OPTIMAL OFDMA RESOURCE ALLOCATION WITH LINEAR COMPLEXITY TO MAXIMIZE ERGODIC WEIGHTED SUM CAPACITY

Ian C. Wong and Brian L. Evans

The University of Texas at Austin, Austin, Texas 78712
Email: {iwong,bevans}@ece.utexas.edu

ABSTRACT

Previous research efforts to optimize OFDMA resource allocation with respect to communication performance have focused on formulations considering only instantaneous per-symbol rate maximization, and on solutions using suboptimal heuristic algorithms. This paper intends to fill gaps in the literature through two key contributions. First, we formulate weighted sum ergodic capacity maximization in OFDMA assuming the availability of perfect channel state information (CSI). Our formulations exploit time, frequency, and multi-user diversity, while enforcing various notions of fairness through weighting factors for each user. Second, we derive algorithms based on a dual optimization framework that solve the OFDMA ergodic capacity maximization problem with $\mathcal{O}(MK)$ complexity per OFDMA symbol for M users and K subcarriers, while achieving data rates shown to be at least 99.9999% of the optimal rate in simulations based on realistic parameters. Hence, this paper attempts to demonstrate that OFDMA resource allocation problems are *not* computationally prohibitive to solve optimally, even when considering ergodic rates.

Index Terms: Multiaccess communication,Information rates, Resource management, Duality, Optimization methods

1. INTRODUCTION

Next-generation broadband wireless system standards, e.g. 3GPP-Long Term Evolution (LTE) [1], consider Orthogonal Frequency Division Multiple Access (OFDMA) as the preferred physical layer multiple access scheme, esp. for the downlink. The problem of assigning the subcarriers, rates, and powers to the different users in an OFDMA system has been an area of active research, (see e.g. [2]). In most of the previous work, the formulation and algorithms only consider instantaneous performance metrics. Thus, the temporal dimension is not being exploited when the resource allocation is performed. Instead of considering only instantaneous data rate, we formulate the problem considering user-weighted *ergodic* capacity. This allows us to exploit the time dimension explicitly in the formulation, and utilize all three degrees of freedom in our system, namely frequency, time, and multiuser dimensions. At the same time, we can enforce certain notions of fairness through the user weights. We focus on the ergodic (Shannon-type) capacity in this paper, and our other paper [3] focuses on the more practical discrete bit and power loading subject to BER constraints.

Furthermore, previous research have assumed that algorithms to find the optimal or near-optimal solution to the problem is too computationally complex for real-time implementation. Hence, the main focus of previous research efforts have been on developing heuristic approaches with typical complexities in the order of $\mathcal{O}(MK^2)$ [2]. Our approach, on the other hand, is based on a dual optimization framework, which is less complex ($\mathcal{O}(MK)$ per iteration, with less than 10 iterations) and achieves relative optimality gaps that are less than 10^{-4} (i.e. achieving 99.9999% of the optimal solution) in typical scenarios, and thus actually allowing us to claim *practical optimality*. Note that the dual optimization approach is also studied in [4], but their focus has been on instantaneous rate optimization.

2. SYSTEM MODEL

We consider a single OFDMA base station with K-subcarriers and M-users indexed by the set $\mathcal{K} = \{1, \ldots, k, \ldots, K\}$ and $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$ (typically $K \gg M$) respectively. We assume an average transmit power of $\bar{P} > 0$, bandwidth B, and noise density N_0. The received signal vector for the mth user at the nth OFDM symbol is given as

$$\boldsymbol{y}_m[n] = \mathbf{G}_m[n]\mathbf{H}_m[n]\boldsymbol{x}_m[n] + \boldsymbol{w}_m[n] \tag{1}$$

where $\boldsymbol{y}_m[n]$ and $\boldsymbol{x}_m[n]$ are the K-length received and transmitted complex-valued signal vectors; $\mathbf{G}_m[n]$ is the diagonal gain allocation matrix with diagonal elements $[\mathbf{G}_m[n]]_{kk} = \sqrt{p_{m,k}[n]}$; $\boldsymbol{w}_m[n] \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \boldsymbol{I}_K)$ with noise variance $\sigma_w^2 = N_0 B/K$ is the white zero-mean, circular-symmetric, complex Gaussian (ZMCSCG) noise vector; and $\mathbf{H}_m[n] = \text{diag}\{h_{m,1}[n], \ldots, h_{m,K}[n]\}$ is the diagonal channel response matrix, where

$$h_{m,k}[n] = \sum_{i=1}^{N_t} g_{m,i}[n] e^{-j2\pi\tau_i k \Delta f}. \tag{2}$$

are the complex-valued frequency-domain wireless channel fading random processes, given as the discrete-time Fourier transform of the N_t time-domain multipath taps $g_{m,i}[n]$ with time-delay τ_i and subcarrier spacing Δf. These taps are modeled as stationary and ergodic discrete-time random processes with tap powers $\sigma_{m,i}^2$, which we assume to be independent across the fading paths i and across users m. Since $g_{m,i}[n]$ is stationary and ergodic, so is $h_{m,k}[n]$. Hence, the distribution of $\boldsymbol{h}_m[n]$ is independent of n through stationarity, and we can replace time averages with ensemble averages in the problem formulations through ergodicity. In the subsequent discussion, we shall drop the index n when the context is clear for notational brevity.

We assume[1] that the time domain channel taps are independent ZMCSCG random variables $g_{m,i} \sim \mathcal{CN}(0, \sigma_{m,i}^2)$ with total power $\sigma_m^2 = \sum_{i=1}^{N_t} \sigma_{m,i}^2$. Then from (2), we have

$$\begin{aligned} \boldsymbol{h}_m &\sim \mathcal{CN}(\mathbf{0}_K, \mathbf{R}_{\boldsymbol{h}_m}) \\ \mathbf{R}_{\boldsymbol{h}_m} &= \mathbf{W}\boldsymbol{\Sigma}_m\mathbf{W}^H \end{aligned} \tag{3}$$

[1] Although the results of this paper are applicable to any fading distribution, we shall prescribe a particular distribution for the fading channels for illustration purposes.

where $\mathbf{W}$ is the $K \times N_t$ DFT matrix with $[\mathbf{W}]_{k,i} = e^{-j2\pi\tau_i k\Delta f}$ and $\mathbf{\Sigma}_m = \mathrm{diag}\{\sigma_{m,1}^2, \ldots, \sigma_{m,N_t}^2\}$ is an $N_t \times N_t$ diagonal matrix of the time-domain path powers. Since we also assume that the fading for each user is independent, then the joint distribution of the stacked fading vector for all users $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_M^T]^T$ is likewise a ZM-CSCG random vector with distribution $\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}_{KM}, \mathbf{R}_{\boldsymbol{h}})$ where $\mathbf{R}_{\boldsymbol{h}}$ is the $KM \times KM$ block diagonal covariance matrix with $\mathbf{R}_{\boldsymbol{h}_m}$ as the diagonal block elements. This is the distribution over which we shall take the weighted sum rate function in the problem formulations. We let $\boldsymbol{\gamma}_m = [\gamma_{m,1}, \ldots, \gamma_{m,k}]^T$ where $\gamma_{m,k} = |h_{m,k}|^2/\sigma_w^2$ denote the instantaneous channel-to-noise ratio (CNR) with mean $\bar{\gamma}_{m,k} = \sigma_m^2/\sigma_w^2$. Note that $\gamma_{m,k}$ for a particular subcarrier k and different users m are independent but not necessarily identically distributed (INID) exponential random variables; and for a particular user m and different subcarriers k are not independent but identically distributed (NIID) exponential random variables. Throughout the paper, we assume that the transmitter has perfect knowledge of $\boldsymbol{\gamma}_m$ for all users, and that the resource allocation decisions are made known to the users through an error-free control channel.

3. ERGODIC WEIGHTED SUM CAPACITY MAXIMIZATION IN OFDMA

3.1. Problem Formulation

The capacity for user m and subcarrier k is given as

$$R(p_{m,k}\gamma_{m,k}) = \log_2(1 + p_{m,k}\gamma_{m,k}) \quad \text{bps/Hz} \tag{4}$$

Denote by $\boldsymbol{p} = [\boldsymbol{p}_1^T, \cdots, \boldsymbol{p}_K^T]^T$ the vector of powers to be determined, where $\boldsymbol{p}_k = [p_{1,k}, \cdots, p_{M,k}]^T$. The exclusive subcarrier assignment restriction in OFDMA can be written as $\boldsymbol{p}_k \in \boldsymbol{\mathcal{P}}_k \subset \mathbb{R}_+^M$,

$$\boldsymbol{\mathcal{P}}_k \equiv \{\boldsymbol{p}_k \in \mathbb{R}_+^M | p_{m,k}p_{m',k} = 0; \forall m \neq m'; m, m' \in \mathcal{M}\} \tag{5}$$

For notational convenience, we let $\boldsymbol{p} \in \boldsymbol{\mathcal{P}} \equiv \boldsymbol{\mathcal{P}}_1 \times \cdots \times \boldsymbol{\mathcal{P}}_K \subset \mathbb{R}_+^{MK}$ denote the space of allowable power vectors for all subcarriers. Since we assumed perfect CSI, we can consider the power allocation vector $\boldsymbol{p}$ as a function of the realization of the fading CNR $\boldsymbol{\gamma} = [\boldsymbol{\gamma}_1^T, \ldots, \boldsymbol{\gamma}_M^T]^T$. The ergodic weighted sum capacity maximization problem can then be written as

$$\begin{aligned} f^* = \max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \quad & \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{m \in \mathcal{M}} w_m \sum_{k \in \mathcal{K}} R\left(p_{m,k}\gamma_{m,k}\right) \right\} \\ \text{s.t.} \quad & \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} p_{m,k} \right\} \leq \bar{P} \end{aligned} \tag{6}$$

where w_m are positive constants such that $\sum w_m = 1$. Theoretically, varying these weights allows us to trace out the ergodic capacity region; algorithmically, varying the weights allows us to prioritize the different users in the system and enforce certain notions of fairness, e.g. proportional fairness and max-min fairness [2]. Note that the objective function in (6) is concave, but the constraint space $\boldsymbol{\mathcal{P}}$ is highly non-convex (it is in fact a discrete space), and is in general very difficult to solve. The next subsection discusses a dual optimization approach that allows us to solve this problem efficiently.

3.2. Dual Optimization Framework

We begin our development by observing that the objective function in (6) is separable across the subcarriers, and is tied together only by the power constraint. In these problems, it is useful to approach the problem using duality principles [5]. The dual problem is defined as

$$g^* = \min_{\lambda \geq 0} \Theta(\lambda) \tag{7}$$

where the dual objective is given by

$$\begin{aligned} \Theta(\lambda) &= \max_{\boldsymbol{p} \in \boldsymbol{\mathcal{P}}} \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{m \in \mathcal{M}} w_m \sum_{k \in \mathcal{K}} R\left(p_{m,k}\gamma_{m,k}\right) \right\} + \\ &\qquad \lambda \left(\bar{P} - \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k \in \mathcal{K}} \sum_{m \in \mathcal{M}} p_{m,k} \right\} \right) \\ &= \lambda\bar{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k \in \mathcal{K}} \max_{\boldsymbol{p}_k \in \boldsymbol{\mathcal{P}}_k} \sum_{m \in \mathcal{M}} w_m R\left(p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k} \right\} \\ &= \lambda\bar{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k \in \mathcal{K}} \max_{m \in \mathcal{M}} \max_{p_{m,k} \geq 0} w_m R\left(p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k} \right\} \end{aligned} \tag{8}$$

where the second equality follows from the separability of variables across subcarriers, and the third from the exclusive subcarrier assignment constraint. We have reduced the problem to a per-subcarrier optimization, and since $K \gg M$, we have significantly decreased the computational burden.

The innermost maximization in (8) has a simple closed-form expression for the optimal powers given as

$$\tilde{p}_{m,k}(\lambda) = \left[\frac{1}{\gamma_{0,m}} - \frac{1}{\gamma_{m,k}} \right]^+ \tag{9}$$

where $[x]^+ = \max(0, x)$ and $\gamma_{0,m} = \frac{\lambda \ln 2}{w_m}$ which is a simple "multi-level water-filling" power allocation with cut-off CNR $\gamma_{0,m}$. Using (9) in (8), the dual problem (7) can be written as

$$g^* = \min_{\lambda \geq 0} \lambda\bar{P} + \sum_{k \in \mathcal{K}} \mathbb{E}_{\boldsymbol{\gamma}_k} \left\{ g_k(\boldsymbol{\gamma}_k, \lambda) \right\} \tag{10}$$

$$g_k(\boldsymbol{\gamma}_k, \lambda) = \max_{m \in \mathcal{M}} \left\{ g_{m,k}(\gamma_{m,k}, \lambda) \right\} \tag{11}$$

$$\begin{aligned} g_{m,k}(\gamma_{m,k}, \lambda) &= w_m R\left(\tilde{p}_{m,k}(\lambda)\gamma_{m,k}\right) - \lambda\tilde{p}_{m,k}(\lambda) \\ &= \begin{cases} \frac{w_m}{\ln 2} \ln\left(\frac{\gamma_{m,k}}{\gamma_{0,m}}\right) - \frac{w_m}{\ln 2} + \frac{\lambda}{\gamma_{m,k}}, & \gamma_{m,k} \geq \gamma_{0,m} \\ 0, & \gamma_{m,k} < \gamma_{0,m} \end{cases} \end{aligned} \tag{12}$$

3.3. Numerical Evaluation of the Expected Dual

Computing the expectation in (10) in a straightforward manner involves an $M-$dimensional integral over the joint pdf of the $M-$length fading vector $\boldsymbol{\gamma}_k$, which is typically too complex to solve using direct numerical integration techniques (e.g. Gaussian quadrature) except for small M, e.g. 2 or 3, since this requires $\mathcal{O}(N^M)$ computations where N is the number of function evaluations required for a one-dimensional integral with the same accuracy [6]. However, if we can somehow compute a closed-form expression for the pdf of the per-subcarrier dual function $g_k(\boldsymbol{\gamma}_k, \lambda)$ (11), then we can reduce the expectation to just a one-dimensional integral that is solvable in $\mathcal{O}(MN)$. Since $\gamma_{m,k}$ for different ms are INID, then (12) is likewise INID for different ms. Thus, (11) is the largest order statistic of INID random variables $g_{m,k}$, which has the following closed-form pdf [7, Sec. 5.2]

$$f_{g_k}(g_k) = \prod_{m \in \mathcal{M}} F_{g_{m,k}}(g_k) \left(\sum_{m \in \mathcal{M}} \frac{f_{g_{m,k}}(g_k)}{F_{g_{m,k}}(g_k)} \right) \tag{13}$$

where $F_{g_{m,k}}(g_{m,k})$ and $f_{g_{m,k}}(g_{m,k})$ are the cdf and pdf of $g_{m,k}$, respectively. In order to derive these distribution functions given the distribution $F_{\gamma_{m,k}}(\gamma_{m,k})$ of $\gamma_{m,k}$, we need an expression for the inverse function of $g_{m,k}(\gamma_{m,k}, \lambda)$. Since $g_{m,k}$ for $\gamma_{m,k} \geq \gamma_{0,m}$ is monotonically increasing and non-negative, there exists a unique inverse function. After some algebraic manipulation, we have

$$-\frac{\gamma_{0,m}}{\gamma_{m,k}} \exp\left(-\frac{\gamma_{0,m}}{\gamma_{m,k}}\right) = -\exp\left(-g_{m,k}\frac{\ln 2}{w_m} - 1\right) \tag{14}$$

Observe that this is in the form of the Lambert-W function $W(x)$ [8], which is the solution to $W(x)\exp(W(x)) = x$. This function is ubiquitous in the physical sciences, and efficient algorithms have been developed for its computation [8]. Thus, we can write

$$W\left(-\exp\left(-g_{m,k}\frac{\ln 2}{w_m} - 1\right)\right) = -\frac{\gamma_{0,m}}{\gamma_{m,k}} \tag{15}$$

which when solved for $\gamma_{m,k}$ gives us

$$\check{\gamma}_{m,k}(g_{m,k}) = \frac{-\gamma_{0,m}}{W\left(-\exp\left(-g_{m,k}\frac{\ln 2}{w_m} - 1\right)\right)}, \quad g_{m,k} \geq 0 \tag{16}$$

Using this expression for the root, we can then derive the distribution functions as [9]

$$\begin{aligned} F_{g_{m,k}}(g_{m,k}) &= F_{\gamma_{m,k}}\left(\check{\gamma}_{m,k}(g_{m,k})\right) \\ f_{g_{m,k}}(g_{m,k}) &= f_{\gamma_{m,k}}\left(\check{\gamma}_{m,k}(g_{m,k})\right) \frac{\check{\gamma}^2_{m,k}(g_{m,k})}{\check{\gamma}_{m,k}(g_{m,k})\frac{w_m}{\ln 2} - \lambda} \end{aligned} \tag{17}$$

Finally, using (17) in (13) and then in (10), our dual problem can now be written as

$$g^* = \min_{\lambda \geq 0} \lambda \bar{P} + \sum_{k \in \mathcal{K}} \int_0^\infty g_k f_{g_k}(g_k) dg_k \tag{18}$$

3.4. Optimal Subcarrier and Power Allocation

Using standard duality arguments (see e.g. [5, Prop. 5.1.2]), the dual objective function in (18) can be shown to be convex and continuously differentiable in the single variable λ. Thus, we could simply take its derivative with respect to λ and set it to zero to find the optimum geometric multiplier λ^*. However, the derivative function requires $\mathcal{O}(M^2)$ computations due to the product terms in the pdf. Thus, it is more efficient to resort to derivative-free line search procedures that only need function evaluations, e.g. Golden-section or Fibonacci search [6].

Once we determine λ^*, we plug it back into the optimal power allocation function and arrive at the following simple user assignment and power allocation for each subcarrier k given as

$$m_k^* = \arg\max_{m \in \mathcal{M}} \left\{w_m R\left(\tilde{p}_{m,k}(\lambda^*)\gamma_{m,k}\right) - \lambda^* \tilde{p}_{m,k}(\lambda^*)\right\} \tag{19}$$

$$p^*_{m,k} = \begin{cases} \tilde{p}_{m,k}(\lambda^*), & m = m_k^* \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

Note that it is possible that the dual optimal powers do not satisfy the total power constraint, since this constraint has been relaxed using the Lagrangian, i.e. it is no longer mathematically enforced. Hence, our final power allocation values should be multiplied by a constant $\eta = \bar{P}/\mathbb{E}_\gamma\left\{\sum_m \sum_k p^*_{m,k}\right\}$ which we plug back into the objective in (6) to arrive at our computed primal optimal value

$$\hat{f}^* = \mathbb{E}_\gamma\left\{\sum_{m \in \mathcal{M}} w_m \sum_{k \in \mathcal{K}} \log_2(1 + \eta\gamma_{m,k}p^*_{m,k})\right\} \tag{21}$$

3.5. Bound on the Relative Duality Gap

The following theorem provides a bound on the relative optimality gap which we can compute in order to assess how far we are from the optimal value.

Theorem 1 *Let $f^* > 0$ and $g^* > 0$ given in (6) and (18) be the optimal values of the primal and dual problems, and let $\hat{f}^* > 0$ given in* (21) *be the computed feasible primal value. Then the relative duality (optimality) gap can be bounded as*

$$0 \leq \frac{g^* - f^*}{f^*} \leq \frac{g^* - \hat{f}^*}{\hat{f}^*} \tag{22}$$

Proof: The left inequality follows directly from the positivity of f^* and the weak duality theorem [5, Prop. 5.1.3. p. 495]. The right inequality is because $\hat{f}^* \leq f^*$, since $\hat{f}^*$ is a feasible primal value and f^* is the optimal feasible primal value. ■

In our numerical results, the power constraints are met almost exactly, resulting in relative optimality gaps that are practically zero ($< 10^{-4}$). Thus, our approach can, for all practical purposes, be considered an optimal solution to the problem. This fortuitous phenomenon is brought about primarily by the separability of the problem, and furthermore by the fact that we have K separable terms (which is typically large) and only a single constraint (average power constraint). This problem structure has been shown to be particularly suitable to dual optimization approaches, and has been noted in [4], and more generally treated with a theoretical justification in [5].

3.6. Complexity Analysis

Once we determine λ^* by solving (18), we do not need to update it as long as the statistics of the fading channel vector $\boldsymbol{\gamma}$ remain the same. Thus, the complexity of resource allocation requires an initial $\mathcal{O}(INMK)$ computations to determine λ^*, where I is the number of iterations for the line search procedure to converge, and N is the number of function evaluations to compute the dual objective integral. In most cases, we can also assume that $\gamma_{m,k}$ for a user m and across subcarriers k are NIID (see Section 2), and thus g_k given in (11) are likewise NIID. This allows us to replace the summation over k in in (18) to a simple multiplication by K, further reducing the initialization complexity to just $\mathcal{O}(INM)$ computations. The optimal resource allocation given in (19)-(20) requires only $\mathcal{O}(MK)$ computations per symbol instance.

4. NUMERICAL RESULTS

We consider an OFDMA system roughly based on a 3GPP-LTE downlink [1], with 128 subcarriers, 76 used subcarriers, 1.25 MHz bandwidth, 1.92 MHz sampling frequency, and a cyclic prefix length of 6 samples. We simulate the frequency-selective Rayleigh fading channel using the ITU-Vehicular A channel model [10]. We generate 10000 IID channel realizations per data point, where for each user's channel realization $\boldsymbol{h}_m$ (2), we generate a complex Gaussian random vector with N_t independent entries, each with variance corresponding to the power delay profile for the corresponding path.

In Fig. 1, we compare the capacity region for 2 users in various SNRs of our ergodic capacity maximization algorithm with instantaneous capacity maximization [4] and constant power allocation [2]. We see that the gain of ergodic maximization is more pronounced for low SNRs and more disparate user weights. This observation is analogous to previous studies in adaptive modulation, e.g. [11],

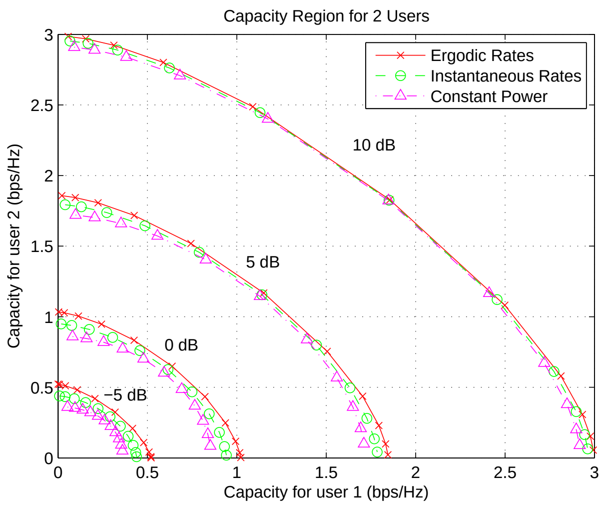

Fig. 1. 2-User capacity Region for ergodic capacity maximization, instantaneous capacity maximization, and constant power allocation.

which concluded that the exploitation of the additional temporal dimension through the ergodic formulation is most useful when other degrees of freedom have been significantly curtailed. In Table 1, we present other relevent metrics for the ergodic and instantaneous rate maximization algorithms. For the ergodic rate maximization, the first main row indicates the average number of function evaluations required to numerically compute the integration of (18) with a tolerance of 10^{-6}, and the second main row indicates the average number of Golden-section search iterations to solve for λ^* in the dual problem (10) with a tolerance of 10^{-4}. The second row for instantaneous rate maximization is the average number of iterations for each channel realization. The third row for both cases is the relative duality gap upper bound computed by (22). Note that the duality gaps are negligible for all practical purposes, and thus both algorithms can be considered optimal. Since the constant power allocation does not involve iterations, it is not included in Table 1.

4.1. Complexity Comparison

Table 2 shows the complexity order of the different resource allocation algorithms. Only the ergodic rate algorithms require initialization. If we use the average numbers given in Table 1, the ergodic rate algorithm is less complex than the instantaneous rate algorithm per symbol on average, as long as the rate of change of the channel fading statistics (roughly at the rate of change of slow fading, e.g. Log-normal shadowing) is much lower than the rate of change of the actual channel realizations (roughly at the rate of fast fading, e.g. Rayleigh fading), such that the initialization is performed less often. One caveat, however, is that the ergodic rate algorithms require information on the channel fading distribution function, which need an additional level of complexity and feedback overhead. Furthermore, the peak-to-average power ratio of the power allocation in the ergodic rates case is typically higher than for instantaneous rates, and even more so for constant power allocation.

Table 1. Relevant Metrics for the Resource Allocation Algorithms

Metric	SNR	Ergodic	Inst.
No. of Fun. Eval. (N)	5 dB	47.91	–
	10 dB	50.09	–
	15 dB	53.73	–
No. of Iterations (I)	5 dB	8.091	8.344
	10 dB	7.727	8.333
	15 dB	7.936	8.539
Relative Gap ($\times 10^{-6}$)	5 dB	7.936	.0251
	10 dB	5.462	.0226
	15 dB	5.444	.0159

Table 2. Complexity for the Resource Allocation Algorithms. M-no. of users, K-no. of subcarriers, N-no. of function evaluations for integration, I-no. of line search iterations.

Algorithm	Initialization	Runtime
Ergodic Rates	$\mathcal{O}(INM)$	$\mathcal{O}(MK)$
Instantaneous Rates	–	$\mathcal{O}(IMK)$
Constant Power	–	$\mathcal{O}(MK)$

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