LOW COMPLEXITY SUBBAND ANALYSIS USING QUADRATURE MIRROR FILTERS

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ABSTRACT

In this article, a novel method of performing subband analysis of digital signals is proposed. Conventional subband decomposition algorithms typically use a binary tree filterbank structure comprised of halfband filters. Due to design limitations of finite length filters, conventional decomposition algorithms typically suffer from interference due to aliasing. While longer halfband filters may reduce aliasing, such filters also increase latency and implementation complexity. Our proposed algorithm uses a novel structure of quadrature mirror filters to ensure aliasing is present outside of the spectral region of interest. Simulation results indicate that, compared to conventional algorithms, the proposed algorithm 1) reduces interference from aliasing by over 30dB, 2) reduces signal processing latency, and 3) reduces implementation complexity.

Index Terms— Subband Decomposition, Quadrature Mirror Filters, Halfband filters

1. INTRODUCTION

Subband decomposition is the process of decomposing a digital signal into multiple component signals, each of which contains a small chunk of the spectral information of the original signal[1]. It is a well investigated topic in the domain of signal processing and has found use in various applications including adaptive filtering[2], acoustic processing and echo cancellation[3, 4], and digital receivers[5, 6, 7]. In these methods, signal processing is performed on the component signals after subband decomposition. In order for these post processing algorithms to perform optimally, it becomes desirable that the subband decomposition stage produce component signals that are distortion free[8, 2].

In this paper, we consider the analysis of a signal into its subband representation, and propose a novel analysis algorithm that reduces interference caused by aliasing, reduces processing latency, and requires fewer computational resources compared to typical subband analysis methods. Section 2 lists typical methods of performing subband decomposition. Sections 3 and 4 explain the conventional and proposed algorithm for subband analysis. Section 5 provides a comparison of the conventional and the proposed algorithm in terms of the performance indicators. Simulation results are given in Section 6, and final conclusions are listed in Section 7.

2. PRIOR WORK

Conventional subband analysis may be performed by using finite impulse response (FIR) and infinite impulse response (IIR) filters in the form of a filterbank structure. Another commonly employed approach to subband analysis utilizes multirate filterbanks in a binary tree structure[9, 10, 11]. Since the binary tree structure has low implementation complexity compared to conventional filterbanks, we will only consider it in this article. Such filterbanks normally require long FIR filters to obtain sufficiently sharp passband to stopband transition to reduce aliasing. These filters introduce substantial delay into the signal path and increase the implementation complexity[12, 13]. Other work has also focused on using allpass polyphase (AP) infinite impulse response (IIR) filters[3] are also used as halfband filters, as they can achieve high discrimination with short filters. However, while IIR filters can achieve high stopband attenuation, they introduce a non-linear phase response and the impact of aliasing even from sharp transition halfband filters is very high.

3. SUBBAND ANALYSIS USING A BINARY TREE FILTERBANK

The core building block in binary tree subband analysis is a two band decomposition shown in Figure 1. Two-band decomposition is defined as dividing an input digital signal into two component signals, each of which contains half of the spectral content of the signal, and each is sampled at half the rate of the original signal. In figure 1, $x_L[n]$ and $x_R[n]$ are sampled at half the rate of $x[n]$. The spectral content of $x[n]$ between normalized frequencies -0.5 to 0 is in $x_L[n]$, and the remainder in $x_R[n]$. Two band decomposers (2BDs) are typically implemented using halfband filters[14] (HBFs), which are a special class of filters known as Quadrature Mirror Filters (QMF) that suppress half of the spectral content of a digital signal. A QMF can be defined by the following key parameters:

Transition Width: The distance in normalized frequency between the stopband-edge frequency, and the passband-edge frequency of the filter[10].

Stopband Ripple: The maximum deviation of the filter from its ideal stopband response of 0.0. The stopband ripple is equal to the attenuation of the spectrum being suppressed by the filter.

Filter Order: The number of taps used in the filter.

Figure 2 shows the frequency response specification of the two HBFs used in 2BD block.

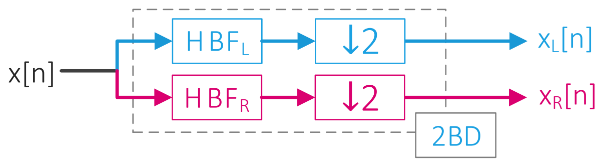


Fig. 1. Two band decomposition using half band filters.

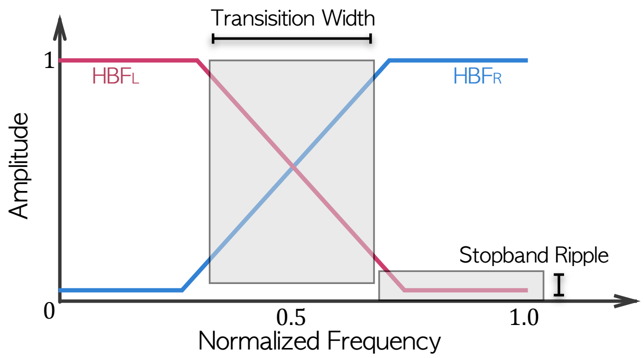


Fig. 2. Halfband filter specifications used in two-band decomposition block.

Figure 3 shows subband decomposition with a binary tree filterbank structure. After N iterations through the 2BD block, the input signal is decomposed into 2^N individual signals each containing an equal size chunk of the original spectral content. In figure 3, a digital signal is decomposed into 8 signals. Each of these output signals contains $\frac{1}{8}^{\text{th}}$ of the spectral content of the input signal, and has a sampling rate $\frac{1}{8}^{\text{th}}$ of the sampling rate of the input signal in order to satisfy the Nyquist criterion.

A perfect HBF would completely remove exactly half of the spectral content, having both stopband ripple and transition width equal to 0. In practical implementation, finite length QMFs and even QMFs implemented using infinite impulse response (IIR) filters have finite stopband ripples and transition width. Table 1 shows the transistion width and stopband ripple for different length FIR QMFs. A finite

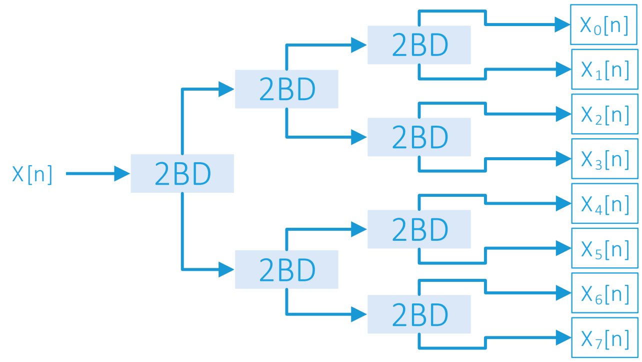


Fig. 3. Subband decomposition algorithm using a binary tree filterbank.

Table 1. Transition width and stopband ripple for various length quadrature mirror filters.

Filter Order	Transition Width	Stopband Ripple
64	0.15	-80dB
96	0.1	-80dB
128	0.075	-80dB

transition width of the QMF followed by decimation results in aliasing interference being present in the output signal of the QMF. The aliasing power gets compounded after each level of the filterbank tree increases, resulting in poor signal to aliased interference ratio at the output signals. This motivates the need for improved algorithms that can reduce the aliased interference present in the decomposed signals.

4. SUBBAND ANALYSIS USING A BINARY TREE FILTERBANK AND DELAYED DECIMATION

In this section, a novel approach to subband decomposition is proposed, using the filterbank structure shown in Figure 6. In this structure, the input signal is first split into two subbands, each containing half of its spectral content, but these subband signals are not down sampled. Therfeore, the subband signals have a sampling rate that is twice the minimum required to fulfill the Nyquist criterion. Henceforth, each stage of the decomposition, the 2BD block requires a filter that has unit response in the first quarter of the spectrum, and stop band attenuation between the normalized frequency 0.75 to 1.0. Therefore the transition band required to prevent aliasing is less than 0.5. This significantly eases design requirements on the filter and quadrature mirror filters that meet this transition width specification can be designed using a few non-zero taps. After the final two-band decomposition stage, the subband signals comprise of relevant spectrum in one half of their band, and aliasing interference in the rest of the band which is filtered using a HBF.

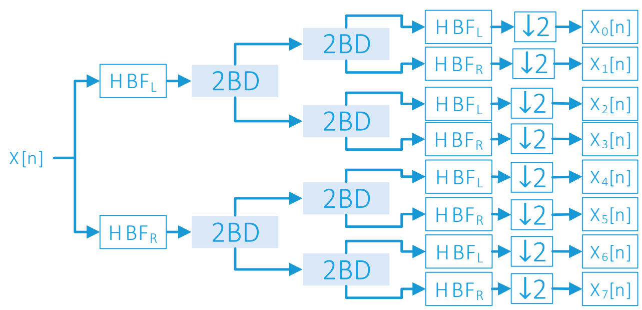


Fig. 4. Subband decomposition using quadrature mirror filterbank

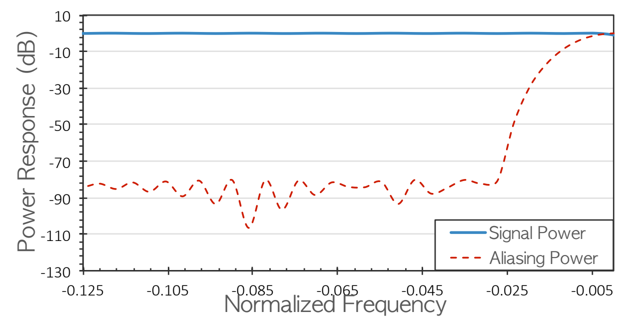


Fig. 5. Signal and aliasing amplitude response of halfband filter used in conventional subband decomposition.

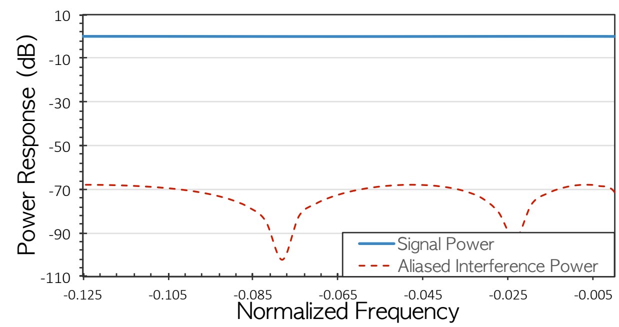


Fig. 6. Signal and aliasing amplitude response of quarterband filter used in proposed subband decomposition.

5. COMPARISON OF DESIGN TRADE-OFFS

In this section, the key design tradeoffs of aliasing performance vs. implementation complexity of the conventional and proposed algorithms are compared. The design objective is to minimize the amount of aliasing induced by the algorithm, while maintaining a low level of implementation complexity. Computational complexity is defined using the number of multiplication operations required by the algorithm. Multiplications directly relate to resource usage in a digital signal processor or field programmable gate array implementation, as well as to the latency of the algorithm.

5.1. Aliasing Performance Analysis

Subband decomposition requires downsampling of the signal by a factor of 2 after passing through each HBF. Since the downsampling procedure does not have any anti-aliasing protection, the spectral component outside the halfband gets aliased on the spectral content of interest. In normalized frequency terms, the signal of interest lies in frequency band between 0.0 to 0.5, and the aliasing originates from 0.5 to 1.0. Ideally, the half band filter should have a passband between 0.0 to 0.5 and stopband between 0.5 to 1.0. This results in a transition width of 0.0, which is not possible in practical design. Therefore, the transition region of practical halfband filters causes aliasing after decimation. Figure 5 shows the halfband filter with a stop band attenuation of 80dB and transition width of 0.05, zoomed into the transition region of the halfband filter, indicating additive interference from aliasing.

In the proposed algorithm, at each stage the signal is sampled at twice the rate of its useful spectral content. After filtering, the useful content of the spectrum is within frequencies 0 to 0.25, and aliasing originates from normalized frequency 0.75 to 1.0. Thus, the QMF should have its passband between 0 to 0.25, and its stopband between 0.75 to 1.0. The resulting transition width evaluates to 0.5, significantly easing design requirements. Similar to Figure 5, the aliasing caused by the proposed filter is shown in Figure 6.

5.2. Implementation Complexity Analysis

A two band decomposition algorithms using FIR filters have been widely studied, and it is a well known result that such algorithms require $\frac{L-1}{4}$ multiplications for a L tap HBF filter[10]. Figure 3 shows that subband decomposition of a signal into 2^N bands requires 2^N halfband filters. This would entail a complexity of $2^N M$, where M is the number of multipliers in each 2BD. However, it is also important to note the rate at which the multipliers are being used. After the first 2BD stage in Figure 3, the decimated signal has half the sampling rate of the original signal. Thus, a single multiplier can operate at twice the rate and perform two computations at each sample, effectively reducing the number of multipliers needed by half. Thus, we are interested in the product of the number of multipliers and the sampling rate at which the multiplier is operating at, giving us the number of multiplications per unit time. At each stage of the decomposition, there are 2^k operational HBFs on a signal sampled at $\frac{1}{2^k}$ times the original sampling rate, effectively using $\frac{2^k}{2^k}M = M$ multiplication units. M multiplication units per stage over N stages gives us a total of MN multipliers used in N-stage subband decomposition. Hence, for a L tap FIR filter per-stage, the implementation complexity is $\frac{N(L-1)}{4}$ multiplication units.

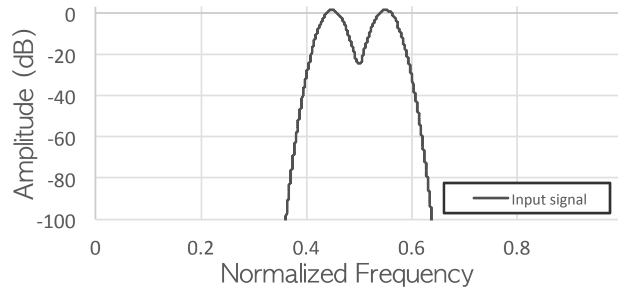
The computational complexity of the proposed algorithm can be estimated in a manner similar to the halfband filter based algorithm. From Figure 4, the first stage of the algorithm performs halfband filtering without the decimation process. The lack of a decimation stage means that certain optimizations used in two band decomposition cannot be used[10], requiring $\frac{L-1}{2}$ multiplications in this stage. The subsequent stages contain quadrature mirror FIR filters, and each 2BD block requires $\frac{L-1}{2}$ multiplications. Since there are $N-1$ such stages, and the k^{th} stage the sampling rate is $\frac{1}{2^{k-1}}$, each stage requires $\frac{(L-1)2^k}{2^{k-1}.2}$ multiplication units. Finally, the HBFs in the last stage require $(L_{\text{HBF}}-1)/2$ multiplication units, where L_{HBF} is the filter length. The total number of multiplication units is $\frac{L-1}{2}+(L-1)(N-1)+\frac{L_{\text{HBF}}-1}{2}$.

6. NUMERICAL SIMULATIONS

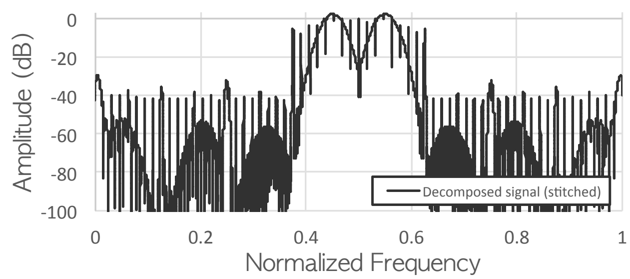
The conventional and proposed algorithms were implemented using National Instruments' LabVIEW programming language. An amplitude modulated digital signal, shown in Figure 6(a) was passed through the subband decomposition blocks. Figures 6(b) and 6(c) show the spectrum of output of the subband decomposition blocks, stitched together to indicate the decomposition process. A significant difference between the aliasing performance of the conventional and the proposed methods is clearly observed. The signal to aliased interference power is calculated as the ratio of the signal power to the total aliased interference power introduced by the subband decomposition process. Furthermore, the proposed algorithm uses a 4 tap FIR filter, compared to the conventional algorithm using a 32 tap FIR filter. Figure 8 shows the tradeoff between the signal to aliasing power ratio and implementation complexity for different filter lengths in the proposed and the conventional algorithms. An ideal implementation would exhibit infinite signal to aliased interference power ratio and require no implementation complexity, consequently, points that are towards the top left of the figure indicate a better design. Figure 8 shows the design superiority of the proposed algorithm compared to conventional algorithm.

7. CONCLUSION

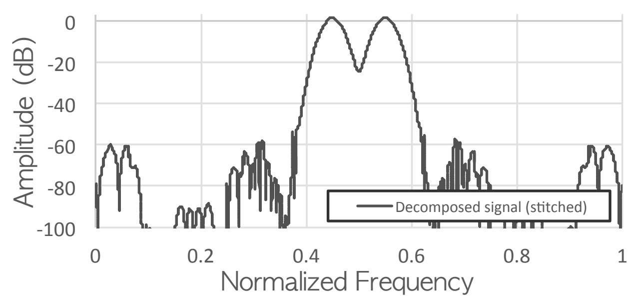
In this article, a novel method of subband decomposition was proposed, using a binary tree filterbank of quadrature mirror filters. The proposed algorithm exhibited suppressed aliasing artifacts compared to conventional subband decomposition algorithms, and also required fewer multiplication resources. Simulation results indicate that the proposed algorithm reduces aliasing levels by over 30dB compared to the conventional algorithm, while using the same amount of implementation resources.



(a) Spectrum of input signal.



(b) Spectrum of subband signals from conventional algorithm.



(c) Spectrum of subband signals from proposed algorithm.

Fig. 7. A digital signal is analyzed using the conventional and proposed subband decomposition algorithms. The spectrum of the subband signals is stitched together.

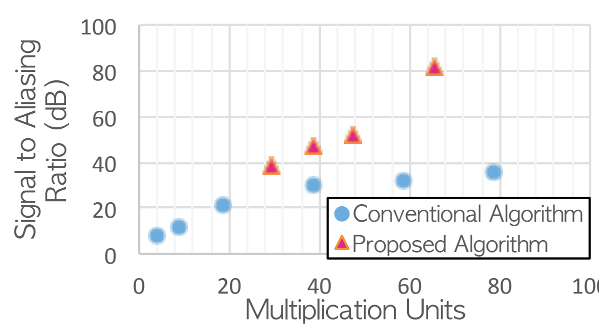


Fig. 8. Design tradeoff between number of implementation complexity vs. aliasing performance in conventional and proposed algorithms.

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