Unit-norm complex-Gaussian $\overset{iid}{\vee}$ samples:

- Amplitude is Rayleigh: $f_\rho(\rho;\sigma) = \frac{\rho}{\sigma^2} e^{-\rho^2/2\sigma^2}$
- Phase is Uniform: $f_\phi(\phi) = \frac{\phi}{2\pi}$

1. Split pdf into N equal probability intervals, i.e.

$$\int_{t_q}^{t_{q+1}} f_\rho(\rho;\sigma)\, d\rho = \frac{1}{N}$$

2. Find centroid of each region:

$$\hat{\rho}_q = \frac{\int_{t_q}^{t_{q+1}} \rho\, f_\rho(\rho;\sigma)\, d\rho}{\int_{t_q}^{t_{q+1}} f_\rho(\rho;\sigma)\, d\rho} \qquad = \frac{1}{N}$$

$$= N \int_{t_q}^{t_{q+1}} \rho\, f_\rho(\rho;\sigma)\, d\rho$$

$$= N \int_{t_q}^{t_{q+1}} \frac{\rho^2}{\sigma^2} e^{-\rho^2/2\sigma^2}\, d\rho$$

$uv - \int v\, du$

Integrate by parts

$u = \rho \qquad dv = \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}}$

$du = d\rho \qquad v = -e^{-\frac{\rho^2}{2\sigma^2}}$

$$= N\left[-\rho e^{-\frac{\rho^2}{2\sigma^2}} - \int e^{-\frac{\rho^2}{2\sigma^2}}\, d\rho\right]\Bigg|_{\rho = t_q}^{\rho = t_{q+1}}$$

// since $t_{q+1} > t_q$ by definition

$$\int_0^{t_{q+1}} e^{-\frac{\rho^2}{2\sigma^2}}\, d\rho - \int_0^{t_q} e^{-\frac{\rho^2}{2\sigma^2}}\, d\rho$$

$$\sqrt{\frac{\pi}{2}}\,\sigma\left[\operatorname{erf}\left(\frac{t_{q+1}}{\sqrt{2\sigma^2}}\right) - \operatorname{erf}\left(\frac{t_q}{\sqrt{2\sigma^2}}\right)\right]$$

$$= N\left[\left(t_q e^{-\frac{t_q^2}{2\sigma^2}} - t_{q+1} e^{-\frac{t_{q+1}^2}{2\sigma^2}}\right) + \sqrt{\frac{\pi}{2}}\,\sigma\left[\operatorname{erf}\left(\frac{t_q}{\sqrt{2\sigma^2}}\right) - \operatorname{erf}\left(\frac{t_{q+1}}{\sqrt{2\sigma^2}}\right)\right]\right]$$