Performance Bounds in OFDM Channel Prediction

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***Abstract*— OFDM channel prediction algorithms have been proposed to combat feedback delay in adaptive OFDM systems. However, no simple closed-form expression that relates prediction mean-square error (MSE) with the design parameters in OFDM channel prediction has been reported in the literature. This paper attempts to fill that gap by deriving simple closed-form asymptotic lower bounds of the MSE for OFDM channel prediction in deterministic doubly-selective fading channels. The bounds relate the best-achievable MSE with the predictor design parameters in a simple manner, thus providing important insight into the design of OFDM channel predictors. We compare the bounds to the deterministic Cramer-Rao bound for parameters consistent with IEEE 802.16e, and show that the asymptotic bounds are reasonably tight for practical scenarios.**

I. Introduction

Adaptive OFDM systems overcome the limitation of conventional OFDM by allowing the transmitter to vary the power, modulation, and code rate on each subcarrier depending on the current channel state information (CSI) [1]. This requires the transmitter to have knowledge of the CSI, which can be obtained through feedback from the receiver's channel estimates, or through its own estimates in a time division duplex (TDD) reciprocal channel. In high mobility environments, in which the Doppler frequency is high and the channel changes rapidly, the CSI used by the transmitter would be outdated due to the processing and feedback delays, thereby causing significant performance degradation [2].

One possible solution to the feedback delay problem is the concept of *channel prediction*, whereby previous estimates of the time-varying channel is used to predict the fading value at a future time instant. In recent years, OFDM channel prediction algorithms [3][4][5][6] have been proposed to combat the feedback delay problem for closed-loop link adaptation in OFDM systems. In the design of these channel prediction algorithms, the ability to compute upper bounds on the performance (or lower bounds on the error) of channel prediction as a function of these parameters is a very useful tool for the systems engineer to make appropriate design decisions. Furthermore, these bounds can also be used as a benchmark for all the different algorithms that have been proposed in the literature.

In [7], approximate mean-square error (MSE) bounds have been derived using a first-order Taylor approximation of the channel prediction MSE expression for a parametric time- and frequency-selective channel model. The Cramer-Rao bound (CRB) is then used to derive a lower bound on this MSE, and a simple approximate expression when the prediction length is large was derived. The effect of the number of time-domain pilot blocks on the MSE was investigated using these expressions. However, the effect of the pilot subcarriers in the frequency domain, and the pilot spacings in both time and frequency dimensions, is not apparent in their expressions.

In this paper, we extend the work of [7] by considering the channel estimation and prediction problem as a 2-dimensional frequency estimation problem. This approach is attractive in OFDM systems where pilot subcarriers in both the time and frequency domains are used. By using a two-dimensional parameterization, we are able to assess the prediction performance for a particular subcarrier and prediction length on 4 parameters – the number of pilots in the time/frequency domains (N_t and N_f), and their corresponding pilot spacings (D_t and D_f). Also in contrast to [7], we use the invariance property of the maximum likelihood estimator (MLE)[1] to derive a simple expression for the CRB on the prediction error, without explicitly resorting to first-order Taylor approximations to the MSE. Interestingly, the resulting expression using either method is identical, thus confirming the result of the previous work in a more rigorous manner.

Although useful in its own right, conventional CRB analysis requires Monte-Carlo generation of channel realizations to obtain results independent of a particular channel. And with the objective of investigating the effect of various channel prediction parameter configurations, this requires performing a Monte-carlo simulation for each parameter configuration. This is a heavy computational burden even for the case of offline analysis, especially when N_t and N_f are large. This motivated us to derive a simple closed-form expression for the lower bound on prediction MSE in the asymptotic case of large N_t and N_f. This expression relates the lowest-achievable MSE for an unbiased estimator with the predictor design parameters in a simple manner, which is independent of any particular channel realization. This provides important insight into the design of OFDM channel predictors, without the need for expensive Monte-Carlo numerical calculations. It is

[1]The invariance property for ML allows us to construct the MLE for a function of parameters $\boldsymbol{\theta}$, i.e. $\mathbf{f} = F(\boldsymbol{\theta})$, by computing the MLE of the parameters and using it in the function, i.e. $\hat{\mathbf{f}} = F(\hat{\boldsymbol{\theta}})$ [8, Ch. 6.3].

also shown through numerical examples that the asymptotic expression is reasonably tight for typical channel parameters with moderate values for N_t and N_f, further motivating the use of these simple expressions.

II. System Model

A. Time- and Frequency-Selective Wireless Channel

The continuous-time impulse response of a time- and frequency-selective wireless channel can be modeled as the superposition of a discrete number of resolvable paths [9]

$$h_c(t,\eta) = \sum_{p=0}^{L-1} \gamma_p(t)\delta(\eta - \eta_p(t)) \tag{1}$$

where $\eta_p(t)$ and $\gamma_p(t)$ are the time-varying delay and complex gain corresponding to the pth path, and L is the number of propagation paths.

We assume that the wireless propagation environment has reflectors and scatterers that are far enough from the receiver, such that the waves incident on the receiving antenna are *plane waves* having a common angle of incidence. We also assume that the receiver travels in a linear motion and with constant velocity, such that the Doppler frequency is time independent.[2] These assumptions allow us to express $\gamma_p(t)$ to be fitted by a linear combination of complex sinusoidal rays

$$\gamma_p(t) = \sum_{r=0}^{M_p-1} \alpha_{r,p} e^{j\nu_{r,p}} e^{j2\pi\xi_{r,p}t} \tag{2}$$

where M_p is the number of rays contributing to the pth path, $\alpha_{r,p}$ is the amplitude, $\nu_{r,p}$ is the phase, and $\xi_{r,p}$ is the Doppler frequency, respectively, for the rth ray in the pth path. A similar model has been used by various researchers for wireless channel characterization and prediction [7] [10] [11].

Substituting (2) into (1), combining indices, and taking its Fourier transform, we get the frequency response of the time-varying channel as

$$H_c(t,f) = \sum_{i=1}^{M} \alpha_i e^{j\nu_i} e^{j2\pi(\xi_i t - \eta_i f)} \tag{3}$$

where $M = \sum_{p=0}^{L-1} M_p$ is the total number of complex sinusoidal rays, and each ray is characterized by the quadruplet $\{\alpha_i, \nu_i, \xi_i, \eta_i\}$. We assume that no two rays share the same pair of time-delay and Doppler frequency values $\{\xi_i, \eta_i\}$ (otherwise they can be combined to form one ray), but different rays may share the same delay or Doppler frequency. Since practical values of ξ_i and η_i are bounded, i.e. $0 \le \xi_i < \xi_{max}$ and $0 \le \eta_i < \eta_{max}$, (3) has finite support in both Doppler and delay domains. Assuming that the OFDM system with symbol period T_{sym} and subcarrier spacing Δf has proper cyclic extension and sample timing, the sampled channel frequency response at the kth tone of the nth OFDM block can be expressed as a linear combination of 2-dimensional complex sinusoidal rays

$$\begin{aligned} H(n,k) &\triangleq H_c(nT_{sym}, k\Delta f) \\ &= \sum_{i=0}^{M-1} \alpha_i e^{j\nu_i} e^{j2\pi(f_i n - \tau_i k)} \end{aligned} \tag{4}$$

where $f_i = \xi_i T_{sym}$ is the normalized Doppler frequency in radians, and $\tau_i = \eta_i \Delta f$ is the normalized frequency shift due to the time delay.

B. OFDM Baseband Model

We assume an OFDM system where the cyclic-prefix time T_{cp} is greater than the maximum delay spread of the channel τ_{max}, the timing and carrier synchronization is perfect, and the used subcarriers are within the flat region of the frequency response of both transmit and receiver pulse-shaping filters, such that the received signal at the kth subcarrier of the nth symbol can be written as

$$\begin{aligned} Y(n,k) &= H(n,k)X(n,k) + W(n,k) \\ &\quad -N_{used}/2 \le k \le N_{used}/2, \quad k \neq 0 \end{aligned} \tag{5}$$

where $W(n,k)$ is a circular-symmetric complex additive white Gaussian noise (AWGN) with variance σ^2.

Furthermore, we assume that there are N_f equally spaced pilot subcarriers in every OFDM symbol inserted within the N_{used} used subcarriers. Let $D_f = \lfloor N_{used}/N_f \rfloor$ denote the frequency spacing in terms of the number of subcarriers between two adjacent pilot subcarriers, where

$$D_f \le 1/(\Delta f \tau_{max}) \tag{6}$$

to avoid aliasing in the frequency domain. We also let $\mathcal{K} \triangleq \{k_0, \ldots, k_{N_f-1}\}$ denote the set of pilot subcarrier indices, where

$$k_q = \left(\frac{N_f}{2} - q\right) D_f, \quad q = 0, \ldots, N_f - 1 \tag{7}$$

We also assume that a block of N_t current and previous OFDM symbols (which we call pilot symbols) equally spaced in time are available for the channel prediction algorithm. Let D_t denote the time spacing in terms of the number of OFDM symbols between pilot symbols, and where

$$D_t \le 1/(2T_{sym} f_{max}) \tag{8}$$

in order to avoid aliasing in the time domain. We also let $\mathcal{N} \triangleq \{n_0, \ldots, n_{N_t-1}\}$ denote the set of pilot symbol indices, where

$$n_l = lD_t, \quad l = 0, \ldots, N_t - 1 \tag{9}$$

Using these $N_t \times N_f$ pilot subcarriers, we can perform a least-squares estimate of the channel at the pilot locations using the received signal $Y(n_l, k_q)$ and the known pilot symbols $X(n_l, k_q)$, given as

$$\begin{aligned} \hat{H}_{LS}(l,q) \quad = \quad & \sum_{i=0}^{M-1} \alpha_i e^{j\phi_i} e^{j(\omega_i l + \varphi_i q)} + \widetilde{W}(l,q), \\ & l = 0, \ldots, N_t - 1 \quad q = 0, \ldots, N_f - 1 \end{aligned} \tag{10}$$

[2] These assumptions are valid in outdoor propagation scenarios, e.g mobile cellular environments; and in considering small time windows of interest, i.e. a few wavelengths.

where

$$\begin{aligned}\phi_i &= \nu_i - 2\pi\tau_i D_f \frac{N_f-1}{2} \\ \omega_i &= 2\pi f_i D_t \\ \varphi_i &= 2\pi\tau_i D_f\end{aligned}$$

The basic premise in OFDM channel prediction is to simply use the least squares estimates of the channel (10) to in turn estimate the parameters in the channel model, i.e. α_i, ϕ_i, ω_i, and φ_i. Channel prediction is then performed through simply extrapolating this model. Note that this idea has been used by several researchers [12] [11], albeit for narrowband, flat-fading channels. In the following section, we derive bounds on the MSE using this approach to OFDM channel prediction.

III. Prediction Mean-square Error Bounds

Observe that (10) is in the standard form of a two-dimensional complex sum-of-sinusoids in additive white Gaussian noise. Estimating the parameters of this model is a classical problem, and is very well studied in the radar, sonar imaging, and other array signal processing literature [13]. Hence, we can use classical results on the CRB for 2-D sinusoidal estimation to derive the CRB for OFDM channel prediction.

A. OFDM Channel Prediction CRB

Let

$$\boldsymbol{\theta} = [\boldsymbol{\theta}_0^T, \ldots, \boldsymbol{\theta}_{M-1}^T]^T \tag{11}$$

with

$$\boldsymbol{\theta}_i = [\alpha_i, \phi_i, \omega_i, \varphi_i]^T \tag{12}$$

denote the $4M$-length vector of channel parameters which we estimate from (10).

Let us rewrite the physical channel model (4) in terms of the parameters we estimate, i.e.

$$H(n,k,\boldsymbol{\theta}) = \sum_{i=0}^{M-1} H_i(n,k,\boldsymbol{\theta}_i) \tag{13}$$

with

$$H_i(n,k,\boldsymbol{\theta}_i) = \alpha_i e^{j\left(\phi_i + \varphi_i \frac{N_f-1}{2}\right)} e^{j\left(\frac{\omega_i}{D_t}n - \frac{\varphi_i}{D_f}k\right)} \tag{14}$$

Note that the dependence of the channel response on the parameter vectors (11) is made explicit in our notation.

Since our channel model is a continuous function of the unknown parameters, by the invariance principle of the maximum likelihood estimate, we can bound the prediction mean-square error using the Cramér-Rao lower bound for function of parameters [8, Ch. 6.4], i.e.

$$\begin{aligned} e(n,k) &= \mathbb{E}\left\{\left|H(n,k,\boldsymbol{\theta}) - H(n,k,\hat{\boldsymbol{\theta}})\right|^2\right\} \\ &\geq \frac{\partial H(n,k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^H \mathbf{I}_{2D}^{-1}(\boldsymbol{\theta}) \frac{\partial H(n,k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \end{aligned} \tag{15}$$

where

$$\frac{\partial H(n,k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{\partial H_1(n,k,\boldsymbol{\theta}_1)}{\partial \boldsymbol{\theta}_1} \\ \vdots \\ \frac{\partial H_{M-1}(n,k,\boldsymbol{\theta}_{M-1})}{\partial \boldsymbol{\theta}_{M-1}} \end{bmatrix} \tag{16}$$

with

$$\frac{\partial H_i(n,k,\boldsymbol{\theta}_i)}{\partial \boldsymbol{\theta}_i} = H_i(n,k,\boldsymbol{\theta}_i) \begin{bmatrix} \frac{1}{\alpha_i} \\ j \\ j\frac{n}{D_t} \\ -j\frac{k-\bar{k}}{D_f} \end{bmatrix} \tag{17}$$

where $\bar{k} = \left(\frac{N_f-1}{2}\right) D_f$ and $\mathbf{I}_{2D}^{-1}(\boldsymbol{\theta})$ is the well-known Cramér-Rao bound matrix for 2-dimensional sinusoidal parameter estimation, which can be found in [14, Appendix A]. An interesting point to note is that [7] and [11] came up with a similar MSE bound as (15), using a different and more tedious approach (first-order Taylor expansion for the MSE function). Thus, using the invariance principle, we were able to verify their result using a simpler approach.

B. Asymptotic CRB

Although the bound (15) can be used to study the effects of various parameters on the prediction MSE, its expression is not readily interpretable. Furthermore, since the bound depends on the actual parameter vector $\boldsymbol{\theta}$, Monte-Carlo simulations that generate realizations of this parameter vector assuming a certain probability distribution is required to assess the performance. And with the objective of investigating the effect of various channel prediction parameter configurations, this requires performing a Monte-carlo simulation for each parameter configuration. Note that approximately $8M^2N_tN_f + O((4M)^3)$ operations are required to compute the CRB for each channel realization. This is a heavy computational burden even for the case of offline analysis, specially when M, N_t, and N_f are large. This motivated us to derive a simple closed-form expression for the lower bound on prediction MSE in the asymptotic case of large N_t and N_f.

Thus, we consider the asymptotic MSE given as

$$\begin{aligned} \tilde{\epsilon}(n,k) &\geq \lim_{\min(N_t,N_f)\to\infty} \epsilon(n,k) \\ &= \frac{\partial H(n,k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}^H \tilde{\mathbf{I}}_{2D}^{-1} \frac{\partial H(n,k,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \end{aligned} \tag{18}$$

where

$$\tilde{\mathbf{I}}_{2D}^{-1} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{K}_M \end{bmatrix} \tag{19}$$

is the block diagonal asymptotic CRB for 2-D superimposed exponential signals, with diagonal elements given by [15]

$$\mathbf{K}_i = \begin{bmatrix} \frac{\sigma^2}{2N_tN_f} & 0 & 0 & 0 \\ 0 & \frac{7\sigma^2}{2\alpha_i^2 N_t N_f} & \frac{-3\sigma^2}{\alpha_i^2 N_t^2 N_f} & \frac{-3\sigma^2}{\alpha_i^2 N_t N_f^2} \\ 0 & \frac{-3\sigma^2}{\alpha_i^2 N_t^2 N_f} & \frac{6\sigma^2}{\alpha_i^2 N_t^3 N_f} & 0 \\ 0 & \frac{-3\sigma^2}{\alpha_i^2 N_t N_f^2} & 0 & \frac{6\sigma^2}{\alpha_i^2 N_t N_f^3} \end{bmatrix} \tag{20}$$

After further simplification, we have

$$\tilde{\epsilon}(n,k) \geq \sigma^2 M \left(\frac{4}{N_t N_f} - \frac{6n}{N_t^2 N_f D_t} + \frac{6n^2}{N_t^3 N_f D_t^2} + \frac{6\left(k-\bar{k}\right)}{N_t N_f^2 D_f} + \frac{6\left(k-\bar{k}\right)^2}{N_t N_f^3 D_f^2} \right) \tag{21}$$

Note that this bound is tightest when N_t and N_f are large, or equivalently the spacing of the frequencies are much larger than the resolution limit, i.e. $|\tau_i - \tau_j| \gg 1/N_f$ and $|f_i - f_j| \gg 1/N_t$. Note also that this bound is actually achievable by the maximum likelihood method [15] and nonlinear-least squares method [16] in the large sample limit.

Using this simple expression for the lower bound on MSE, we could easily deduce the impact of the various parameters on the MSE, e.g.

- The bound increases linearly with increasing noise variance σ^2 and number of 2-D sinusoidal rays M. This is intuitively satisfying, and agrees with previous results that dense multipath channel environments, i.e. M large, are the hardest to predict [11].
- The contribution to the overall MSE from the error variances corresponding to the estimate of frequencies ω_i and φ_i grow quadratically with n and $|k|$, emphasizing the importance of estimating these accurately.
- In general, N_t and N_f and the downsampling factors D_t and D_f should be chosen as large as possible to decrease the MSE bound, but are of course subject to limitations imposed by other system consideration like complexity, training overhead, validity of the linear model, etc.

IV. Numerical Results

The purpose of this section is to study the effect of the various parameters on the OFDM prediction MSE bounds using numerical evaluation on a computer. In order to be as concrete and as realistic as possible, we provide numerical results for an outdoor mobile OFDM system based on the IEEE 802.16e mobile broadband wireless access standard [17], whose parameters are given in Table I.

We consider a wireless channel based on the ITU Vehicular A [18] power delay profile, which is a sparse multipath model with 6 taps, each tap having Clarke's classical U-shaped spectrum [9]. We assume that each path is composed of 16 complex sinusoidal rays, whose amplitude, phase, and Doppler frequency parameters are generated using the modified Jake's simulation model [19].

TABLE I

Simulation Parameters

Parameter	Value
Bandwidth (B)	5 MHz
Sampling Frequency (f_s)	5.712 Ms/s
Carrier Frequency (f_c)	2.4 GHz
Mobile Velocity (v)	75 km/hr
Maximum Doppler Frequency (f_{max})	166.67 Hz
N_{fft}	512
N_{used}	426

Using the above channel parameters, the downsampling factors should be chosen as $D_f \leq 35$ (6) and $D_t \leq 29$ (8) to avoid aliasing in both domains. We also require the pilot subcarriers in the frequency domain to be within the used subcarriers $N_f D_f \leq N_{used} = 426$. The restriction on $N_t D_t$ however, requires more knowledge on the exact scattering environment, since this value should be confined to the interval over which the linear approximation (2) is valid. In [20], a rough approximation to this interval based on the accumulated phase error in the sinusoidal model is given as

$$T = \sqrt{\frac{\lambda r}{3v^2}} \tag{22}$$

where λ is the wavelength, r is the distance of the mobile to the closest scatterer, and v is the mobile velocity. We estimate the distance of the scatter as $r = \tau_{rms} c$ where τ_{rms} is the root-mean-square delay spread [9], and $c = 3 \times 10^8$ m/s is the speed of light. Using these approximations, the interval over which the channel model is valid is approximately 17λ or 1024 OFDM symbols. Thus, we require $N_t D_t + n \leq 1024$, where n is the prediction length.

Figure 1 shows the MSE bounds for subcarrier $k = N_{used}/2$ and prediction length $n = D_t$ versus the number of pilots used in time (N_t) and frequency (N_f) domains. The upper surface is the CRB, and the lower surface is the Asymptotic CRB. We set $N_t D_t = 1024$ and $N_f D_f = 426$ to satisfy parameter restrictions, relaxing the integer requirement on D_t and D_f. The MSE clearly decreases as we increase both N_t and N_f. For values of $N_t < 40$ and for values of $N_f < 10$, the surface is not plotted since the Fisher information matrix in this case is singular. This is because we have $4M = 4 \times 6 \times 16 = 384$ parameters to estimate, requiring $N_t N_f > 384$ in order for the problem to be statistically well-posed. Furthermore, note that for reasonable values of N_t and N_f, the asymptotic Cramer-Rao bound is close to the CRB, and the two bounds become closer as we increase N_t and N_f as expected.

Figure 2 shows the MSE bounds for all the used subcarriers versus the D_t, where the prediction length $n = 60$ ($\approx 1\lambda$), and $N_t = 50$, $N_f = 42$, $D_f = 10$. Once again, the upper surface is the CRB, and the lower surface is the asymptotic CRB. Note that increasing D_t while holding N_t fixed decreases the MSE as expected. This is an important concept since increasing D_t does not cost extra in terms of computations, and is thus a good tool to improve MSE performance without increasing complexity. The asymptotic CRB is also tight in this case.

Figure 3 shows the MSE bounds for all subcarriers versus prediction length expressed in wavelengths, where the upper and lower surfaces correspond to the CRB and asymptotic CRB respectively. We set $N_t = 100$, $D_t = 19$, $N_f = 141$, and $D_f = 3$. These parameters coincide with the 2 ms frame size for IEEE 802.16e, and assuming that we perform an estimate in every preamble of the frame which has 141 pilots spaced 3 subcarriers apart. Note the quadratic nature of the MSE versus subcarrier index, showing that the edge subcarriers have the highest prediction MSE. This can be thought of as an alternative explanation for the so called *edge effect* in OFDM

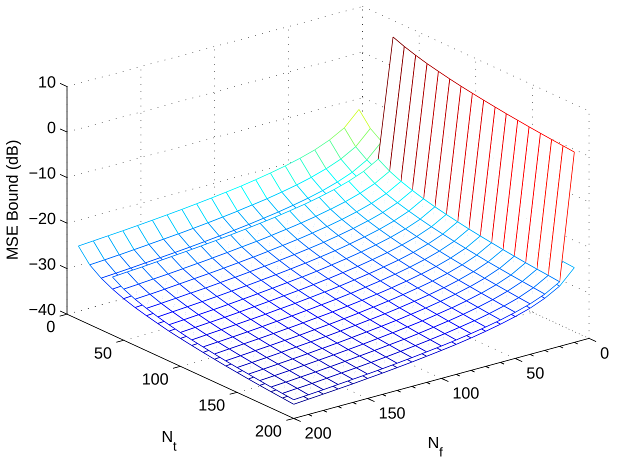

Fig. 1. MSE bounds for subcarrier $k = N_{used}/2$ and prediction length $n = D_t$ versus the number of pilots used in time (N_t) and frequency (N_f) domains. The upper surface is the Cramer-Rao bound, and the lower surface is the asymptotic CRB. We set $N_t D_t = 1024$ and $N_f D_f = 426$ to satisfy parameter restrictions, relaxing the integer requirement on D_t and D_f.

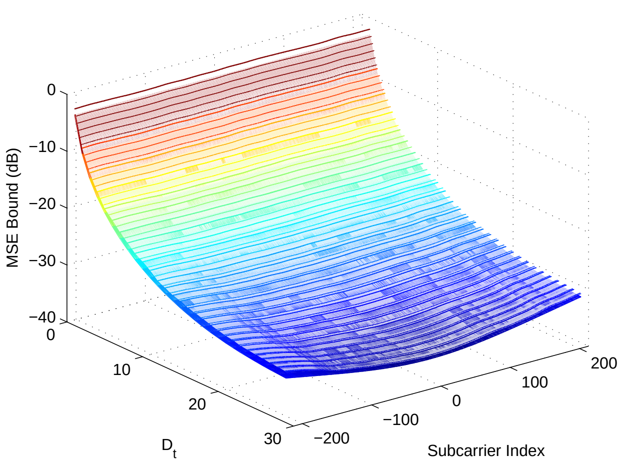

Fig. 2. MSE bounds for all subcarriers for various D_t, for a prediction length $n = 60$ ($\approx 1\lambda$) and $N_t = 100$, $N_f = 42$, $D_f = 10$. The upper surface is the CRB, and the lower surface is the Asymptotic CRB.

channel estimation. We also note the quadratic increase of the MSE bound with prediction length.

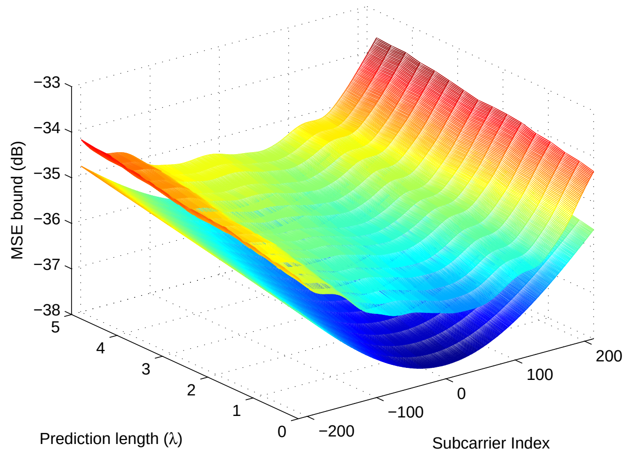

Fig. 3. MSE bounds for all subcarriers versus prediction length expressed in wavelengths, for $N_t = 100$, $D_t = 19$, $N_f = 141$, $D_f = 3$. The upper surface is the CRB, and the lower surface is the Asymptotic CRB.

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