OFDMA Downlink Resource Allocation for Ergodic Capacity Maximization with Imperfect Channel Knowledge

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Abstract— **In this paper, we derive an optimal resource allocation algorithm for ergodic weighted-sum capacity maximization in OFDMA systems assuming the availability of only partial (imperfect) CSI. Using a dual optimization framework, we show that the optimal power allocation is multi-level waterfilling based on the conditional expected value of the channel-to-noise ratio, and the optimal user is selected based on the maximum weighted conditional expected capacity penalized by the required power. The algorithm is shown to have $\mathcal{O}(MK)$ complexity for an OFDMA system with K used subcarriers and M active users, and achieves relative duality gaps of less than 10^{-5} (99.99999% optimal).**

I. Introduction

The problem of assigning the subcarriers, rates, and powers to the different users in an OFDMA system has been an area of active research, (see e.g. [2][3][4][5]). A common assumption in previous work is that the channel state information (CSI) of the users are known perfectly. This assumption is quite unrealistic due to channel estimation errors, and more importantly, channel feedback delay. Thus, in this paper, we focus on the case where only imperfect (partial) CSI is available.

The effect of imperfect CSI for rate maximization in wireless systems has been quite well studied for single-user OFDM systems [6]. However, no work to the best of the authors's knowledge considered the multiuser OFDM case. In the multiuser OFDM (or OFDMA) case, the difficulty arises from the fact that the exclusive subcarrier assignment restriction (i.e. only one user is allowed to transmit on each subcarrier) renders the problem to be combinatorial in nature. Fortunately, recent work on optimal resource allocation for rate maximization in OFDMA systems with perfect CSI [2] [4] [5] have shown that using a dual optimization approach, the problem can be solved with just $\mathcal{O}(MK)$ complexity per symbol for an OFDMA system with M active users and K used subcarriers. Furthermore, our solution results in relative optimality gaps of less than 10^{-4} in typical scenarios, thereby supporting us to claim *practical optimality*. Using a similar dual optimization approach, we relax the assumption of perfect CSI and formulate and solve the problem assuming the availability of imperfect CSI. We show that by using the dual optimization framework, we can solve the imperfect CSI problem with relative optimality gaps of less than 10^{-5} in cases of practical interest. The discrete rate allocation scenario was studied in a similar fashion in [3].

II. System Model

We consider a single OFDMA base station with K-subcarriers and M-users indexed by the set $\mathcal{K} = \{1, \ldots, k, \ldots, K\}$ and $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$ (typically $K \gg M$) respectively. We assume an average transmit power of $\bar{P} > 0$, bandwidth B, and noise density N_0. The received signal vector for the mth user at the nth OFDM symbol is given as

$$\boldsymbol{y}_m[n] = \mathbf{G}_m[n]\mathbf{H}_m[n]\boldsymbol{x}_m[n] + \boldsymbol{w}_m[n] \tag{1}$$

where $\boldsymbol{y}_m[n]$ and $\boldsymbol{x}_m[n]$ are the K-length received and transmitted complex-valued signal vectors; $\mathbf{G}_m[n]$ is the diagonal gain allocation matrix with diagonal elements $[\mathbf{G}_m[n]]_{kk} = \sqrt{p_{m,k}[n]}$; $\boldsymbol{w}_m[n] \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \boldsymbol{I}_K)$ with noise variance $\sigma_w^2 = N_0 B/K$ is the white zero-mean, circular-symmetric, complex Gaussian (ZMC-SCG) noise vector; and $\mathbf{H}_m[n] = \mathrm{diag}\{\boldsymbol{h}_m[n]\}$ is the diagonal channel response matrix, where $\boldsymbol{h}_m = [h_{m,1}[n], \ldots, h_{m,K}[n]]^T$ and where

$$h_{m,k}[n] = \sum_{i=1}^{N_t} g_{m,i}[n] e^{-j2\pi\tau_i k\Delta f}. \tag{2}$$

are the complex-valued frequency-domain wireless channel fading random processes, given as the discrete-time Fourier transform of the N_t time-domain multipath taps $g_{m,i}[n]$ with time-delay τ_i and subcarrier spacing Δf. The $g_{m,i}[n]$ are the time-domain fading channel taps modeled as stationary and ergodic discrete-time random processes, with identical normalized temporal autocorrelation function

$$r_m[\Delta] = \frac{1}{\sigma_{m,i}^2}\mathbb{E}\{g_{m,i}[n]g_{m,i}^*[n+\Delta]\} \tag{3}$$

with tap powers $\sigma_{m,i}^2$, which we assume to be independent across the fading paths i and across users m. Since $g_{m,i}[n]$ is stationary, $h_{m,k}[n]$ is also stationary, and the distribution of $\boldsymbol{h}_m[n]$ is independent of symbol index n.

Assuming that the time domain channel taps are independent ZMCSCG random variables $g_{m,i} \sim \mathcal{CN}(0, \sigma_{m,i}^2)$, then from (2),

$$\begin{aligned} \boldsymbol{h}_m &\sim \mathcal{CN}(\mathbf{0}_K, \boldsymbol{\Sigma}_{\boldsymbol{h}_m}) \\ \boldsymbol{\Sigma}_{\boldsymbol{h}_m} &= \mathbf{W}\boldsymbol{\Sigma}_{\boldsymbol{g}_m}\mathbf{W}^H \end{aligned} \tag{4}$$

where $\mathbf{W}$ is the $K \times N_t$ DFT matrix with $[\mathbf{W}]_{k,i} = e^{-j2\pi\tau_i k\Delta f}$ and $\boldsymbol{\Sigma}_{\boldsymbol{g}_m} = \text{diag}\{\sigma_{m,1}^2, \ldots, \sigma_{m,N_t}^2\}$ is an $N_t \times N_t$ diagonal matrix of the time-domain path covariances. We model partial CSI as

$$\boldsymbol{h}_m = \hat{\boldsymbol{h}}_m + \hat{\boldsymbol{e}}_m \tag{5}$$

where $\hat{\boldsymbol{h}}_m \sim \mathcal{CN}\left(\mathbf{0}_K, \boldsymbol{\Sigma}_{\boldsymbol{h}_m} - \hat{\boldsymbol{\Sigma}}_m\right)$ is the estimated channel vector and $\hat{\boldsymbol{e}} \sim \mathcal{CN}(\mathbf{0}_K, \hat{\boldsymbol{\Sigma}}_m)$ is the estimation error vector with

$$\begin{aligned} \hat{\boldsymbol{\Sigma}}_m = \boldsymbol{\Sigma}_{\boldsymbol{h}_m} - \\ (\mathbf{r}_m^T \otimes \boldsymbol{\Sigma}_{\boldsymbol{h}_m})(\mathbf{R}_m \otimes \boldsymbol{\Sigma}_{\boldsymbol{h}_m} + \sigma_e^2\mathbf{I})^{-1}(\mathbf{r}_m^T \otimes \boldsymbol{\Sigma}_{\boldsymbol{h}_m})^H \end{aligned}$$

as the error covariance matrix for a Pth order MMSE predictor for the channel with pilot spacing D_t, where $[\mathbf{R}_m]_{i,j} = r_m[(i-j)D_t]$, $\mathbf{r}_m^T = [r_m[D_t], \ldots, r_m[PD_t]]$, and $\otimes$ is the Kronecker product.

We assume that the marginal fading distribution on subcarrier k conditioned on the estimated channel is a non-zero mean complex Gaussian random variable given as $h_{m,k}|\hat{h}_{m,k} \sim \mathcal{CN}(\hat{h}_{m,k}, \hat{\sigma}_{m,k}^2)$ where $\hat{h}_{m,k}$ is the estimated complex channel gain and $\hat{\sigma}_{m,k}^2$ is the estimation error variance for that subcarrier. Thus, the channel-to-noise ratio (CNR) $\gamma_{m,k} = |h_{m,k}|^2/\sigma_w^2$ conditioned on $\hat{\gamma}_{m,k} = |\hat{h}_{m,k}|^2/\sigma_w^2$ is in turn a non-central Chi-squared distributed random variable with two degrees of freedom with pdf [7, Eq. 2-1-118]

$$\begin{aligned} f_{\gamma_{m,k}}(\gamma_{m,k}|\hat{\gamma}_{m,k}) = \\ \frac{1}{\rho_{m,k}} e^{-\frac{\hat{\gamma}_{m,k}+\gamma_{m,k}}{\rho_{m,k}}} I_0\left(\frac{2}{\rho_{m,k}}\sqrt{\hat{\gamma}_{m,k}\gamma_{m,k}}\right) \end{aligned} \tag{6}$$

where I_0 is the zeroth-order modified Bessel function of the first kind, and $\rho_{m,k} = \hat{\sigma}_{m,k}^2/\sigma_w^2$ is the ratio of the estimation error variance to the ambient noise variance.

III. Capacity Maximization with Partial CSI

A. Problem Formulation

The capacity for user m and subcarrier k is given as

$$R(p_{m,k}\gamma_{m,k}) = \log_2(1 + p_{m,k}\gamma_{m,k}) \quad \text{bps/Hz} \tag{7}$$

Denote by $\boldsymbol{p} = [\boldsymbol{p}_1^T, \cdots, \boldsymbol{p}_K^T]^T$ the vector of powers to be determined, where $\boldsymbol{p}_k = [p_{1,k}, \cdots, p_{M,k}]^T$. The exclusive subcarrier assignment restriction can be written as $\boldsymbol{p}_k \in \boldsymbol{\mathcal{P}}_k \subset \mathbb{R}_+^M$, where

$$\boldsymbol{\mathcal{P}}_k \equiv \{p_{m,k} \geq 0 | p_{m,k}p_{m',k} = 0; \forall m \neq m'\} \tag{8}$$

We let $\boldsymbol{p} \in \boldsymbol{\mathcal{P}} \equiv \boldsymbol{\mathcal{P}}_1 \times \cdots \times \boldsymbol{\mathcal{P}}_K \subset \mathbb{R}_+^{MK}$ denote the space of allowable power vectors for all subcarriers.

We assume that we have knowledge of the imperfect CNR vector $\hat{\boldsymbol{\gamma}} = [\hat{\gamma}_1^T, \ldots, \hat{\gamma}_K^T]^T$, $\hat{\boldsymbol{\gamma}}_k = [\hat{\gamma}_{1,k}, \ldots, \hat{\gamma}_{M,k}]^T$; corresponding to an estimate of the actual CNR realization $\boldsymbol{\gamma} = [\boldsymbol{\gamma}_1^T, \ldots, \boldsymbol{\gamma}_K^T]^T$, $\boldsymbol{\gamma}_k = [\gamma_{1,k}, \ldots, \gamma_{M,k}]^T$. Thus, our power allocation vector $\boldsymbol{p}$ can only be a function of $\hat{\boldsymbol{\gamma}}$. The ergodic weighted sum rate maximization problem assuming partial CSI is then

$$\begin{aligned} f^* = \max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \sum_{m \in \mathcal{M}} w_m \sum_{k \in \mathcal{K}} \mathbb{E}\left\{ R\left(p_{m,k}\gamma_{m,k}\right) \middle| \, \hat{\gamma}_{m,k}\right\} \\ \text{s.t.} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} p_{m,k} \leq \bar{P} \end{aligned} \tag{9}$$

where w_m are positive constants such that $\sum w_m = 1$. Theoretically, varying these weights allows us to trace out the ergodic capacity region assuming partial CSI; algorithmically, varying the weights allows us to prioritize the different users in the system and enforce certain notions of fairness, e.g. proportional fairness and max-min fairness [2].

B. Dual Optimization Framework

We begin our development by observing that the objective function in (9) is separable across the subcarriers, and is tied together only by the power constraint. We will approach this problem using dual optimization techniques [8]. The dual problem for (9) is defined as

$$g^* = \min_{\lambda \geq 0} \Theta(\lambda) \tag{10}$$

where the dual objective is given as

$$\Theta(\lambda) = \max_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} \sum_{m\in\mathcal{M}} w_m \sum_{k\in\mathcal{K}} \Bigg[\mathbb{E}\left\{ R\left(p_{m,k}\gamma_{m,k}\right) \middle| \hat{\gamma}_{m,k} \right\} + \lambda \left(\bar{P} - \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} p_{m,k} \right) \Bigg] \tag{11a}$$

$$= \lambda \bar{P} + \sum_{k\in\mathcal{K}} \max_{\boldsymbol{p}_k\in\boldsymbol{\mathcal{P}}_k} \tag{11b}$$

$$\sum_{m\in\mathcal{M}} \left(w_m \mathbb{E}\left\{ R\left(p_{m,k}\gamma_{m,k}\right) \middle| \hat{\gamma}_{m,k} \right\} - \lambda p_{m,k} \right)$$

$$= \lambda \bar{P} + \sum_{k\in\mathcal{K}} \max_{m\in\mathcal{M}} \tag{11c}$$

$$\max_{p_{m,k}\geq 0} \left(w_m \mathbb{E}\left\{ R\left(p_{m,k}\gamma_{m,k}\right) \middle| \hat{\gamma}_{m,k} \right\} - \lambda p_{m,k} \right)$$

where (11b) follows from the separability of the variables across subcarriers, and (11c) from the exclusive subcarrier assignment constraint. We have reduced the problem to a per-subcarrier optimization and, since $K \gg M$, significantly decreased the computational burden.

We denote the optimal power allocation function for the innermost per-user and per-subcarrier problem in (11c) as $\hat{p}_{m,k}(\lambda)$, which can be found using simple differentiation as

$$\hat{p}_{m,k}(\lambda) = \begin{cases} p_{m,k} : \mathbb{E}\left\{ \frac{\gamma_{m,k}}{1+p_{m,k}\gamma_{m,k}} \middle| \hat{\gamma}_{m,k} \right\} = \gamma_{0,m}, \\ \qquad\qquad \mathbb{E}\{\gamma_{m,k}|\hat{\gamma}_{m,k}\} \geq \gamma_{0,m} \\ 0 \qquad , \mathbb{E}\{\gamma_{m,k}|\hat{\gamma}_{m,k}\} < \gamma_{0,m} \end{cases} \tag{12}$$

where $\gamma_{0,m} = \frac{\lambda \ln 2}{w_m}$. This can be interpreted as a *multi-level water-filling* with cut-off CNR $\gamma_{0,m}$ similar to [2, Eq. 9], except that the cut-off is now based on the conditional mean of the CNR given its estimate, instead of the actual CNR. Using the pdf in (6), the conditional mean is simply $\mathbb{E}\left\{ \gamma_{m,k} \middle| \hat{\gamma}_{m,k} \right\} = \hat{\gamma}_{m,k} + \rho_{m,k}$. Note that when we have perfect CSI, $\rho_{m,k} = 0$ and (12) actually reduces to the multi-level waterfilling equation for perfect CSI in [2, Eq. 9]. There is no closed form solution to (12), but it can be solved using numerical integration of the expectation, and a zero-finding procedure like bisection method [9] to find the power allocation.

Plugging (12) into (11c) and then in (10), we arrive at the following dual problem in the single variable λ

$$g^* = \min_{\lambda\geq 0} \lambda \bar{P} + \sum_{k\in\mathcal{K}} \max_{m\in\mathcal{M}} \left(w_m \mathbb{E}\left\{ R\left(\hat{p}_{m,k}(\lambda)\gamma_{m,k}\right) \middle| \hat{\gamma}_{m,k} \right\} - \lambda \hat{p}_{m,k}(\lambda) \right) \tag{13}$$

Using standard duality arguments (see e.g. [8, Prop. 5.1.2]), the objective in (13) can be shown to be convex in the single variable λ, but is actually not continuously differentiable due to the presence of the $\max$ function. Hence, powerful derivative-based minimization methods such as Newton's method cannot be used. Fortunately, we can use derivative-free single-dimensional line search methods that only need function evaluations, e.g. Golden-section or Fibonacci search [9] to find the optimum multiplier λ^*.

C. Optimal Subcarrier and Power Allocation

The optimal multiplier λ^* determines the optimal cutoff SNR $\gamma^*_{0,m} = \frac{\lambda^* \ln 2}{w_m}$. This can then be plugged back into the power allocation function (12) to arrive at

$$m^*_k = \arg\max_{m\in\mathcal{M}} \left[\mathbb{E}\left\{ w_m R\left(\hat{p}_{m,k}(\lambda^*)\gamma_{m,k}\right) \middle| \hat{\gamma}_{m,k} \right\} - \lambda^* \hat{p}_{m,k}(\lambda^*) \right]$$

$$p^*_{m,k} = \begin{cases} \hat{p}_{m,k}(\lambda^*), & m = m^*_k \\ 0, & m \neq m^*_k \end{cases} \tag{14}$$

Note, however, that it is possible that the candidate power allocation does not satisfy the total power constraint, since the constraint is not enforced explicitly. Hence, our final power allocation values should be multiplied by a constant $\eta = \bar{P} / \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} p^*_{m,k}$ which we plug back into the objective in (9) to arrive at our computed primal optimal value

$$\hat{f}^* = \sum_{m\in\mathcal{M}} w_m \sum_{k\in\mathcal{K}} \mathbb{E}\left\{ \log_2(1 + \eta p^*_{m,k}\gamma_{m,k}) \middle| \hat{\gamma}_{m,k} \right\} \tag{15}$$

Unfortunately, the above procedure is still highly computationally intensive, since for each candidate λ in the line search iterations, we need to compute MK power allocation values (12), each of which requires a zero-finding routine where a function value evaluation involves numerical integration to compute the expectation. Although both the line search and the zero-finding routines typically converge within very few iterations (< 10 in our experiments), the numerical integration procedure requires a lot more iterations (> 50), and hence is the main computational bottleneck. We shall overcome this problem using a closed-form approximation to the expectation in the power allocation function.

D. Power Allocation Function Approximation

Our approach to approximating the expectation in (12) is to use a Gamma distribution to approximate the non-central Chi-squared distribution of $\gamma_{m,k}|\hat{\gamma}_{m,k}$ (6), which

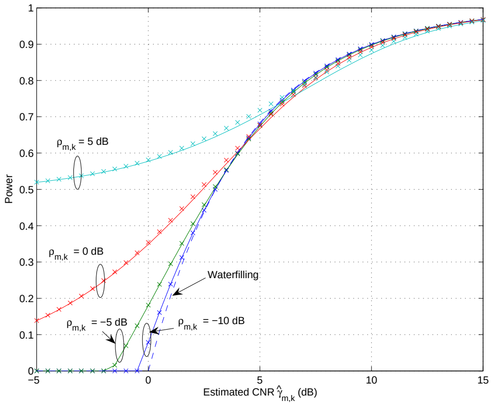

Fig. 1. Power allocation as a function of estimated CNR ($\hat{\gamma}$) with $\gamma_0 = 1$ for various $\rho_{m,k} = \hat{\sigma}^2_{m,k}/\sigma^2_w$. A solid line indicates the optimal power allocation, 'x's denote the approximation, and the dashed line correspond to the simple waterfilling allocation.

is known to approximate the body of this pdf quite well [10, p. 55]. This approximation is given by

$$f_{\gamma_{m,k}}(\gamma_{m,k}|\hat{\gamma}_{m,k}) \approx \frac{\beta^\alpha}{\Gamma(\alpha)} \gamma_{m,k}^{\alpha-1} e^{-\beta\gamma_{m,k}} \qquad (16)$$

where $\alpha = (K_{m,k}+1)^2/(2K_{m,k}+1)$ is the Gamma pdf shape parameter with $K_{m,k} = \frac{\hat{\gamma}_{m,k}}{\rho_{m,k}}$ as the specular to diffuse power ratio, equivalent to the $K-$factor in a Ricean pdf; and $\beta = \alpha/(\hat{\gamma}_{m,k} + \rho_{m,k})$ is the Gamma pdf rate parameter. Using this pdf, we can use [11, Section 3.383.10] to arrive at the following closed form approximation to the integral

$$\begin{aligned} \mathbb{E}&\left\{ \frac{\gamma_{m,k}}{1+p_{m,k}\gamma_{m,k}} \middle| \hat{\gamma}_{m,k} \right\} \\ &\approx \frac{\beta^\alpha}{\Gamma(\alpha)} \int_0^\infty \frac{\gamma_{m,k}^\alpha}{1+p_{m,k}\gamma_{m,k}} e^{-\beta\gamma_{m,k}} d\gamma_{m,k} \qquad (17) \\ &= \frac{\alpha}{p_{m,k}} \left(\frac{\beta}{p_{m,k}}\right)^\alpha e^{\frac{\beta}{p_{m,k}}} \Gamma\left(-\alpha, \frac{\beta}{p_{m,k}}\right) \end{aligned}$$

where $\Gamma(a,x)$ is the incomplete Gamma function [11, Section 8.350]. Using (17) in (12) to solve for $p_{m,k}$, we are able to closely approximate the power allocation function $\hat{p}_{m,k}$. We plot the power allocation function using the Gamma pdf approximation and the actual Chi-squared pdf in Fig. 1 with $\gamma_0 = 1$ for various $\rho_{m,k} = \hat{\sigma}^2_{m,k}/\sigma^2_w$. Note that the approximation error is negligible, with a normalized mean-squared error of 5×10^{-5} and maximum error of 2.7×10^{-4}, while the computation of the approximation is almost $300\times$ faster than the actual using very crude computational time measurements in Matlab 7.2 (`tic-toc`).

E. Bound on the Relative Duality Gap

If we let $f^* > 0$ and $g^* > 0$ be the optimal values of the primal and dual problems given in (9) and (13), and let $\hat{f}^* > 0$ given in (15) be the computed feasible primal value, the relative duality (optimality) gap can be bounded as

$$0 \leq \frac{g^* - f^*}{f^*} \leq \frac{g^* - \hat{f}^*}{\hat{f}^*} \qquad (18)$$

The left inequality follows directly from the non-negativity of f^* and the weak duality theorem [8, Prop. 5.1.3. p. 495], and the right inequality follows from $\hat{f}^* \leq f^*$ since $\hat{f}^*$ is a feasible primal value and f^* is the optimal feasible primal value. In our numerical results, we show that the resulting optimality gaps using our algorithm are practically zero ($< 10^{-5}$). Thus, our approach can, for all practical purposes, be considered an optimal solution to the problem. This fortuitous phenomenon is brought about primarily by the separability of the problem, and furthermore by the fact that we have K separable terms (which is typically large) and only a single constraint (average power constraint). This problem structure has been shown to be particularly suitable to dual optimization approaches, and has been noted in [12] (for the instantaneous optimization case), and more generally treated in [8].

F. Complexity Analysis

In each search iteration for λ in (13), we need to compute MK candidate power allocation functions given by (12) and (17). Each power allocation value calculation requires a zero-finding routine, e.g. bisection or Newton search [9], which we assume requires I_p function evaluations to converge. After determining the power allocation value, we then use it in the ergodic capacity integral in (13), which we assume requires I_c function evaluations to compute. Finally, assuming that we require I_λ line search iterations to solve for the optimum λ, the overall complexity is $\mathcal{O}(I_\lambda MK(I_p+I_c))$. Ignoring the constants I_λ, I_p, and I_c, the complexity is just $\mathcal{O}(MK)$.

IV. Results and Discussion

We present several numerical examples to substantiate our theoretical claims. Our simulations are roughly based on a 3GPP-LTE downlink [1] system with parameters given in Table I. We simulate the frequency-selective Rayleigh fading channel using the ITU-Vehicular A channel model [1]. We assume Clarke's U-shaped power spectrum [10] for each multipath tap, resulting in the temporal autocorrelation function $r_m[\Delta] =$

TABLE I

SIMULATION PARAMETERS

Parameter	Value	Parameter	Value
Subcarriers	64	Vehicular speed	120 kph
Used Subcarriers	33	Doppler frequency	289 Hz
Bandwidth	1.25 MHz	Prediction filter length	4
Sampling Freq.	1.92 MHz	Pilot spacing	7
Carrier Freq.	2.6 GHz	CP Length	6 samples

TABLE II

OTHER RELEVANT METRICS

SNR	No. of Iterations (I_λ)	Relative Gap ($\times 10^{-6}$)
5	8.599	8.40
10	8.501	5.68
15	8.686	4.12

$J_0(2\pi\Delta F_d D_t(K_{fft} + L_{cp})/F_s)$ where $J_0(x)$ is the zeroth-order Bessel function of the first kind [13, Ch. 9]. To simulate imperfect CSI, we generate IID realizations of $\hat{\boldsymbol{h}}_m$ and its prediction error vector $\hat{\boldsymbol{e}}_m$ as discussed in Section II. This allows us to also generate the "actual" channel $\boldsymbol{h}_m$ for the perfect CSI cases using (5).

In Fig. 2, we show the 2-user capacity region for continuous rate allocation with imperfect CSI (Imperfect CSI-Optimal) with 5000 channel realizations per data point. We also show the capacity region using optimal instantaneous rate resource allocation assuming perfect CSI (Perfect CSI-Optimal), which is essentially multi-level waterfilling (MWF) [2]; and the capacity region when we simply use MWF on the imperfect CSI (Imperfect CSI-MWF). Note that in all cases, rate maximization with imperfect CSI through channel prediction performs quite close to the case with perfect CSI. More important, Imperfect CSI-MWF performs similar to Imperfect CSI-Optimal. This can be explained by noticing that the optimal power allocation assuming imperfect CSI is almost equal to the waterfilling curve (see Fig. 1) except for very low estimated CNR. However, due to the effect of frequency and multiuser diversity, the subcarrier is typically assigned to the user with the highest CNR; thus, the power allocation is quite often almost identical to performing waterfilling on the imperfect CSI. A similar observation was made in [6] for the single user scenario.

Table II shows the other relevant metrics of the optimal resource algorithms. The first column shows the average number of line-search iterations it took to converge to a tolerance of 10^{-4}. The second column shows the resulting relative duality gaps given by (18). We can see that the duality gaps are virtually zero, and thus both algorithms can be considered optimal for all practical purposes.

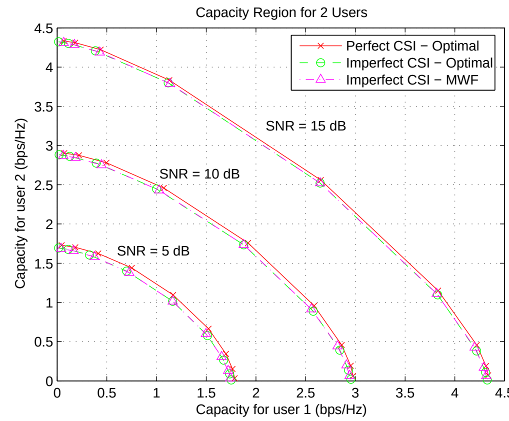

Fig. 2. 2-user capacity region for the optimal resource allocation with imperfect CSI, optimal allocation with perfect CSI, and using multi-level waterfilling (MWF) on the imperfect CSI.

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