Matrix Gain Model For Vector Color Error Diffusion

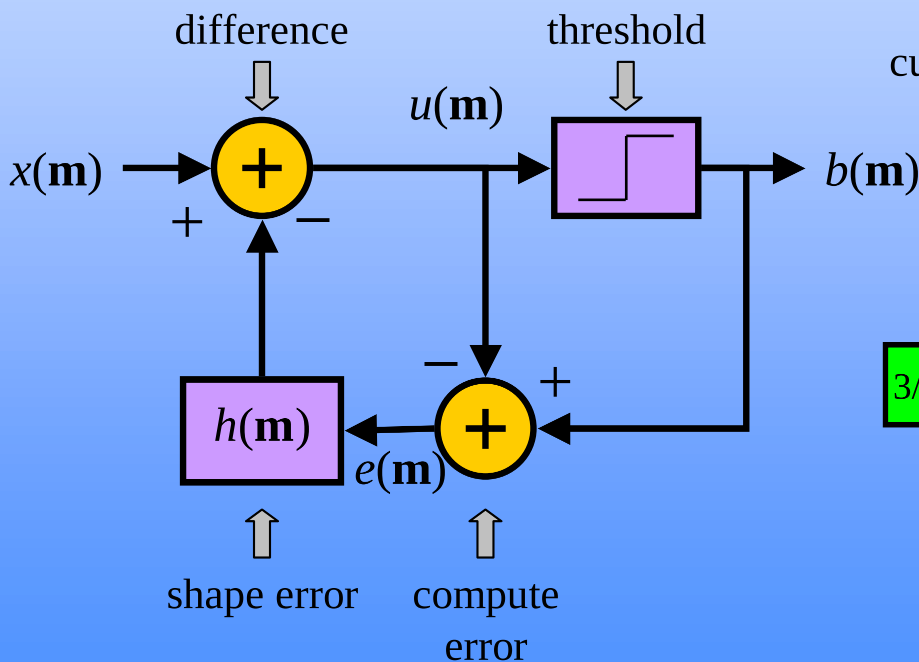
Niranjan Damera-Venkata
and
Brian L. Evans
Embedded Signal Processing Laboratory
The University of Texas at Austin
Austin TX 78712-1084

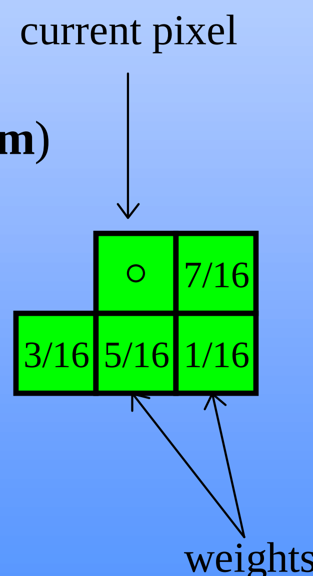
Outline

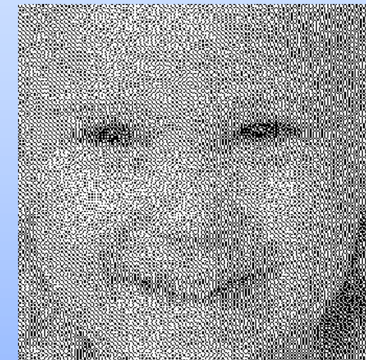
- **Digital halftoning**
  - Modeling grayscale error diffusion halftoning
  - Modeling color error diffusion halftoning
- **Matrix gain model for color error diffusion**
  - Validating the signal shaping predicted by the model
  - Validating the noise shaping predicted by the model
- **Conclusions**

Grayscale Error Diffusion

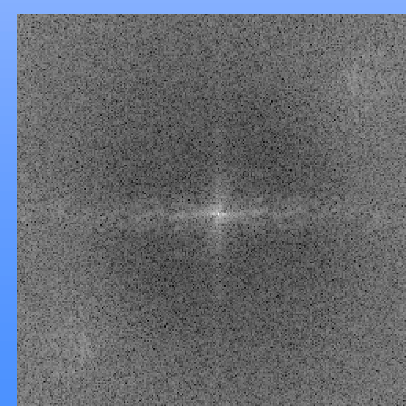
- **Shape quantization noise into high frequencies**
- **Two-dimensional sigma-delta modulation**
- **Design of error filter is key to high quality**







Error Diffusion



Spectrum

Modeling Grayscale Error Diffusion

- **Sharpening is caused by a correlated error image** **[Knox, 1992]**

Floyd-Steinberg

Jarvis

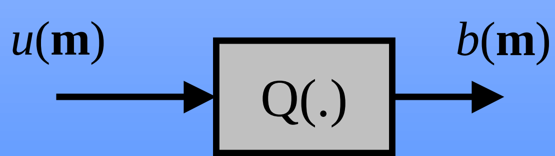
Error images

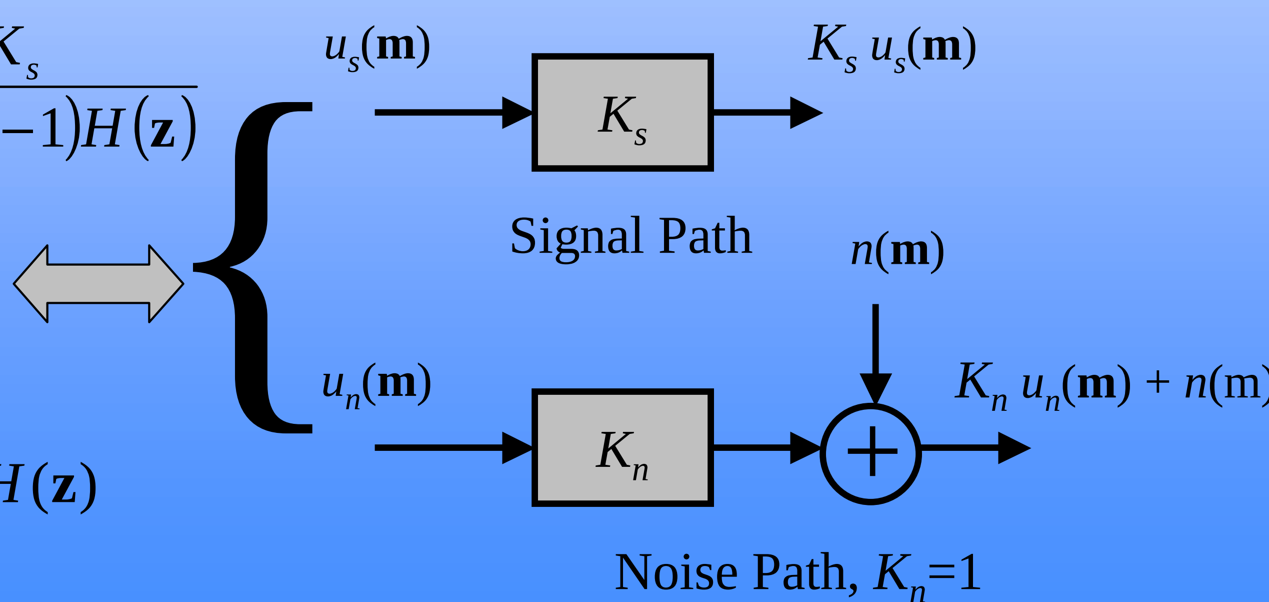
Halftones

Modeling Grayscale Error Diffusion

- **Apply sigma-delta modulation analysis to two dimensions**
  - Linear gain model for quantizer in 1-D [Ardalan and Paulos, 1988]
  - Linear gain model for grayscale image [Kite, Evans, Bovik, 2000]
  - Signal transfer function (STF) and noise transfer function (NTF)
  - 1 – H(**z**) is highpass so H(**z**) is lowpass

$$STF = \frac{B_s(\mathbf{z})}{X(\mathbf{z})} = \frac{K_s}{1 + (K_s - 1) H(\mathbf{z})}$$



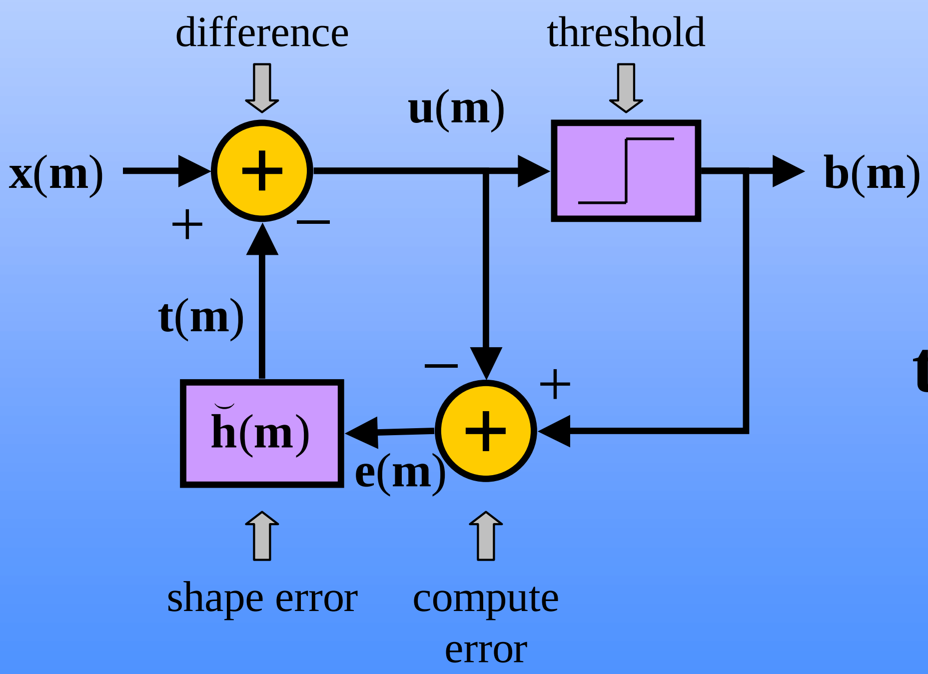


$$NTF = \frac{B_n(\mathbf{z})}{N(\mathbf{z})} = 1 - H(\mathbf{z})$$

Vector Color Error Diffusion

- **Error filter has matrix-valued coefficients**
- **Algorithm for adapting matrix coefficients**

[Akarun, Yardimci, Cetin 1997]



$$\mathbf{t}(\mathbf{m}) = \sum_{\mathbf{k}\in\wp} \underbrace{\breve{\mathbf{h}}(\mathbf{k})}_{\text{matrix}} \; \underbrace{\mathbf{e}(\mathbf{m}-\mathbf{k})}_{\text{vector}}$$

The Matrix Gain Model

- **Replace scalar gain with a matrix**

$$\breve{\mathbf{K}}_s = \arg\min_{\breve{\mathbf{A}}} E\left\{ \left\| \mathbf{b}(\mathbf{m}) - \breve{\mathbf{A}}\,\mathbf{u}(\mathbf{m}) \right\|^2 \right\} = \breve{\mathbf{C}}_{\mathbf{bu}} \breve{\mathbf{C}}_{\mathbf{uu}}^{-1}$$

$$\breve{\mathbf{K}}_n = \breve{\mathbf{I}}$$

u(**m**) quantizer input
b(**m**) quantizer output

  - Noise is uncorrelated with signal component of quantizer input
  - Convolution becomes matrix–vector multiplication in frequency domain

$$\mathbf{B}_n(\mathbf{z}) = \left(\breve{\mathbf{I}} - \breve{\mathbf{H}}(\mathbf{z})\right) \mathbf{N}(\mathbf{z})$$

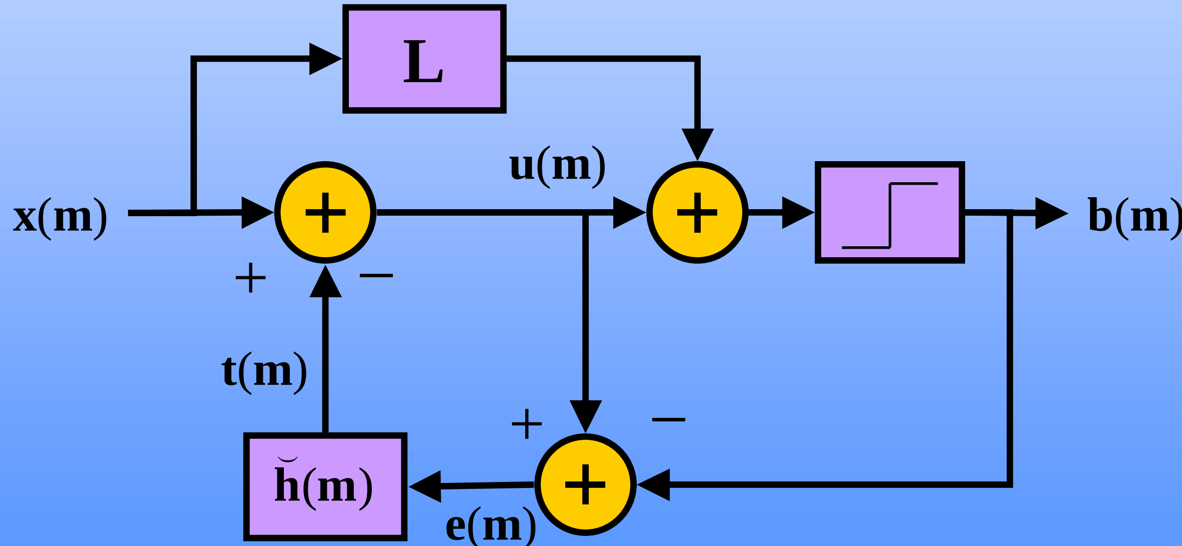
Noise component of output

$$\mathbf{B}_s(\mathbf{z}) = \breve{\mathbf{K}}\left(\breve{\mathbf{I}} + \breve{\mathbf{H}}(\mathbf{z})\left(\breve{\mathbf{K}} - \breve{\mathbf{I}}\right)\right)^{-1} \mathbf{X}(\mathbf{z})$$

Signal component of output

How to Construct an Undistorted Halftone

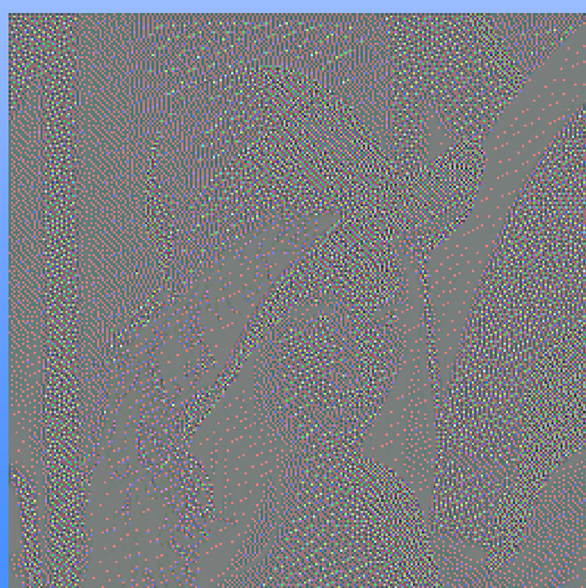
- **Pre-filter with inverse of signal transfer function to obtain undistorted halftone** $\breve{\mathbf{G}}(\mathbf{z}) = \left[\breve{\mathbf{I}} + \breve{\mathbf{H}}(\mathbf{z})\left(\breve{\mathbf{K}} - \breve{\mathbf{I}}\right)\right]\breve{\mathbf{K}}^{-1}$
- **Pre-filtering is equivalent to the following when** $\breve{\mathbf{L}} = \breve{\mathbf{K}}^{-1} - \breve{\mathbf{I}}$



Modified error diffusion

Validation #1 by Constructing Undistorted Halftone

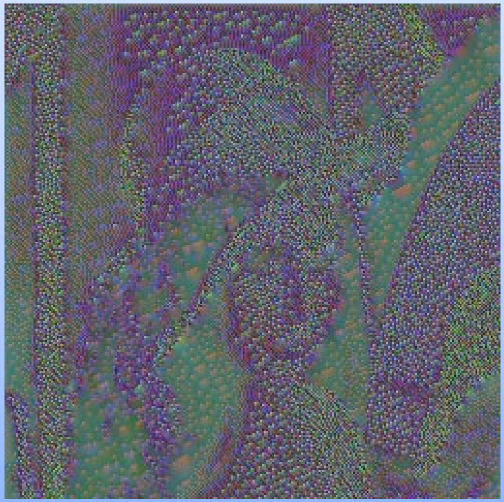
- **Generate linearly undistorted halftone**
- **Subtract original image from halftone**
- **Since halftone should be "undistorted", the residual should be uncorrelated with the original**



Correlation matrix of residual image (undistorted halftone minus input image) with the input image

$$\breve{\mathbf{C}}_{\mathbf{rx}} = \begin{bmatrix} 0.0052 & 0.0009 & 0.0040 \\ 0.0054 & 0.0023 & 0.0020 \\ 0.0058 & 0.0011 & 0.0027 \end{bmatrix}$$

Validation #2 by Knox's Conjecture



$$\mathbf{E}_s(\mathbf{z}) = 0$$

$$\mathbf{E}_n(\mathbf{z}) = \mathbf{N}(\mathbf{z})$$

Correlation matrix for an error image and input image for an error diffused halftone

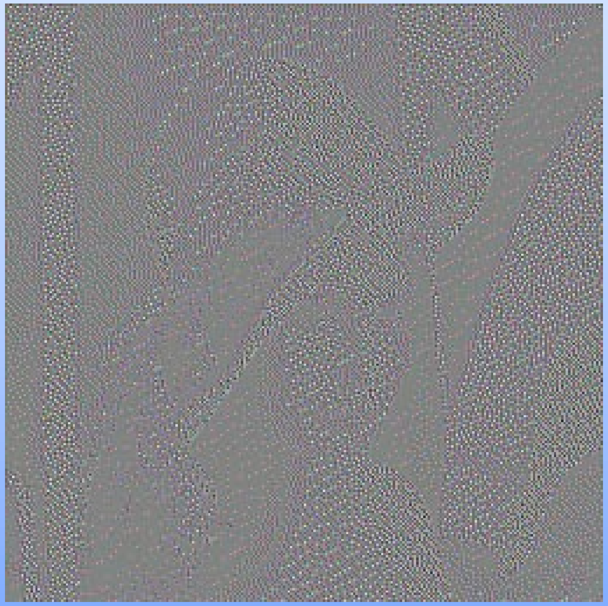
$$\breve{\mathbf{C}}_{\mathbf{ex}} = \begin{pmatrix} 0.3204 & 0.2989 & 0.0999 \\ 0.2787 & 0.3295 & 0.1605 \\ 0.2063 & 0.2952 & 0.1836 \end{pmatrix}$$

Correlation matrix for an error image and input image for an undistorted halftone

$$\breve{\mathbf{C}}_{\mathbf{ex}} = \begin{pmatrix} 0.0455 & 0.0235 & 0.0122 \\ 0.0493 & 0.0144 & 0.0164 \\ 0.0428 & 0.0142 & 0.0150 \end{pmatrix}$$

Validation #3 by Distorting Original Image

- **Validation by constructing a linearly distorted original**
  - Pass original image through error diffusion with matrix gain substituted for quantizer
  - Subtract resulting color image from color halftone
  - Residual should be shaped uncorrelated noise

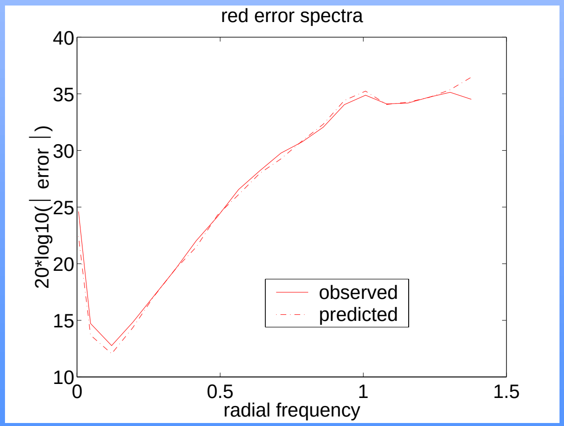


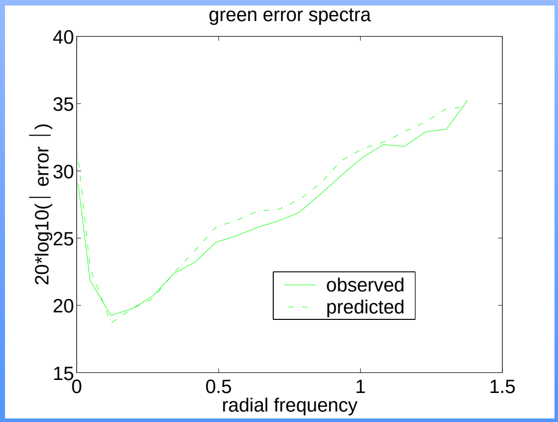
Correlation matrix of residual image (halftone minus distorted input image) with the input image

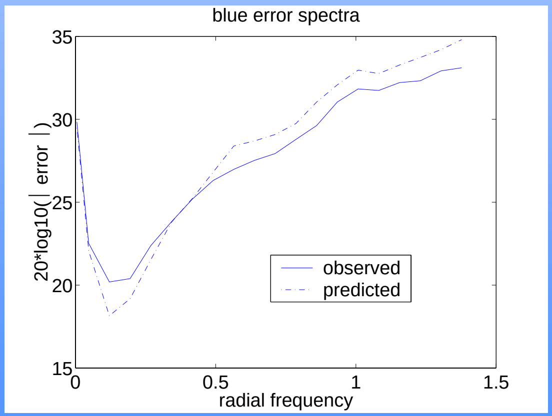
$$\breve{\mathbf{C}}_{\mathbf{rx}} = \begin{pmatrix} 0.0067 & 0.0007 & 0.0051 \\ 0.0065 & 0.0039 & 0.0049 \\ 0.0082 & 0.0040 & 0.0062 \end{pmatrix}$$

Validation #4 by Noise Shaping

- **Noise process is error image for an undistorted halftone**
- **Use model noise transfer function to compute noise spectrum**
- **Subtract actual halftone from modeled halftone and compute actual noise spectrum**







Conclusions

- **Modeling of color error diffusion in the frequency domain using a coupled matrix gain and noise injection approach**
- **Linearizes error diffusion**
- **Predicts linear distortion and noise shaping effects**
- **Signal frequency distortion may be ‘‘cancelled’’**
- **Filters may be designed for optimum noise shaping**