Reverse Link Analysis for Full-Duplex Cellular Networks with Low Resolution ADC/DAC[1]

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Outline

Introduction

- Full duplex: transmission and reception in same time/frequency resource block
- Benefits
  - Double spectral efficiency
  - Reduce latency
  - Enhance reliability/coverage
- Challenge: Loop-back self-interference
  - Transmitted signal received by co-located receiver
  - Saturates receiver analog-to-digital converters (ADCs)
  - ADC saturation results in poor spectral efficiency
- Full duplex applications
  - Machine-to-machine communications
  - Integrated access and backhaul
- Proposed in 3GGP Release 17 cellular standard

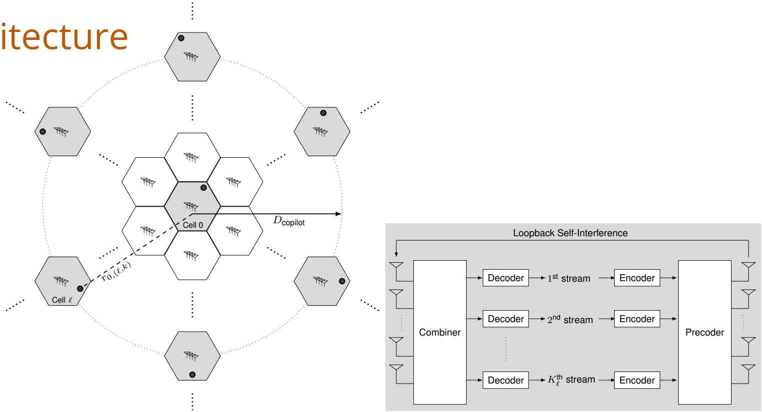
Introduction

- Multi-antenna systems reduce hefty power consumption by reducing
  - Number of RF processing chains using hybrid analog/digital beamforming
  - Data converter resolution
- Solution for full-duplex multiantenna basestation with low-resolution converters
  - Use degrees of freedom in the all-digital beamformer design due to number of antennas to suppress self-interference
- Contributions
  - Provide unified framework for cellular reverse link (uplink)
  - Derive signal to quantization plus interference plus noise ratio (**sqinr**).
  - Quantify effects on outage probability and spectral efficiency due to
    - Quantization error
    - Pilot contamination
    - Self interference
    - Number of users
    - Overhead

Outline

- Introduction
- Network Model
  - Architecture
  - Model
  - Signal
  - SQINR
  - Special Cases
- System Performances
  - Performance Measures
  - Numerical Results
- Conclusion

Architecture



(a) (b)

Figure: Fig. 1a Hexagonal lattice network. The cell of interest and the cells belonging to subset $\mathcal{C}$, i.e., all the ones reusing the same pilot dimensions, are shaded and a copilot user equipment in each cell is indicated by a circle. Also indicated is the distance $r_{\mathbf{0},(\ell,k)}$ between the cell of interest cell 0 and the k-th user equipment served by the cell ℓ. In addition, indicated is the distance $D_{\mathbf{copilot}}$ between the cell of interest and its first tier of copilot cells. Fig. 1b Basic abstraction of a FD BS: The uplink user equipment (UE) sends the data to the BS independently from the data intended to the downlink UE sent from the BS. Since the BS transmits and receives simultaneously at the same resource blocks, SI leakage is created in the form of a loopback from TX to RX sides of the BS.

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Model

1. Network geometry: Hexagonal lattice.
2. Cellular systems (multicell and multiuser).
3. BSs and UEs operate in FD and Half-Duplex (HD), respectively.
4. Massive number of antennas $N_\mathsf{a} \gg 1$ at the BS and UEs are equipped with a single antenna.
5. BSs operate in low-resolution ADC/DAC.
6. Additive Quantization Noise Model (AQNM): $\mathbf{y}_\mathbf{q} = \alpha \mathbf{y} + \mathbf{q}$.
7. Large-scale fading: Pathloss and Lognormal shadowing.
8. Small-scale fading: Rayleigh.
9. Matched filter receiver at the BS.
10. Pilot contamination: pilots reuse per cell.
11. Channel hardening.

Outline

Signal

- Upon the data transmission from the users, the BS observes the following signal

$$\begin{aligned} \boldsymbol{y}_q^u =& \alpha_u \sum_{\ell} \sum_{k=0}^{K_\ell - 1} \sqrt{G_{\ell,k} P_{\ell,k}} \boldsymbol{h}_{\ell,k} s_{\ell,k} + \alpha_u \sqrt{P_{\mathsf{SI}}} \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{q}_d \\ & + \alpha_u \alpha_d \sqrt{P_{\mathsf{SI}}} \sum_{k=0}^{K-1} \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f}_k s_k^d + \boldsymbol{q}_u + \alpha_u \boldsymbol{v} \end{aligned}$$

- The BS applies the receive filter ($\boldsymbol{w}_k$) to extract the signal of the k-th uplink UE as

$$\begin{aligned} y_{q,k}^u =& \underbrace{\alpha_u \sqrt{G_k P_k} \mathbb{E}[\boldsymbol{w}_k^* \boldsymbol{h}_k] s_k}_{\text{Desired Signal}} + \underbrace{\alpha_u \sqrt{G_k P_k} \left(\boldsymbol{w}_k^* \boldsymbol{h}_k - \mathbb{E}[\boldsymbol{w}_k^* \boldsymbol{h}_k] \right) s_k}_{\text{Channel Estimation Error}} + \underbrace{\alpha_u \sum_{\mathrm{k} \neq k} \sqrt{G_k P_{\mathrm{k}}} \boldsymbol{w}_k^* \boldsymbol{h}_k s_{\mathrm{k}}}_{\text{Intra-Cell Interference}} + \underbrace{\alpha_u \sum_{\ell \in \mathcal{C}} \sqrt{G_{\ell,k} P_{\ell,k}} \boldsymbol{w}_k^* \boldsymbol{h}_{\ell,k} s_{\ell,k}}_{\text{Pilot Contamination}} \\ & + \underbrace{\alpha_u \sum_{\substack{\ell \neq 0 \\ \ell \notin \mathcal{C}}} \sum_{\mathrm{k}=0}^{K_\ell - 1} \sqrt{G_{\ell,k} P_{\ell,\mathrm{k}}} \boldsymbol{w}_k^* \boldsymbol{h}_{\ell,k} s_{\ell,\mathrm{k}} + \alpha_u \sum_{\ell \in \mathcal{C}} \sum_{\substack{\mathrm{k}=0 \\ \mathrm{k} \neq k}}^{K_\ell - 1} \sqrt{G_{\ell,k} P_{\ell,\mathrm{k}}} \boldsymbol{w}_k^* \boldsymbol{h}_{\ell,k} s_{\ell,\mathrm{k}}}_{\text{Inter-Cell Interference}} + \underbrace{\alpha_u \alpha_d \sqrt{P_{\mathsf{SI}}} \sum_{\mathrm{k}=0}^{K-1} \boldsymbol{w}_k^* \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{f}_{\mathrm{k}} s_{\mathrm{k}}^d}_{\text{Self-Interference due to Full-Duplexing}} \\ & + \underbrace{\alpha_u \sqrt{P_{\mathsf{SI}}} \boldsymbol{w}_k^* \boldsymbol{H}_{\mathsf{SI}} \boldsymbol{q}_d + \boldsymbol{w}_k^* \boldsymbol{q}_u}_{\text{Aggregate AQNM}} + \underbrace{\alpha_u \boldsymbol{w}_k^* \boldsymbol{v}}_{\text{Filtered Noise}} \end{aligned}$$

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SQINR

Corollary

The matched filter receiver $\boldsymbol{w}_k^{\mathsf{MF}}$ has the following properties:

1. $\mathbb{E}\left[\|\boldsymbol{w}_k^{\mathsf{MF}}\|^2\right] = N_{\mathsf{a}}$.
2. $\mathbb{E}\left[\|\boldsymbol{w}_k^{\mathsf{MF}}\|^4\right] = N_{\mathsf{a}}^2 + N_{\mathsf{a}}$.
3. $\mathbb{E}\left[\left|\boldsymbol{w}_k^{\mathsf{MF}*}\boldsymbol{h}_{\ell,\mathrm{k}}\right|^2\right] = N_{\mathsf{a}}$.

Theorem

The output SQINR of the k-th user is given by the following equation.

$$\overline{\mathsf{sqinr}}_k^{\mathsf{MF}} = \frac{\alpha_u^2\left(\frac{P_k}{P_u}\mathsf{SNR}_k^u\right)^2 N_{\mathsf{a}}^2}{\left(1+\frac{P_k}{P_u}\mathsf{SNR}_k^u+\sum_{\ell\in\mathcal{C}}\frac{P_{\ell,k}}{P_u}\mathsf{SNR}_{\ell,k}^u\right)\overline{\mathsf{den}}^{\mathsf{MF}}}$$

$$\begin{aligned}\overline{\mathsf{den}}^{\mathsf{MF}} =& \alpha_u^2 N_{\mathsf{a}}\left(1+\sum_{\ell}\sum_{\mathrm{k}=0}^{K_\ell^u-1}\frac{P_{\ell,\mathrm{k}}}{P_u}\mathsf{SNR}_{\ell,\mathrm{k}}^u\right)+\alpha_u^2 N_{\mathsf{a}}^2\frac{\sum_{\ell\in\mathcal{C}}\left(\frac{P_{\ell,k}}{P_u}\mathsf{SNR}_{\ell,k}^u\right)^2}{1+\frac{P_k}{P_u}\mathsf{SNR}_k^u+\sum_{\ell\in\mathcal{C}}\frac{P_{\ell,k}}{P_u}\mathsf{SNR}_{\ell,k}^u}+\alpha_u^2\alpha_d(1-\alpha_d)K^d N_{\mathsf{a}}^2\mathsf{INR}\\ &+\alpha_u^2\alpha_d^2 K^d N_{\mathsf{a}}^2\mathsf{INR}+N_{\mathsf{a}}\alpha_u(1-\alpha_u)\left[2\frac{P_k}{P_u}\mathsf{SNR}_k^u+\sum_{\mathrm{k}\neq k}\frac{P_{\mathrm{k}}}{P_u}\mathsf{SNR}_{\mathrm{k}}^u+\sum_{\ell\neq 0}\sum_{\mathrm{k}}\frac{P_{\ell,\mathrm{k}}}{P_u}\mathsf{SNR}_{\ell,\mathrm{k}}^u+\alpha_d N_{\mathsf{a}}\mathsf{INR}+1\right]\end{aligned}$$

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Special Cases

Corollary

To further characterize the spectral efficiency, we derive a new bound using the following formula. Assuming statistical independence between x and y, we have

$$\mathbb{E}\left[\log\left(1+\frac{x}{y}\right)\right] \cong \log\left(1+\frac{\mathbb{E}[x]}{\mathbb{E}[y]}\right)$$

Proposition

Considering a single-cell multiuser system (without any inter-cell interference) with perfect CSI, Corollary (above) entails the results for reverse link in[a]*.*

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[a] Jianxin Dai, Juan Liu, Jiangzhou Wang, Junxi Zhao, Chonghu Cheng, and Jin-Yuan Wang. Achievable Rates for Full-Duplex Massive MIMO Systems With Low-Resolution ADCs/DACs. In: *IEEE Access* 7 (2019), pp. 24343–24353. DOI: `10.1109/ACCESS.2019.2900273`.

Special Cases

Proposition

With channel hardening, without full-duplexing ($\boldsymbol{H}_{\mathrm{SI}} = \mathbf{0}$), with full-resolution and matched filter receiver, the output SINR of the k-th uplink user is given by the following equation.

$$\overline{\mathrm{sinr}}_k^{\mathrm{MF}} = \frac{\frac{N_{\mathbf{a}}}{1+\frac{P_k}{P_u}\mathrm{SNR}_k^u+\sum_{\ell\in\mathcal{C}}\frac{P_{\ell,k}}{P_u}\mathrm{SNR}_{\ell,k}^u}\left(\frac{P_k}{P_u}\mathrm{SNR}_k^u\right)^2}{1+\sum_{\ell}\sum_{k=0}^{K_\ell^u-1}\frac{P_{\ell,k}}{P_u}\mathrm{SNR}_{\ell,k}^u+\frac{N_{\mathbf{a}}}{1+\frac{P_k}{P_u}\mathrm{SNR}_k^u+\sum_{\ell\in\mathcal{C}}\frac{P_{\ell,k}}{P_u}\mathrm{SNR}_{\ell,k}^u}\sum_{\ell\in\mathcal{C}}\left(\frac{P_{\ell,k}}{P_u}\mathrm{SNR}_{\ell,k}^u\right)^2}$$

Remark

Note that this Proposition entails the same result for reverse link derived by [Eq. (10.63)] in[a].

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[a] Robert W. Heath Jr. and Angel Lozano. *Foundations of MIMO Communication*. Cambridge University Press, 2018. DOI: `10.1017/9781139049276`.

Special Cases

Lemma

When the power (SNR_k^u) *of the k-th uplink user goes to infinity, the output SINR converges to* N_a.

Lemma

Neglecting the pilot contamination, the output SINR expression can be reduced to

$$\overline{\mathsf{sinr}}_k^{\mathsf{MF}} \approx \frac{N_\mathsf{a}\left(\frac{P_k}{P_u}\mathsf{SNR}_k^u\right)^2}{\left(1+\frac{P_k}{P_u}\mathsf{SNR}_k^u\right)\left(1+\sum_\ell\sum_{k=0}^{K_\ell^u-1}\frac{P_{\ell,k}}{P_u}\mathsf{SNR}_{\ell,k}^u\right)}$$

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Performance Measures

- The effective spectral efficiency of the k-th uplink UE is given by

$$\frac{\mathcal{I}_k^{\mathsf{eff}}}{B} = \left(1 - \beta \frac{N_{\mathsf{p}}}{N_{\mathsf{c}}}\right) \log\left(1 + \overline{\mathsf{sqinr}}_k\right), \; k = 0, \dots, K-1$$

  where N_{p}, N_{c}, $\beta \in [0,1]$ and B are number of pilots, number of coherence tiles, fraction of pilot overhead and bandwidth, respectively.

- Once a transmission strategy is specified, the corresponding cumulative distribution function (CDF) or outage probability for rate R (bit/s/Hz) is then

$$P_{\mathsf{out}}(\mathsf{SNR}, R) = \mathbb{P}[\mathcal{I}(\mathsf{SNR}) < R].$$

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Numerical Results

Table: System Parameters.

Parameter	Value
Bandwidth	20 MHz
Pathloss Exponent (η)	4
Shadowing (σ_{dB})	8 dB
Uplink Transmit Power	200 mW
SI Power (P_{SI})	40 W
SI Channel Power (μ_{SI})	10 dB
Thermal Noise Spectral Density	-174 dBm/Hz
Noise Figure	3 dB
BS Antennas Gain	30 dB
Number of Antennas (N_{a})	100
Uplink/Downlink Users per Cell (K_{ℓ})	10
Number of Pilots per Cell (N_{p})	$3K_{\ell}$
Fraction of Pilot Overhead (β)	0.5
Fading Coherence Tile (N_{c})	20,000 (Pedestrians)
ADC/DAC resolution	3 bits

Heath Jr. and Lozano 2018; Geordie George, Angel Lozano, and Martin Haenggi. Massive MIMO Forward Link Analysis for Cellular Networks. In: *IEEE Transactions on Wireless Communications* 18.6 (June 2019), pp. 2964–2976. ISSN: 1558-2248. DOI: `10.1109/twc.2019.2907584`.

CDF

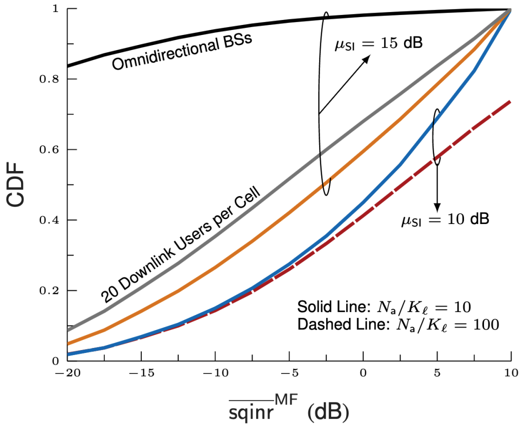

Figure: Reverse link results: Effects of the antennas gain, SI channel power and the number of downlink UEs on the CDF of the SQINR. Unless otherwise stated, the number of downlink UEs per cell is 10 users and the antennas array gain is 30 dB. The difference between Red and Blue curves is the the value of the ratio $N_{\mathbf{a}}/K_{\ell}$. The difference between the gray and orange curves is the number of downlink users per cell. The black curve is simulated following the default value but except with 0 dB of antenna gain.

Effective Spectral Efficiency

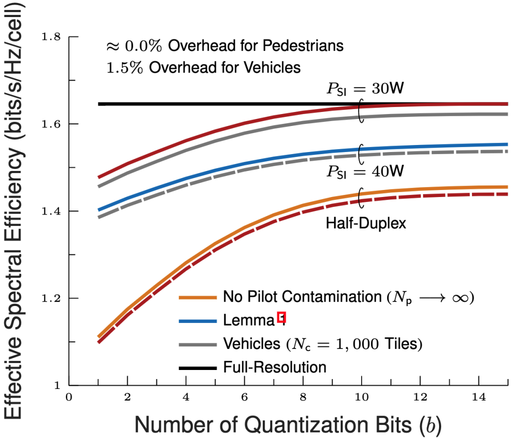

Figure: Reverse link results: Effects of SI power, overhead, ADC/DAC resolution, duplexing mode and pilot contamination on the spectral efficiency. The dashed red is simulated for half-duplex and accounting for pilot contamination unlike the solid yellow curve. The dashed gray curve is simulated following the default value as defined in the Table of simulation parameters. The Hexagonal grid is assumed for this simulation. The solid red curve is considered for pedestrians case unlike the solid gray curve is considered for vehicular scenario.

Conclusion

- Considered full-duplex multiantenna basestations
    - All-digital beamformers
    - Low-resolution data converters
    - Arranged on hexagonal lattice
- Developed a unified framework for reverse link analysis
    - Derived outage probability and spectral efficiency based on received signal, interference, quantization error, and thermal noise
    - Derived special cases
- Designed digital beamformers
    - Used degrees of freedom due to massive number of antennas
    - Compensated quantization error and inter-user interference
- Full duplex system outperforms half-duplex mode in effective spectral efficiency
    - Gives evidence of feasibility of full-duplex cellular networks

Thanks for your attention
Questions ?

The University of Texas at Austin
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