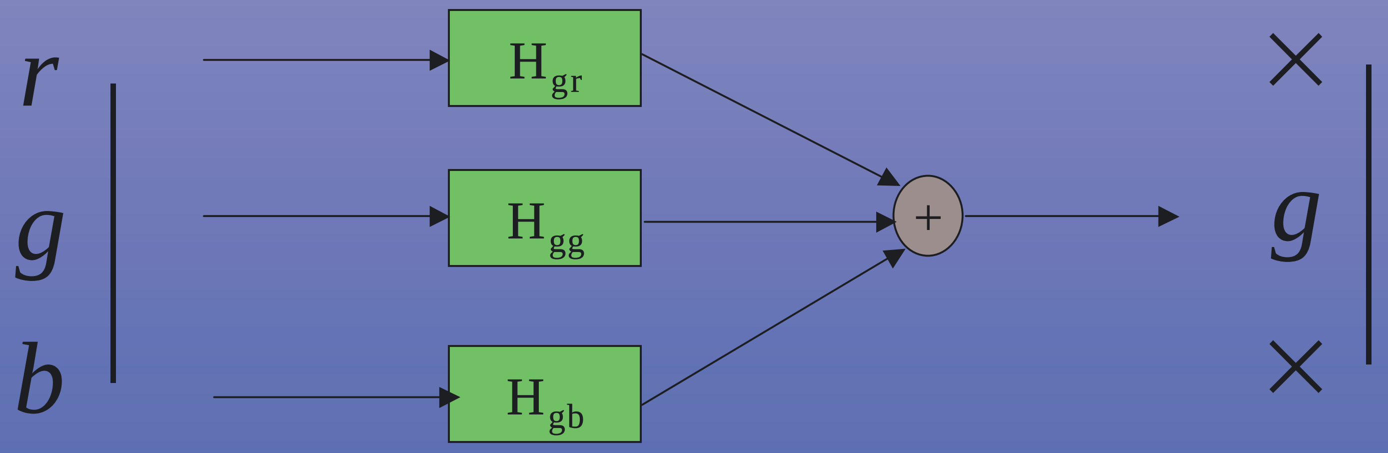
Sample Images and optimum
coefficients for sRGB monitor
available at:
http://signal.ece.utexas.edu/~damera/col-vec.html

Floyd-Steinberg

Optimum Filter

Implementation of Vector Color Error Diffusion

$$\breve{\mathbf{H}}(\mathbf{z}) = \begin{bmatrix} H_{rr}(\mathbf{z}) & H_{rg}(\mathbf{z}) & H_{rb}(\mathbf{z}) \\ H_{gr}(\mathbf{z}) & H_{gg}(\mathbf{z}) & H_{gb}(\mathbf{z}) \\ H_{br}(\mathbf{z}) & H_{bg}(\mathbf{z}) & H_{bb}(\mathbf{z}) \end{bmatrix}$$



Conclusions

- **Design of “optimal” color noise shaping filters**
  - We use the matrix gain model [*Damera-Venkata and Evans, 2001*]
    - Predicts shaped color halftone noise
  - HVS could be modeled as a general LSI system
  - Solve for best error filter that minimizes visually weighted average color halftone noise energy
- **Future work**
  - Above optimal solution does not guarantee “optimal” dot distributions
    - Tone dependent error filters for optimal dot distributions
  - Improve numerical stability of descent procedure