Optimizing Communication Performance of Low-Resolution ADC Systems with Hybrid Beamforming

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WHAT STARTS HERE CHANGES THE WORLD

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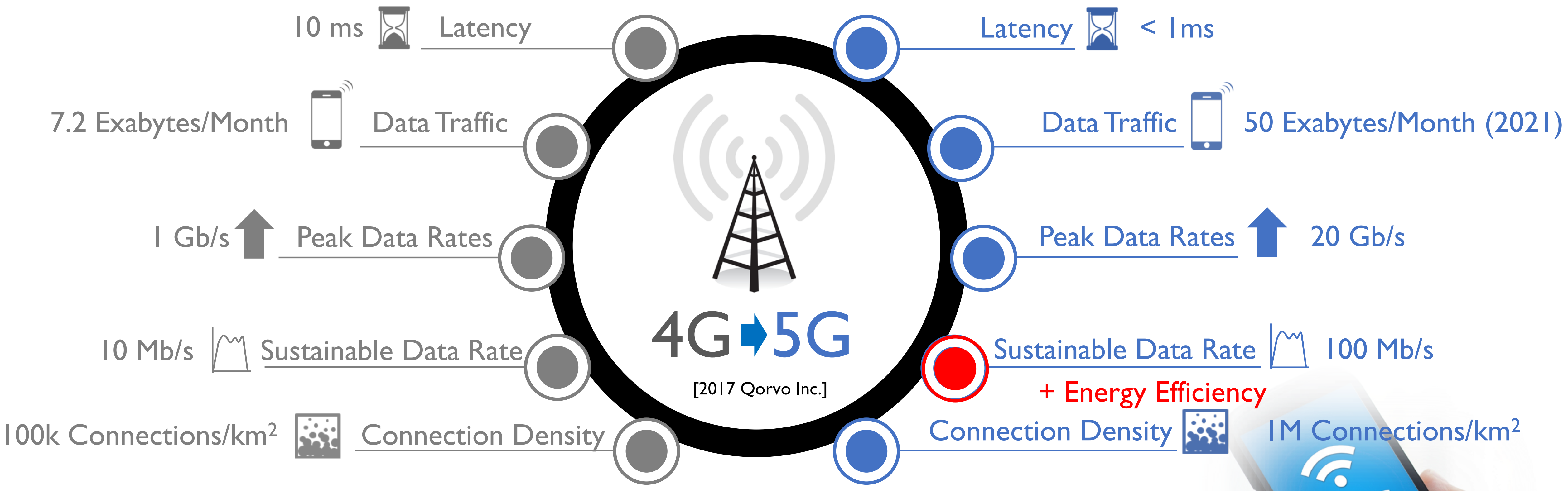
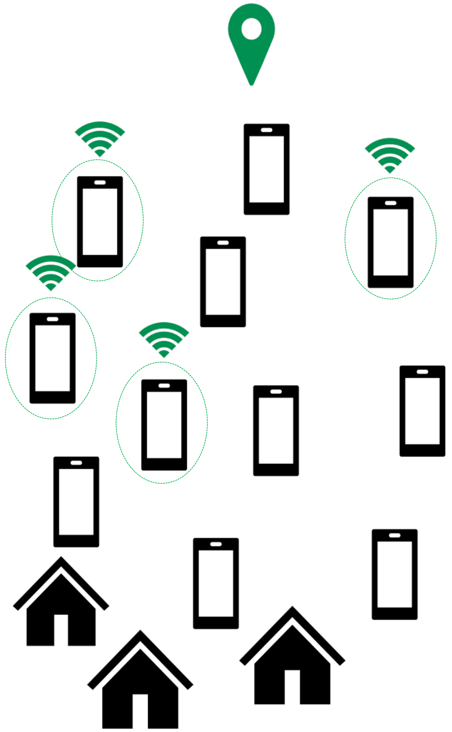
Vision for 5G Communications



[https://www.qorvo.com/design-hub/blog/getting-to-5g-comparing-4g-and-5g-system-requirements]

OVERVIEW

User devices

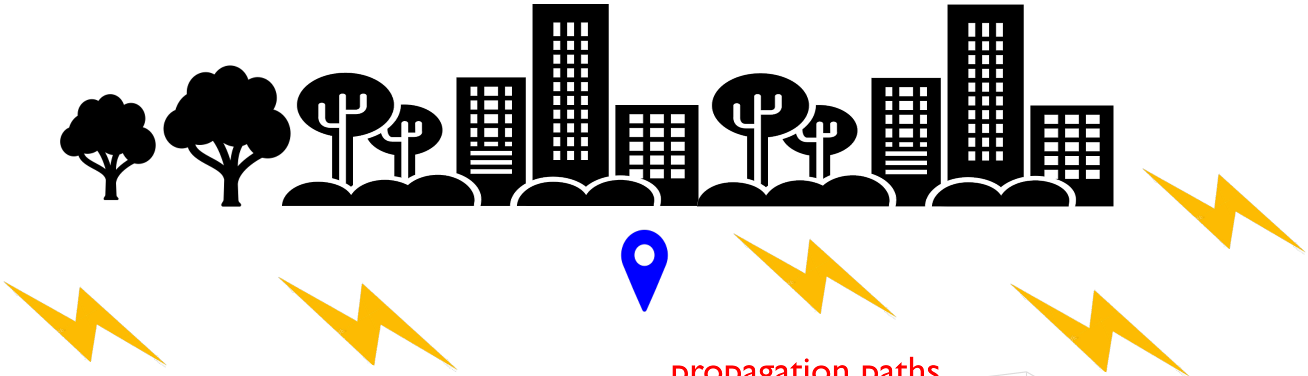


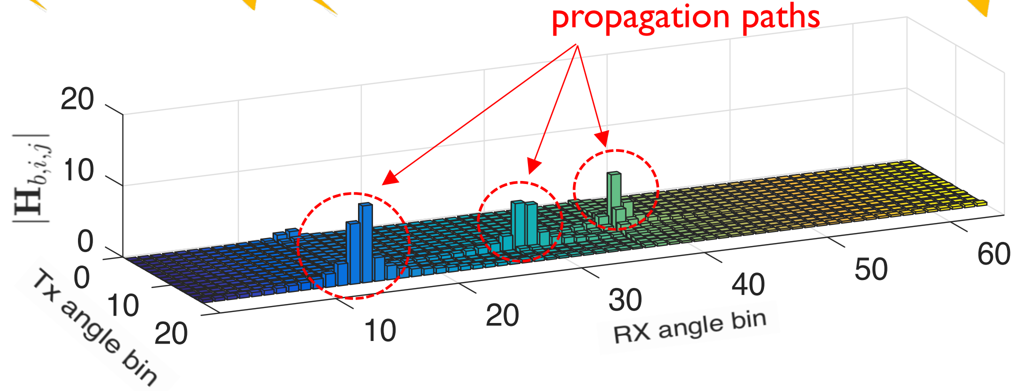
- ✓ Multiple users
- ✓ Single antenna
- ✓ Low-resolution ADCs

User scheduling

Millimeter wave channel

[Andrews14]

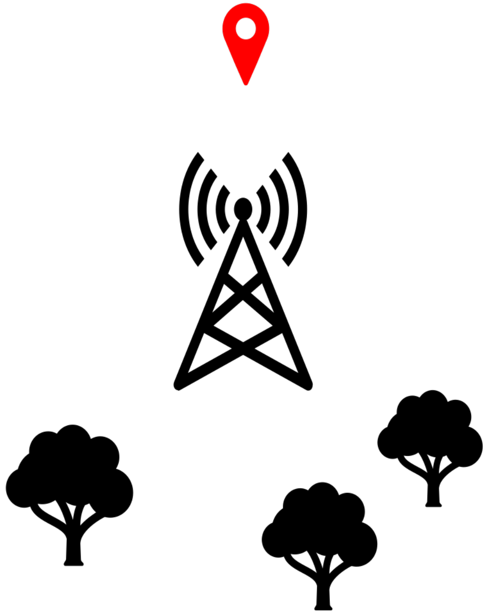




[Pi&Khan11, Akdeniz&Rappaport14]

- ✓ High frequency: 30 – 300 GHz
- ✓ Large bandwidth: 100MHz – 1GHz
- ✓ Sparse in angular (beam) domain
- ✓ Severe large scale fading (pathloss & shadowing)

Base station (BS)



- ✓ Many antennas (64+)
- ✓ Hybrid analog/digital BF*
- ✓ Low-resolution ADCs

Advanced BS design

*BF: beamforming

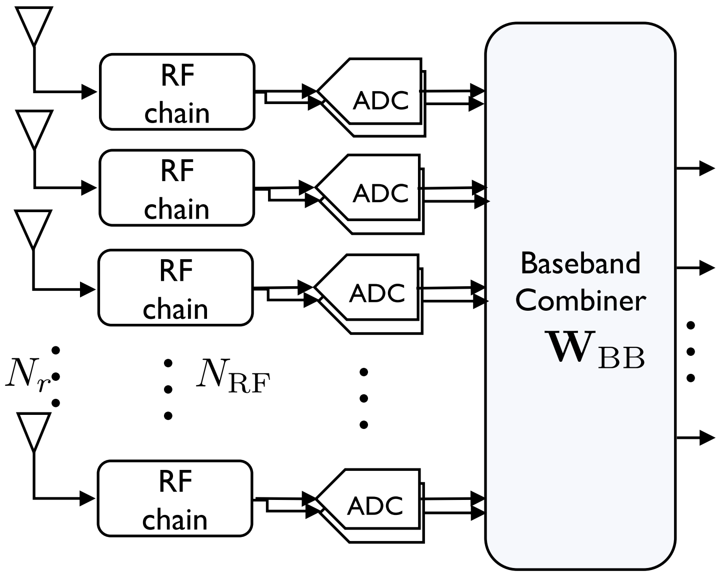
CHALLENGE IN MILLIMETER WAVE COMMUNICATION

- Excessive power consumption
  : caused by *large number of antennas/RF components* and *high sampling rate* of mmWave systems

Conventional Solution (A)

Conventional Solution (B)

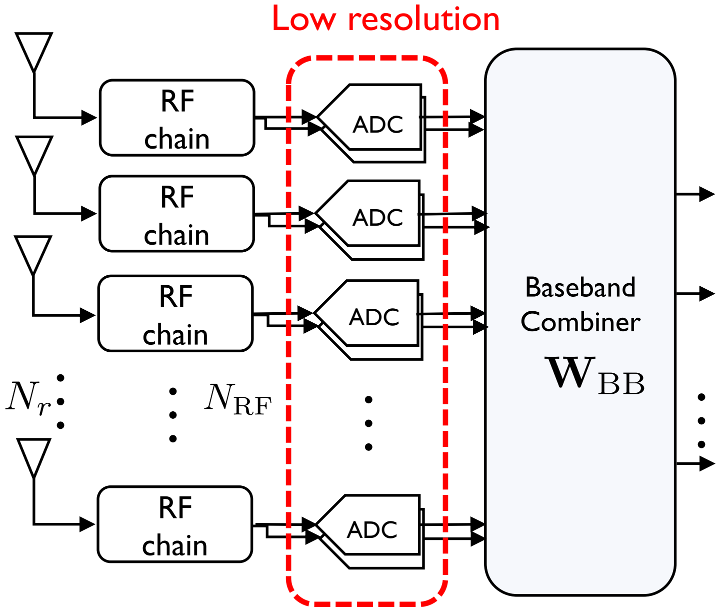
Conventional MIMO receiver



- Full-resolution ADC (12 – 16 bits)
- Full RF chains ($N_{RF} = N_r$)

$$$$$

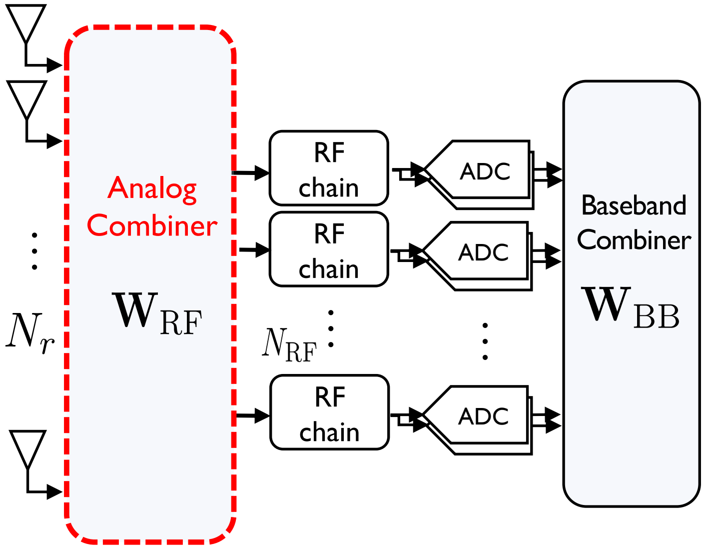
Low-resolution ADC receiver



- Low-resolution ADC (<<12bits)
- Full RF chains ($N_{RF} = N_r$)

$$$

Hybrid beamforming receiver

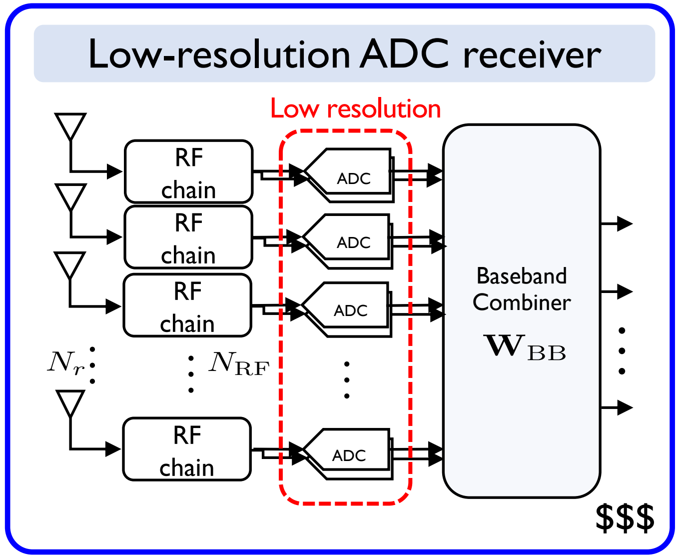


- Full-resolution ADC (12 – 16 bits)
- Fewer RF chains ($N_{RF} << N_r$)

$$$

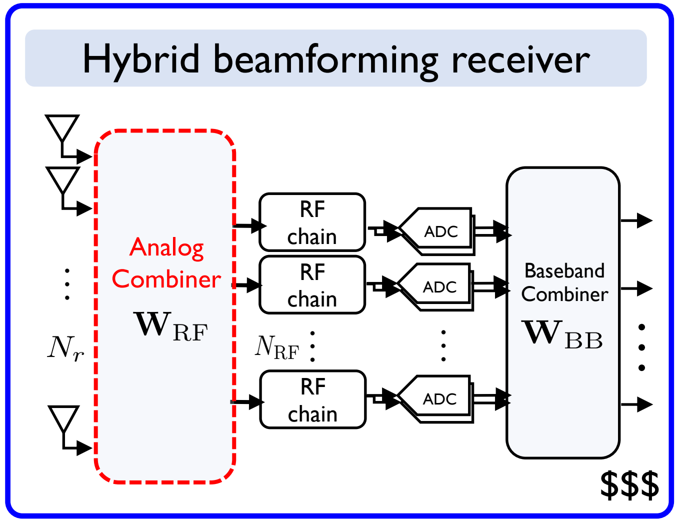
Low Power Receiver Architecture

Sol A

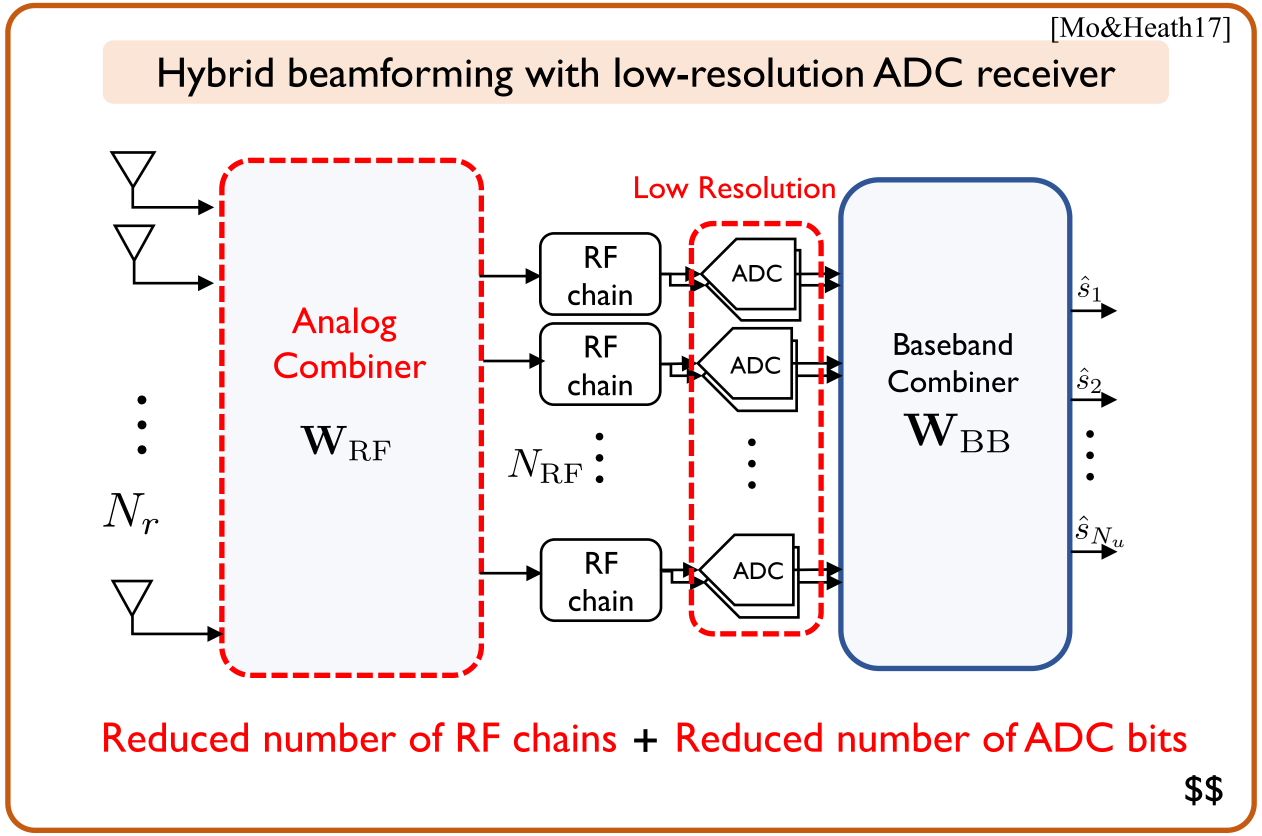


+

Sol B



Considered receiver (BS) architecture



Problem

Naïve combination of low-resolution ADCs and hybrid beamforming
: applying techniques for hybrid BF with perfect quantization does not work well

CONTRIBUTIONS

- How to improve new system performance?

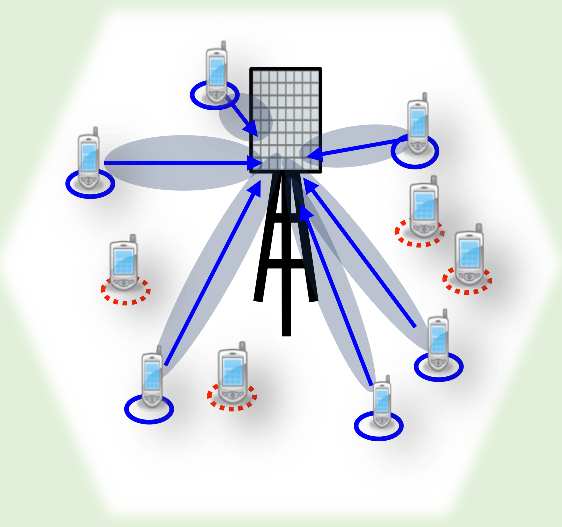
optimize architecture and/or technique to efficiently *reduce quantization error*

MAC perspective

PHY perspective

User scheduling

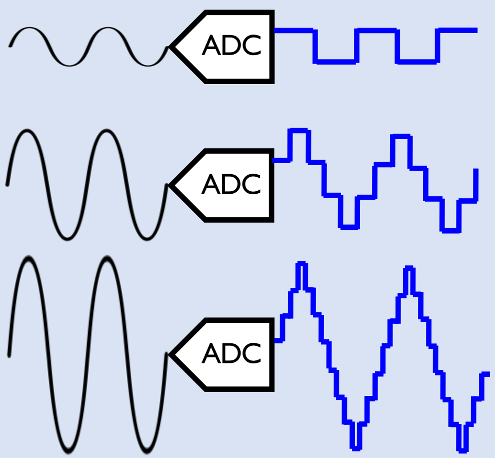
Contribution 1



Optimize: set of users

Resolution-adaptive ADCs

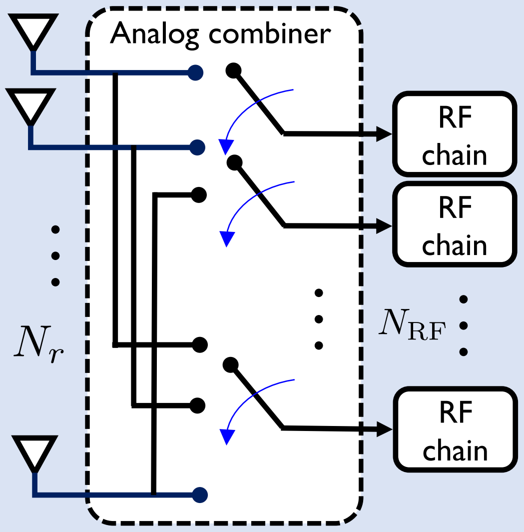
Contribution 2



Optimize: ADC bits

Antenna selection

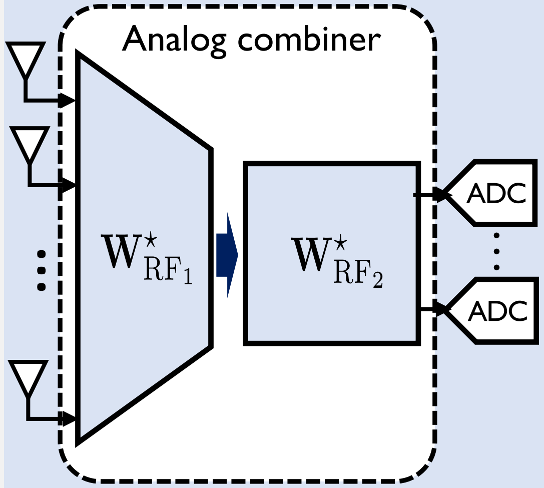
Contribution 3



Switch-based

Two-stage analog combiner

Contribution 4



Phase shifter-based

Optimize: analog beamformer

MAC: medium access control layer PHY: physical layer

Contribution 1

UPLINK USER SCHEDULING FOR HYBRID RECEIVERS WITH LOW-RESOLUTION ADCS

Discussed in the PhD Qualifying Exam and Included in the PhD Dissertation

Related publications:

[1]. Jinseok Choi, Gilwon Lee, and Brian L. Evans, "Millimeter-Wave MIMO User Scheduling for Low-Resolution ADC Systems", *IEEE Transactions on Wireless Communications,* vol. 18, no. 4, pp. 2401-2414, Apr. 2019.

[2]. Jinseok Choi, and Brian L. Evans, "User Scheduling for Millimeter Wave MIMO Communications with Low-Resolution ADCs", *IEEE International Conference on Communications,* May 20-24, 2018, Kansas City, MO, USA.

System Model

- Multi-user MIMO uplink system
  - Single cell environment with K users with single antenna
  - Selects $N_u \le N_{RF}$ users to serve
  - Uniform linear array (ULA) antennas
  - DFT-based analog combining
  - Zero-forcing digital equalizer

- Millimeter wave channel model with limited scattering [Akdeniz&Rappaport14]

$$\mathbf{h}_k = \sqrt{\frac{N_r}{L_k}} \sum_{\ell=1}^{L_k} g_{\ell,k} \mathbf{a}(\theta_{\ell,k})$$

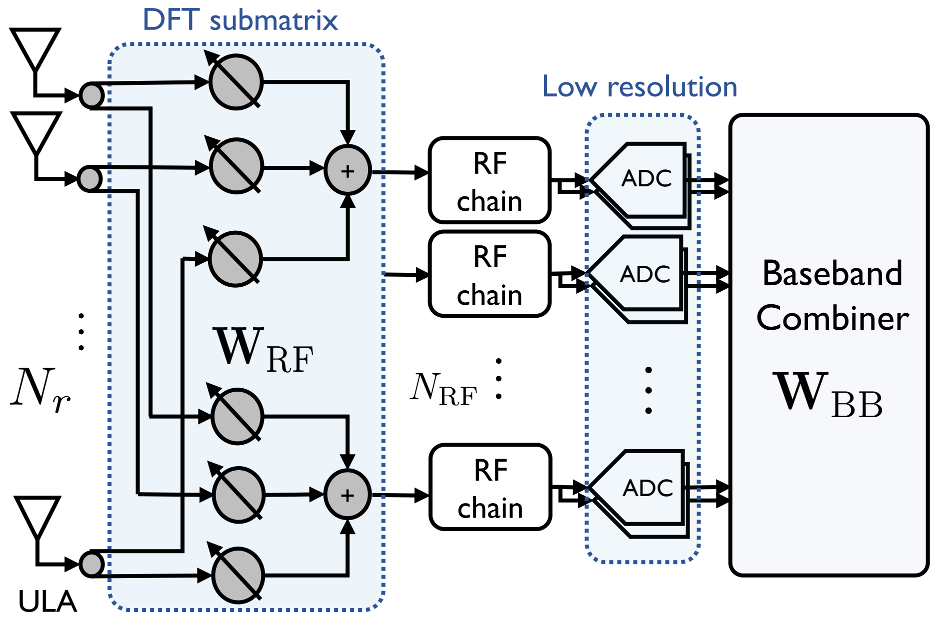
channel path gain

angle of arrival (AoA)

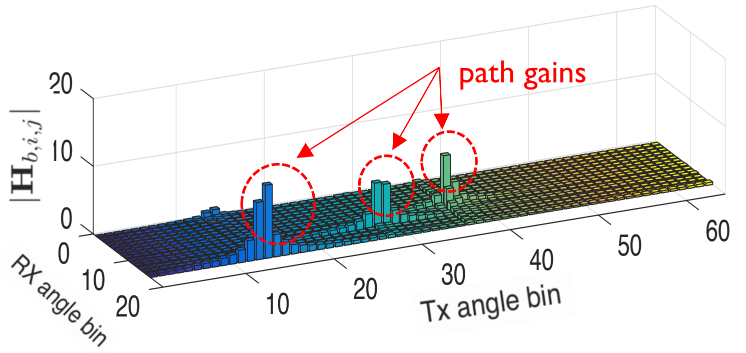
array response vector (ARV)

: small L_k sparse in beam domain

- Beam domain projection by using $\boldsymbol{W}_{RF}$
  - $\mathbf{A} = \left[\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_{N_r})\right]$ ➡ DFT matrix
  - Beam domain projection: $\mathbf{h}_{\mathrm{b}} = \mathbf{W}_{\mathrm{DFT}}^H \mathbf{h}$



Considered receiver architecture



Beam domain channel

Notations

$\mathbf{A}$: matrix

$\mathbf{a}$: column vector

Motivation & Problem Formulation

- Non-negligible quantization error
  - Achievable rate of user k

$$r_k(\mathbf{H}_\mathrm{b}) = \log_2\left(1 + \frac{\alpha^2 p_u}{\underbrace{\alpha^2\|\mathbf{w}_{\mathrm{zf},k}\|^2}_{\text{AWGN}} + \underbrace{\mathbf{w}_{\mathrm{zf},k}^H \mathbf{R}_{\mathbf{qq}}(\mathbf{H}_\mathrm{b})\mathbf{w}_{\mathrm{zf},k}}_{\text{Quantization noise (QN)}}}\right)$$

AWGN: minimized by previous criteria

(1) $\mathbf{h}_{\mathrm{b},k} \perp \mathbf{h}_{\mathrm{b},k'},\ \ k \neq k'$

(2) $\text{maximize } \|\mathbf{h}_{\mathrm{b},k}\|^2$

QN: requires additional condition

(1), (2) cannot minimize quantization error

$\mathbf{R}_{\mathbf{qq}}(\mathbf{H}_\mathrm{b}) = \alpha\beta\,\mathrm{diag}(p_u \mathbf{H}_\mathrm{b}\mathbf{H}_\mathrm{b}^H + \mathbf{I}_{N_\mathrm{RF}})$

- Maximum sum rate and fairness problems
  - Maximum sum rate user scheduling

$$\mathcal{P}1: \quad \mathcal{S}^\star = \underset{\mathcal{S}\subset\{1,\ldots,K\}:|\mathcal{S}|\leq N_u}{\mathrm{argmax}} \sum_{k\in\mathcal{S}} r_k(\mathbf{H}_\mathrm{b}(\mathcal{S}))$$

set of scheduled users

beam domain channel of users in *S*

  - Proportional fairness (PF) scheduling

weight of user *k* at time *t*

$$\mathcal{P}2: \quad k^\star = \underset{k}{\mathrm{argmax}}\, r_k(t)/\mu_k(t)$$

indicator function

where $\mu_k(t+1) = (1-\delta)\mu_k(t) + \delta r_k(t)\mathbf{1}_{\{k\in\mathcal{S}_t\}}$

: first-order auto-regressive filter

regression rate parameter in (0, 1)

New Scheduilng Criteria & Simulations

❑ New scheduling criteria

1. Unique *AoAs for channel paths:

$$\mathcal{L}_{\mathcal{S}(k)} \cap \mathcal{L}_{\mathcal{S}(k')} = \emptyset \text{ if } k \neq k'.$$

index set of nonzero channel gains

2. Equal power spread within each channel:

$$|h_{\mathrm{b},i,\mathcal{S}(k)}| = \sqrt{\gamma_{\mathcal{S}(k)}/L_{\mathcal{S}(k)}} \text{ for } i \in \mathcal{L}_{\mathcal{S}(k)}.$$

number of nonzero channel gains for user S(k)

total channel gain for user S(k)

❑ Proposed user scheduling methods

	Set Filtering	Selection	Update
Max Sum:	[Yoo&Goldsmith06] Semi-orthogonality $\frac{\lvert \mathbf{f}^H_{\mathcal{S}(i)} \mathbf{h}_{\mathrm{b},k} \rvert}{\lVert \mathbf{f}_{\mathcal{S}(i)} \rVert \lVert \mathbf{h}_{\mathrm{b},k} \rVert} < \epsilon$	Approximated SINR $\max_k \mathrm{SINR}_k\left(\mathbf{H}_\mathrm{b}(\mathcal{S} \cup \{k\})\right)$	
Prop Fair:	[Lee&Sung16] Spatial orthogonality $\lvert \mathcal{B}_{\mathcal{S}(i)} \cap \mathcal{B}_k \rvert \leq N_{\mathrm{OL}}$	Approximated weighted rate $\max_k \frac{\log_2(1+\mathrm{SINR}_k(\mathbf{H}_\mathrm{b}(\mathcal{S}_t \cup \{k\})))}{\mu_k(t)}$	AR filter $\mu_k(t+1)$ $\forall k \in \mathcal{S}_t$

where $\mathrm{SINR}_k\left(\mathbf{H}_\mathrm{b}(\mathcal{S} \cup \{k\})\right) \approx \frac{\alpha p_u \lVert \mathbf{h}_{\mathrm{b},k} \rVert^4}{(1-\alpha)\mathbf{h}^H_{\mathrm{b},k} \mathbf{D}\left(\mathbf{H}_\mathrm{b}(\mathcal{S} \cup \{k\})\right) \mathbf{h}_{\mathrm{b},k}}$

*AoA: angle of arrivals

Fig. 1: Rate vs. Tx power

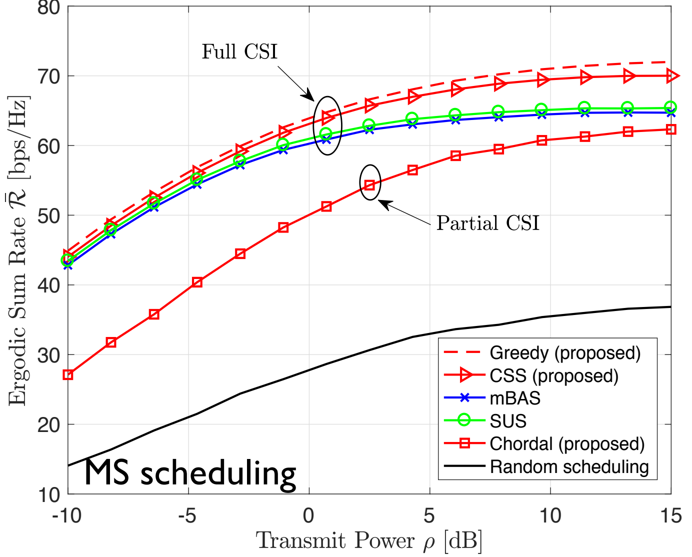
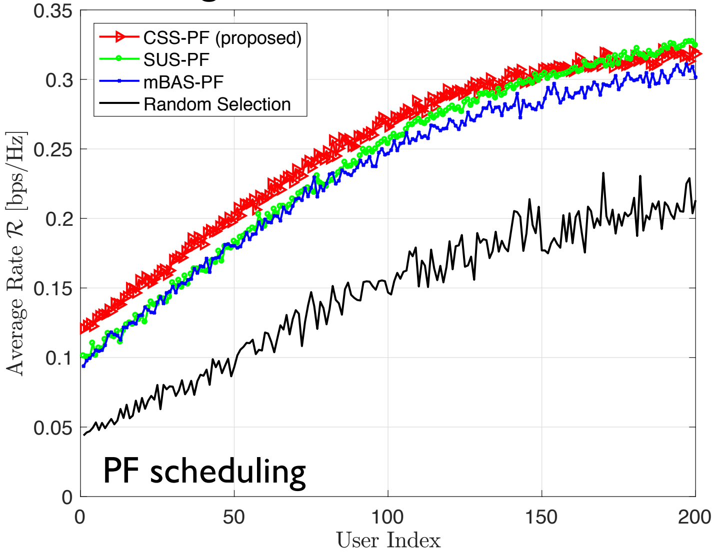


Fig. 2: Rate vs. User index



SUMMARY

Channel structural scheduling criteria

I. Unique *AoAs for channel paths:

$$\mathcal{L}_{\mathcal{S}(k)} \cap \mathcal{L}_{\mathcal{S}(k')} = \emptyset \text{ if } k \neq k'.$$

II. Equal power spread within each channel:

$$|h_{\mathrm{b},i,\mathcal{S}(k)}| = \sqrt{\gamma_{\mathcal{S}(k)}/L_{\mathcal{S}(k)}} \text{ for } i \in \mathcal{L}_{\mathcal{S}(k)}.$$

Full CSI

Partial

Channel structure-based scheduling

Semi-orthogonality

$$\frac{|\mathbf{f}_{\mathcal{S}(i)}^H \mathbf{h}_{\mathrm{b},k}|}{\|\mathbf{f}_{\mathcal{S}(i)}\|\|\mathbf{h}_{\mathrm{b},k}\|} < \epsilon$$

Spatial orthogonality

$$|\mathcal{B}_{\mathcal{S}(i)} \cap \mathcal{B}_k| \leq N_{\mathrm{OL}}$$

Maximum SINR scheduling

$$\max_{k \in \mathcal{K}_i} \mathrm{SINR}_k$$

Chordal distance-based scheduling

Filtering $d_{\mathrm{cd}}\left(\mathcal{S}_{\mathrm{cd}}(i-1), k\right)/\sqrt{L_{min}} < d_{\mathrm{th}}$

Selection $\mathcal{S}_{\mathrm{cd}}(i) = \arg\max_{k \in \mathcal{U}} d_{\mathrm{cd}}\left(\mathcal{S}_{\mathrm{cd}}(i-1), k\right)$

Sum rate analysis

$$\bar{\mathcal{R}}_1 = \frac{N_u}{\ln 2}\left(e^{\frac{1}{p_u N_r}}\Gamma\left(0, \frac{1}{p_u N_r}\right) - e^{\frac{1}{p_u(1-\alpha)N_r}}\Gamma\left(0, \frac{1}{p_u(1-\alpha)N_r}\right)\right)$$

$$\bar{\mathcal{R}}_2^{lb} \approx \frac{N_u}{\ln 2}\left(e^{\frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u\alpha N_r + p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}}\Gamma\left(0, \frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u\alpha N_r + p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}\right) - e^{\frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}}\Gamma\left(0, \frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}\right)\right)$$

Contribution 2

Bit Allocation for Hybrid Beamforming Receivers with Resolution-Adaptive ADCs

Discussed in the PhD Qualifying Exam and Included in the PhD dissertation

Related publications:

[1]. Jinseok Choi, Brian L. Evans, and Alan Gatherer, "Resolution-Adaptive Hybrid MIMO Architectures for Millimeter Wave Communications", *IEEE Transactions on Signal Processing,* vol. 65, no. 23, pp. 6201-6216, Dec. 2017.

[2]. Jinseok Choi**,** Brian L. Evans, and Alan Gatherer, "ADC Bit Allocation under a Power Constraint for MmWave Massive MIMO Communication Receivers", *Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Mar. 5-9, 2017, New Orleans, LA, USA.

[3]. Jinseok Choi, Junmo Sung, Brian L. Evans, and Alan Gatherer, "ADC Bit Optimization for Spectrum- and Energy-Efficient Millimeter Wave Communications", *IEEE Global Communications Conf.,* Dec. 4-8, 2017, Singapore.

SYSTEM MODEL

- Multi-user MIMO uplink system
  - Single cell environment
  - Serve $N_u \leq N_{RF}$ users with single antenna
  - Uniform linear array (ULA) antennas
  - DFT-based analog combining
  - Resolution-adaptive ADCs

- Millimeter wave channel model with limited scattering [Akdeniz&Rappaport14]

large scale fading gain

channel path gain

$$\mathbf{h}_k = \sqrt{\gamma_k}\sum_{\ell=1}^{L_k} g_{\ell,k}\mathbf{a}(\theta_{\ell,k})$$

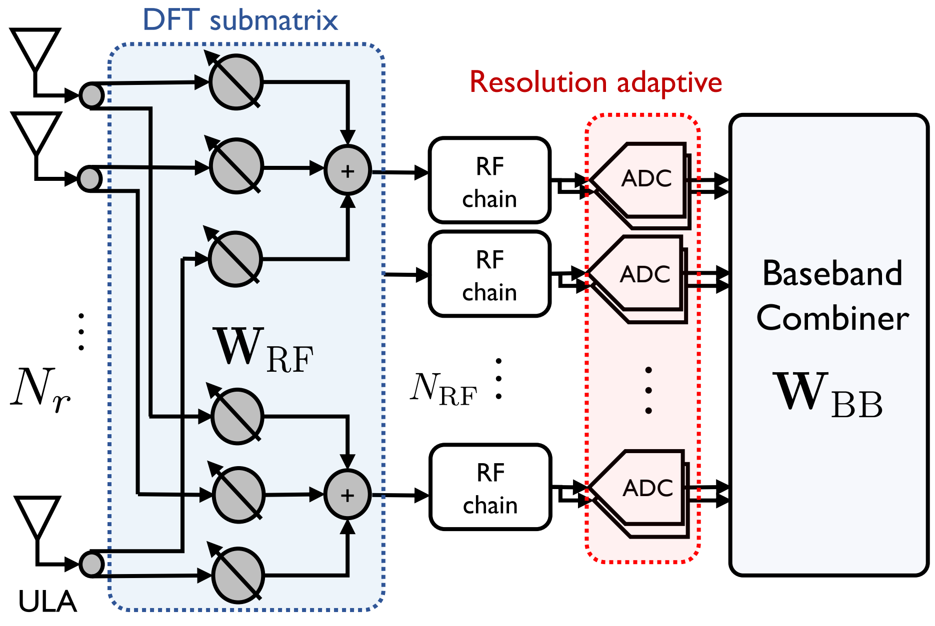
: small L_k
sparse in beam domain

angle of arrival (AoA)

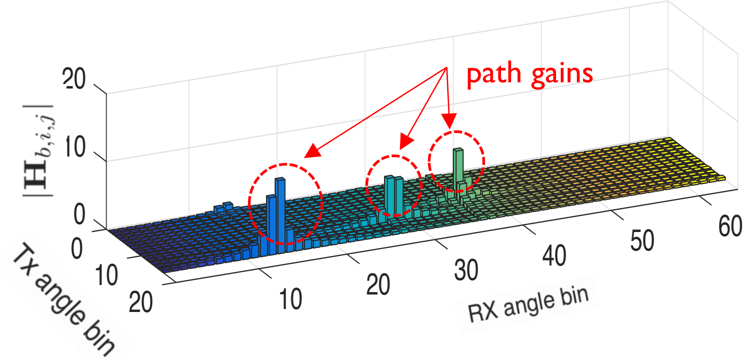
array response vector (ARV)

- Beam domain projection by using $\boldsymbol{W}_{RF}$
  - $\mathbf{A} = \left[\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_{N_r})\right]$ ➡ DFT matrix
  - Beam domain projection: $\mathbf{h}_{\mathrm{b}} = \mathbf{W}_{\mathrm{DFT}}^H\mathbf{h}$

Notations
$\mathbf{A}$: matrix
$\mathbf{a}$: column vector



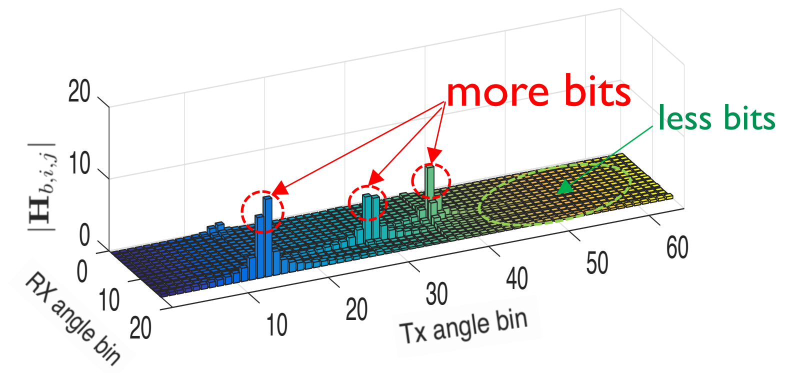
Considered receiver architecture



Beam domain channel

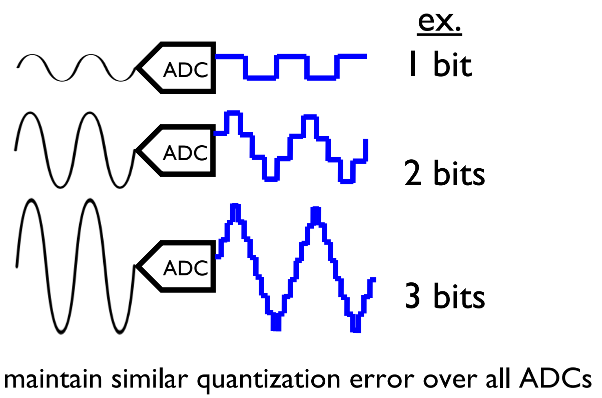
MOTIVATION & PROBLEM FORMULATION

- Selective bit allocation
  - Sparse beam domain mmWave channel



fewer quantization bits to **smaller** channel gain

More quantization bits to **larger** channel gain



- Minimizing quantization error [Gersho&Gray12]
  - Mean squared quantization error (MSQE)

$$\mathcal{E}_{x_i}(b_i) = \mathbb{E}[|x_i - x_{\mathrm{q},i}|^2] \approx \frac{\pi\sqrt{3}}{2} p_u \|[\mathbf{H}_{\mathrm{b}}]_{i,:}\|^2 2^{-2b_i}$$

quantization bits

quantized desired signal

desired signal

  - Relaxed minimum MSQE problem in high SNR

$$\mathcal{P}1: \quad \mathbf{b}_1^\star = \underset{\boxed{\mathbf{b}\in\mathbb{R}^{N_{\mathrm{RF}}}}}{\operatorname{argmin}} \sum_{i=1}^{N_{\mathrm{RF}}} \mathcal{E}_{x_i}(b_i) \quad \text{s.t.} \quad \sum_{i=1}^{N_{\mathrm{RF}}} P_{\mathrm{ADC}}(b_i) \le N_{\mathrm{RF}} P_{\mathrm{ADC}}(\bar{b})$$

real number relaxation

:ADC total power constraint

Convex optimization

Bit Allocation Solution & Simulations

- Near optimal bit allocation (solution for $\mathcal{P}1$)
  - Minimizes total MSQE in high SNR

associated channel gain

$$b_i^\star = \bar{b} + \log_2\left(\frac{N_{\rm RF}\|[\mathbf{H}_{\rm b}]_{i,:}\|^{\frac{2}{3}}}{\sum_{j=1}^{N_{\rm RF}}\|[\mathbf{H}_{\rm b}]_{j,:}\|^{\frac{2}{3}}}\right)$$

: more bit to larger channel

channel gain for all RF chains

  - Minimizes generalized mutual information in low SNR

- Worst case analysis
  - Approximated lower bound of achievable rate

$$\tilde{R}_n = \log_2\left(1 + \frac{p_u\gamma_n\alpha\left(\lambda_L^2 + 2\lambda_L + 2e^{-\lambda_L}\right)}{\eta}\right)$$

where $\eta = \left(\lambda_L + e^{-\lambda_L}\right)\left(1 + 2p_u\gamma_n(1-\alpha) + \left(\lambda_L + e^{-\lambda_L}\right)\frac{p_u}{N_{\rm RF}}\sum_{\substack{k=1\\k\neq n}}^{N_u}\gamma_k\right)$

- System parameters

[Akdeniz&Rappaport14]

Cell radius	200 m	Noise figure	5 dB	# User	8
Min. distance	30 m	Equalizer	MRC	Tx power	20 dBm
Carrier freq.	28 GHz	# Antennas	256		
Bandwidth	1 GHz	# RF chains	128		

Fig. 3: Rate vs. Constraint bits

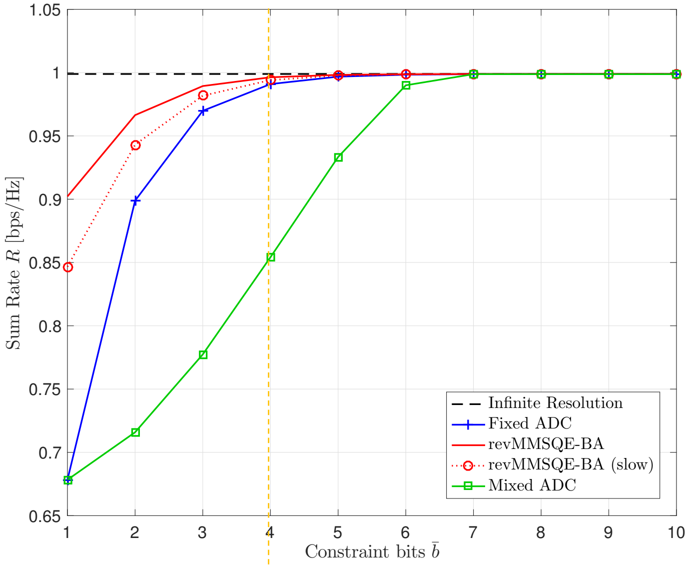
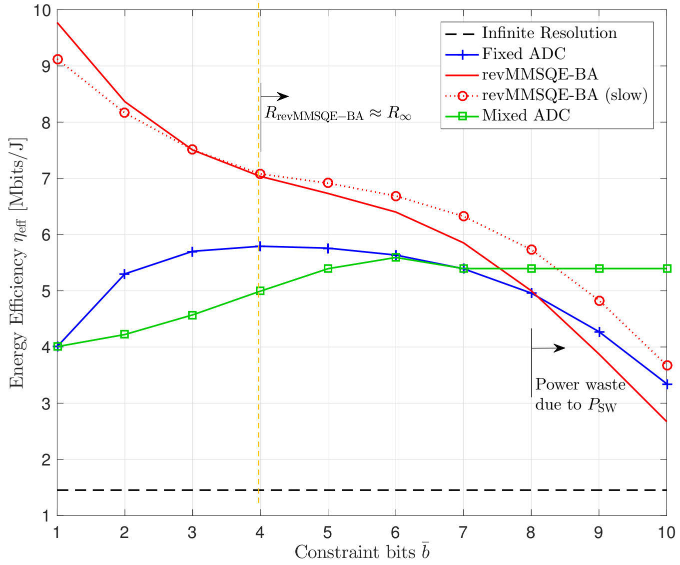


Fig. 4: Energy efficiency vs. Constraint bits



SUMMARY

Adaptive Bit Allocation

✓ Main assumption

ADC changes its resolution depending on channel

✓ Main results

Resolution-adaptive ADC with bit-allocation solution

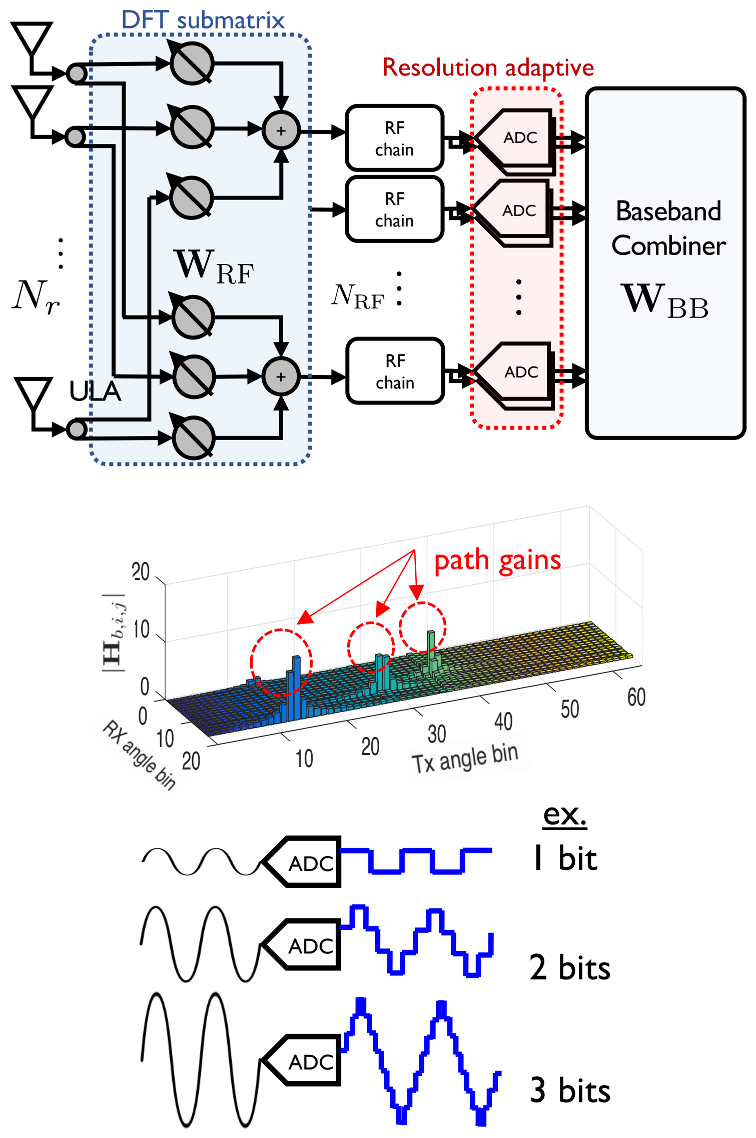
High SNR: minimizes mean squared quantization error

Low SNR: maximizes generalized MI

Performance analysis: lower bound

✓ Takeaway Message

Selective bit allocation achieves high SE* and EE*



maintain similar quantization error over all ADCs

*SE: spectral efficiency *EE: energy efficiency

Contribution 3

BASE STATION ANTENNA-SELECTION FOR LOW-RESOLUTION ADC SYSTEMS

Partially Discussed in the PhD Qualifying Exam

Related publications:

[1] Jinseok Choi, Junmo Sung, Narayan Prasad, Xiao-Feng Qi, Brian L. Evans, and Alan Gatherer, "Base Station Antenna Selection for Low-Resolution ADC Systems", *IEEE Transactions on Communications* (submitted).

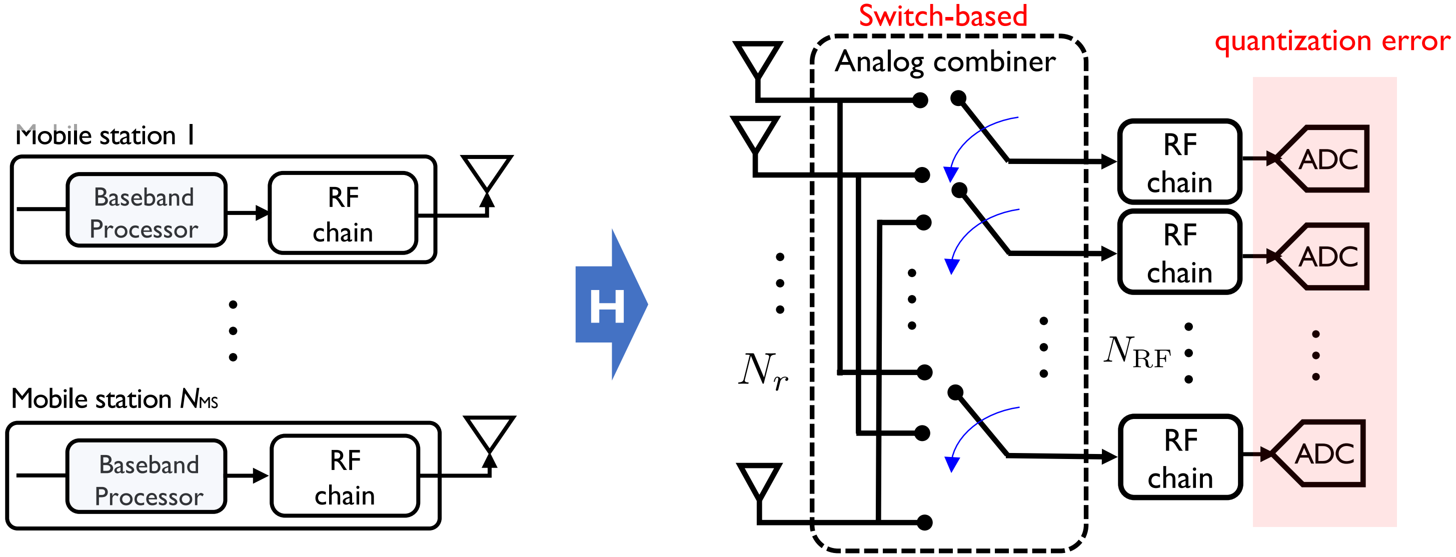
[2] Jinseok Choi, Brian L. Evans, and Alan Gatherer, "Antenna Selection for Large-Scale MIMO Systems with Low-Resolution ADCs", *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing,* Apr. 15-20, 2018, Calgary, Alberta, Canada,

MOTIVATION & UPLINK SYSTEM MODEL

- Switch-based analog beamforming
  - Lower implementation cost and complexity compared to phase shifters [Méndez-Rial16]
  - Moderate performance in reducing the number of RF chains with small loss [Gao15]

New design/analysis is necessary due to coarse quantization

- Uplink system model
  - N_{MS} users are equipped with single antenna
  - BS selects antenna subset with known CSI
  - BS employs low-resolution ADCs
  - Narrowband channel assumption



Considered uplink communication

Problem Formulation

[Fletcher07]

- Mutual information with selected antennas under additive quantization noise model (AQNM)

$$\mathcal{R}(\mathbf{H}_{\mathcal{K}}) = \log_2 \left| \mathbf{I}_{N_{\mathrm{RF}}} + p_u \alpha^2 \left(\alpha^2 \mathbf{I}_{N_{\mathrm{RF}}} + \mathbf{R}_{\mathbf{qq}} \right)^{-1} \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H \right|$$

set of selected antennas

quantization noise covariance matrix

$$\mathbf{R}_{\mathbf{qq}} = \alpha(1-\alpha)\,\mathrm{diag}(p_u \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H + \mathbf{I}_{N_{\mathrm{RF}}})$$

- Uplink maximum mutual-information problem
  - Maximum mutual-information selection for narrowband system

$$\mathcal{P}1: \quad \mathcal{R}(\mathbf{H}_{\mathcal{K}^\star}) = \max_{\mathcal{K}} \log_2 \left| \mathbf{I}_{N_{\mathrm{RF}}} + p_u \alpha^2 \left(\alpha^2 \mathbf{I}_{N_{\mathrm{RF}}} + \mathbf{R}_{\mathbf{qq}} \right)^{-1} \mathbf{H}_{\mathcal{K}} \mathbf{H}_{\mathcal{K}}^H \right|$$

  - Challenges

    (1) Large number of antennas at BS*: Exhaustive search (X) vs. Greedy search (O)

    (2) Greedy is suboptimal: performance bound is necessary

*BS: base station

UPLINK BS ANTENNA SELECTION METHOD

- Decomposition of mutual information
  - At $(n+1)$th antenna selection stage

$$\mathcal{R}(\mathbf{H}_{n+1}) = \mathcal{R}(\mathbf{H}_n) + \log_2\left(1 + \frac{p_u\alpha}{d_{\mathcal{K}(n+1)}} c_{\mathcal{K}(n+1),n}\right)$$

matrix determinant lemma

Generalized greedy selection criterion

$$J = \underset{j}{\mathrm{argmax}} \frac{c_{j,n}}{d_j}$$

gain
penalty
tradeoff

- Computational complexity reduction
  - Matrix inversion in gain computation

$$c_{j,n} = \mathbf{f}_j^H\left(\mathbf{I} + p_u\alpha\mathbf{H}_n^H\mathbf{D}_n^{-1}\mathbf{H}_n\right)^{-1}\mathbf{f}_j$$

★

matrix inversion lemma

$$\mathbf{Q}_{n+1} = \left(\mathbf{I} + p_u\alpha\mathbf{H}_{n+1}^H\mathbf{D}_{n+1}^{-1}\mathbf{H}_{n+1}\right)^{-1}$$

$$\mathbf{Q}_{n+1} = \mathbf{Q}_n - \mathbf{a}\mathbf{a}^H \quad ★$$

where $\mathbf{a} = \left(c_{J,n} + \frac{d_J}{p_u\alpha}\right)^{-1/2}\mathbf{Q}_n\mathbf{f}_J$

$$\mathbf{Q}_0 = \mathbf{I}_{N_u}$$

scalar

Simplified gain update

$$c_{j,n+1} = \mathbf{f}_j^H\mathbf{Q}_{n+1}\mathbf{f}_j = \mathbf{f}_j^H\left(\mathbf{Q}_n - \mathbf{a}\mathbf{a}\right)^H\mathbf{f}_j$$

$$= c_{j,n} - |\mathbf{f}_j^H\mathbf{a}|^2$$

vector inner product
reuse previous gain

Quantization-aware fast antenna selection (QFAS)

PERFORMANCE ANALYSIS: LOWER BOUND

- Submodular function

Definition

- Function with diminishing return property : $f(\mathcal{A} \cup \{v\}) - f(\mathcal{A}) \geq f(\mathcal{B} \cup \{v\}) - f(\mathcal{B})$ where $\mathcal{A} \subseteq \mathcal{B}$

Theorem (Lower bound) [Nemhauser78]

- Normalized nonnegative and monotone submodular function *f*
- $\mathcal{A}_{\rm G}$: set obtained by selecting one at a time with largest marginal increase
- $\mathcal{A}^\star$: optimal set with same size as $\mathcal{A}_{\rm G}$

Lower bound of *f* with $\mathcal{A}_{\rm G}$

$$f(\mathcal{A}_{\rm G}) \geq (1 - 1/e) f(\mathcal{A}^\star)$$

- Performance lower bound of proposed greedy selection

Corollary 1 (Lower bound of QFAS)

Mutual information achieved by proposed greedy-based antenna selection is lower bounded by

$$\mathcal{R}(\mathcal{K}_{\rm qfas}) \geq \left(1 - \frac{1}{e}\right) \mathcal{R}(\mathcal{K}^\star)$$

optimal antenna subset

Proof sketch

- ✓ Define $\mathbf{\Gamma}_{\mathcal{K}} = \mathbf{I}_{N_r} + \rho \alpha_b^2 (\alpha_b^2 \mathbf{I}_{N_r} + \mathbf{R}_{\mathbf{q}^{\rm ul}\mathbf{q}^{\rm ul}})^{-1/2} \mathbf{H}_{\mathcal{K}}^{\rm ul} \mathbf{H}_{\mathcal{K}}^{{\rm ul}\,H} (\alpha_b^2 \mathbf{I}_{N_r} + \mathbf{R}_{\mathbf{q}^{\rm ul}\mathbf{q}^{\rm ul}})^{-1/2}$ and $\mathbf{x}_{\mathcal{K}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Gamma}_{\mathcal{K}})$
- ✓ Use submodularity of entropy function w.r.t. selected antennas $h(\mathbf{x}_{\mathcal{K}}) = \ln |\pi e \mathbf{\Gamma}_{\mathcal{K}}| = N_r \ln(\pi e) + \mathcal{R}(\mathcal{K}) \ln 2$
- ✓ Show submodular, normalized, nonnegative, and monotone function

Wideband Uplink Antenna Selection

❑ Uplink wideband OFDM* system

- Quantized signal for subcarrier *n* under AQNM* [Fletcher07]

$$\mathbf{z}_n = \alpha_b \sqrt{\rho} \mathbf{G}_{\mathcal{K},n} \mathbf{s}_n + \mathbf{v}_n .$$

quantization gain < 1

thermal noise + quantization noise

Frequency domain channel

$$\mathbf{G}_{\mathcal{K},n} = \sum_{\ell=0}^{L-1} \mathbf{H}_{\mathcal{K},\ell} e^{-\frac{j2\pi(n-1)\ell}{N_{\mathrm{sc}}}}$$

- Mutual-information for subcarrier *n*

$$\mathcal{R}_n(\mathcal{K}) = \log_2 \left| \mathbf{I}_{N_r} + \rho\alpha_b^2 (\alpha_b^2 \mathbf{I}_{N_r} + \mathbf{R}_{\mathbf{q}_n \mathbf{q}_n})^{-1} \mathbf{G}_{\mathcal{K},n} \mathbf{G}_{\mathcal{K},n}^H \right|$$

Quantization noise variance

$$\mathbf{R}_{\mathbf{q}_n \mathbf{q}_n} = \alpha_b(1-\alpha_b)\mathrm{diag}\{\rho \mathbf{B}_{\mathcal{K}} \mathbf{B}_{\mathcal{K}}^H + \mathbf{I}_{N_r}\}$$

$$\mathbf{B}_{\mathcal{K}} = [\mathbf{H}_{\mathcal{K},0}, \mathbf{0}, \cdots, \mathbf{0}, \mathbf{H}_{\mathcal{K},L-1}, \cdots, \mathbf{H}_{\mathcal{K},1}]$$

❑ Greedy antenna selection and performance bound

- Maximum mutual-information problem for OFDM system

$$\mathcal{P}2: \quad \mathcal{K}_{\mathrm{ofdm}}^{\star} = \underset{\mathcal{K}\subseteq\mathcal{S}:|\mathcal{K}|=N_r\geq N_{\mathrm{MS}}}{\mathrm{argmax}} \sum_{n=1}^{N_{\mathrm{sc}}} \mathcal{R}_n(\mathcal{K})$$

all subcarriers share same antenna subset

- Simplified greedy antenna selection method without matrix inversion

$$J = \underset{j\in\mathcal{S}\setminus\mathcal{K}_t}{\mathrm{argmax}} \sum_{n=1}^{N_{\mathrm{sc}}} \log_2\left(1 + \frac{\rho\alpha_b}{d_j} c_{n,t}(j)\right)$$

gain update w/o matrix inversion

Corollary 2 (Lower bound of QFAS)

$$\sum_{n=1}^{N_{\mathrm{sc}}} \mathcal{R}_n(\mathcal{K}_{\mathrm{qfas}}) \geq \left(1-\frac{1}{e}\right) \sum_{n=1}^{N_{\mathrm{sc}}} \mathcal{R}_n(\mathcal{K}_{\mathrm{ofdm}}^{\star})$$

Proof: submodularity is closed under addition

*OFDM: orthogonal frequency-division multiplexing

*AQNM: additive quantization noise model

SIMULATION RESUTLS: UPLINK OFDM SYSTEM

❑ Simulated algorithms

- **QFAS**: quantization-aware fast antenna selection (proposed)
- **FAS**: fast antenna selection without quantization
- **NBS**: norm-based selection

$$k^\star = \max_k \|[\mathbf{H}]_{k,:}\|$$

- **QMCMC-AS**: quantization-aware adaptive-MCMC* based on MIS*
  - Near numerical upper bound
  - High complexity: iterations with multiple sampling (ex. 60, 120)

❑ System parameters [Akdeniz&Rappaport14]

Cell radius	200 m	Noise figure	5 dB
Min. distance	20 m	# channel paths	3
Carrier freq.	28 GHz	# channel delay taps	4
Bandwidth	100MHz	# subcarriers	64
BS antenna gain	15 dBi	# ADC bits	3

Fig. 5	Fig. 6
$N_{BS} = 32$	$N_{BS} = 128$
$N_{MS} = 8$	$N_{MS} = 12$
$N_r = 8$	$\rho = 30$ dBm

Effective in low-resolution ADC system with high performance

Fig. 5: MI vs. Tx power

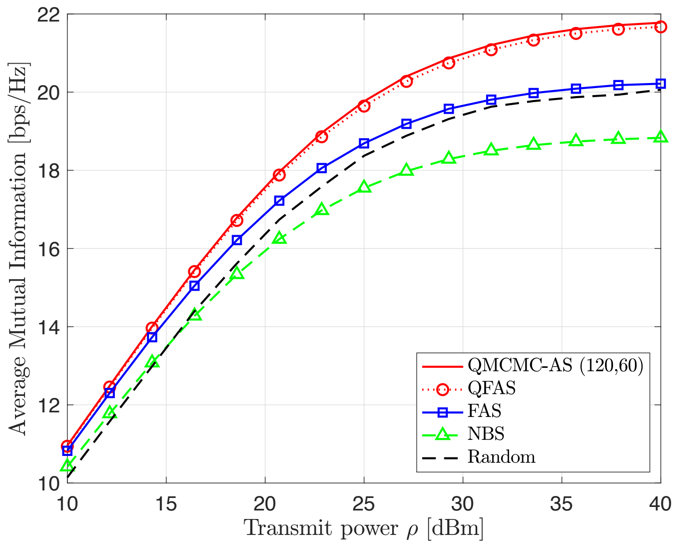
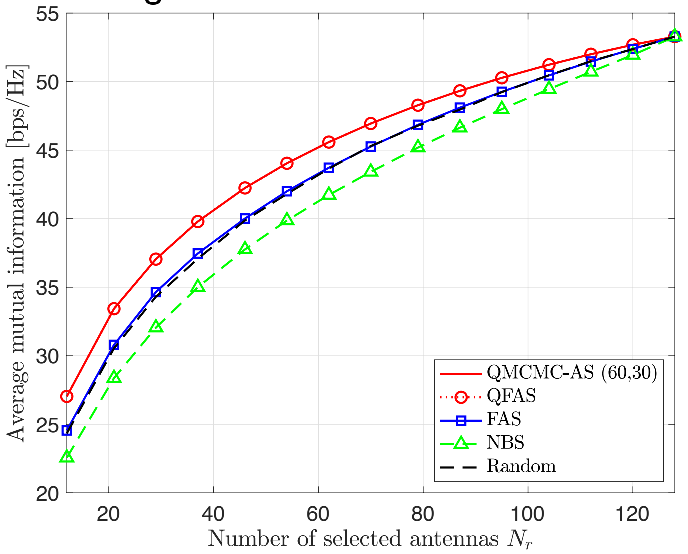


Fig. 6: MI vs. # selected antennas



*MCMC: Markov chain Monte Carlo *MIS: metropolized independence sampler

DOWNLINK BS ANTENNA SELECTION

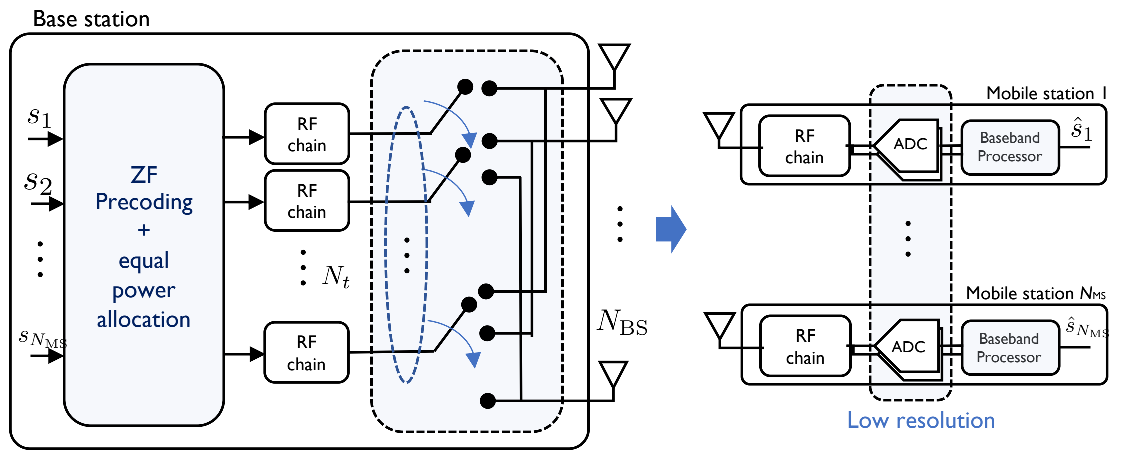
❑ System model

- Zero-forcing (ZF) precoder with CSI known at BS
- Equal power allocation (EPA)
- Users employ low-resolution ADCs
- Achievable sum rate:

Tx symbol power

$$\mathcal{R}(\mathcal{T}) = N_{\rm MS} \log_2 \left(1 + \frac{\alpha_b p_{\mathcal{T}}}{1 + (1-\alpha_b) p_{\mathcal{T}}}\right)$$

set of selected antennas



Considered downlink communication

❑ Antenna selection problem

- Maximum rate antenna selection problem

$$\mathcal{P}3: \max_{\mathcal{T} \subseteq \mathcal{S}: N_{\rm MS} \le |\mathcal{T}| \le N_t} \mathcal{R}(\mathcal{T})$$

total transmit power

$$\max_{\mathcal{T}} p_{\mathcal{T}} = \frac{P}{\mathrm{tr}\left(\mathbf{W}_{\rm BB}^H(\mathcal{T})\mathbf{W}_{\rm BB}(\mathcal{T})\right)} = \frac{P}{\mathrm{tr}\left((\mathbf{H}_{\mathcal{T}}\mathbf{H}_{\mathcal{T}}^H)^{-1}\right)}$$

: needs to be large and orthogonal

Equivalent to antenna selection in perfect quantization system

Sum Rate Analysis

- How many antennas?
  - More antennas do not always provide higher rate due to limited transmit power

 # In perfect quantization system $b = \infty$

: higher maximum rate with more antennas for ZF precoding with equal power allocation [Lin&Tsai12]

$$\mathcal{R}(\mathcal{T}_{\mathrm{opt1}};\infty) < \mathcal{R}(\mathcal{T}_{\mathrm{opt2}};\infty), \quad \text{if } |\mathcal{T}_{\mathrm{opt1}}| < |\mathcal{T}_{\mathrm{opt2}}|$$

 # In coarse quantization system $b \neq \infty$

Corollary 3 (Monotonicity)

Higher rate with more antennas for ZF-EPA*

$$\mathcal{R}(\mathcal{T}_1;b) < \mathcal{R}(\mathcal{T}_2;b), \quad \text{if } \mathcal{T}_1 \subset \mathcal{T}_2$$

Theorem 1

Higher maximum rate with more antennas for ZF-EPA

$$\mathcal{R}(\mathcal{T}_{\mathrm{opt1}};b) < \mathcal{R}(\mathcal{T}_{\mathrm{opt2}};b), \quad \text{if } |\mathcal{T}_{\mathrm{opt1}}| < |\mathcal{T}_{\mathrm{opt2}}|$$

Proof sketch

(a) Define $\mathcal{R}_D(\bar{\mathcal{T}}) = \mathcal{R}(\mathcal{T}_2) - \mathcal{R}(\mathcal{T}_1)$ where $\mathcal{T}_1 \subset \mathcal{T}_2 \subseteq \mathcal{S}$ and $\bar{\mathcal{T}} = \mathcal{T}_2 - \mathcal{T}_1$

(b) Show $\mathcal{R}_D(\bar{\mathcal{T}}) > 0$ by using matrix inversion lemma and [Lemma 2, Lin&Tsai12]

(c) Show $\mathcal{R}(\mathcal{T}_{\mathrm{opt1}}) < \mathcal{R}(\mathcal{T}_2) \leq \mathcal{R}(\mathcal{T}_{\mathrm{opt2}})$ where $\mathcal{T}_{\mathrm{opt1}} \subset \mathcal{T}_2$ and $|\mathcal{T}_{\mathrm{opt1}}| < |\mathcal{T}_2| = |\mathcal{T}_{\mathrm{opt2}}|$

from (a), (b)

*ZF-EPA: zero-forcing precoding and equal power allocation

Sum Rate Analysis

- How much transmit power?
  - More power provides higher rate, but maybe less efficient

 # In perfect quantization system $b = \infty$

: rate loss $\mathcal{R}_D(\bar{\mathcal{T}}) = \mathcal{R}(\mathcal{T}_2) - \mathcal{R}(\mathcal{T}_1)$ increases with tx power and upper bounded by [Lin&Tsai12]

matrix of channel eigenvalues

$$\mathcal{R}_D(\bar{\mathcal{T}}) \le N_t \log\left(1 + \frac{\mathrm{tr}(\mathbf{\Lambda}_{\bar{\mathcal{T}}})}{\mathrm{tr}(\mathbf{H}_{\mathcal{T}_2}\mathbf{H}_{\mathcal{T}_2}^H)^{-1}}\right)$$

where $\mathcal{T}_1 \subset \mathcal{T}_2 \subseteq \mathcal{S}$ and $\bar{\mathcal{T}} = \mathcal{T}_2 - \mathcal{T}_1$

 # In coarse quantization system $b \neq \infty$

Corollary 4 (Vanishing loss)

Rate loss converges to zero with tx power:

$$\mathcal{R}_D(\bar{\mathcal{T}}) \to 0 \quad \text{as } P \to \infty$$

Corollary 5 (Maximum loss)

Maximum rate loss occurs at following tx power:

$$P_D^{\max} = \sqrt{\frac{\mathrm{tr}((\mathbf{H}_{\mathcal{T}_2}\mathbf{H}_{\mathcal{T}_2}^H)^{-1})\mathrm{tr}((\mathbf{H}_{\mathcal{T}_1}\mathbf{H}_{\mathcal{T}_1}^H)^{-1})}{1-\alpha_b}}$$

- Tx power can compensate for rate loss due to using less antennas
- $P_D^{\max}$ can be good reference point

Similar analysis also holds for downlink OFDM systems

SIMULATION RESUTLS: DOWNLINK OFDM SYSTEM

- System parameters

[Akdeniz&Rappaport14]

Cell radius	200 m
Min. distance	20 m
Carrier freq.	28 GHz
Bandwidth	100MHz
BS antenna gain	15 dBi
Noise figure	5 dB
# channel paths	3
# channel delay taps	4
# subcarriers	64

Fig. 5	Fig. 6
$N_{BS} = 64$	$N_{BS} = 128$
$N_{MS} = 8$	$N_{MS} = 12$
$P = 50$ dB	$N_t = 16$
$b = 3, 4, 5$	$b = 3$

Fig. 7: Rate vs. # selected antennas

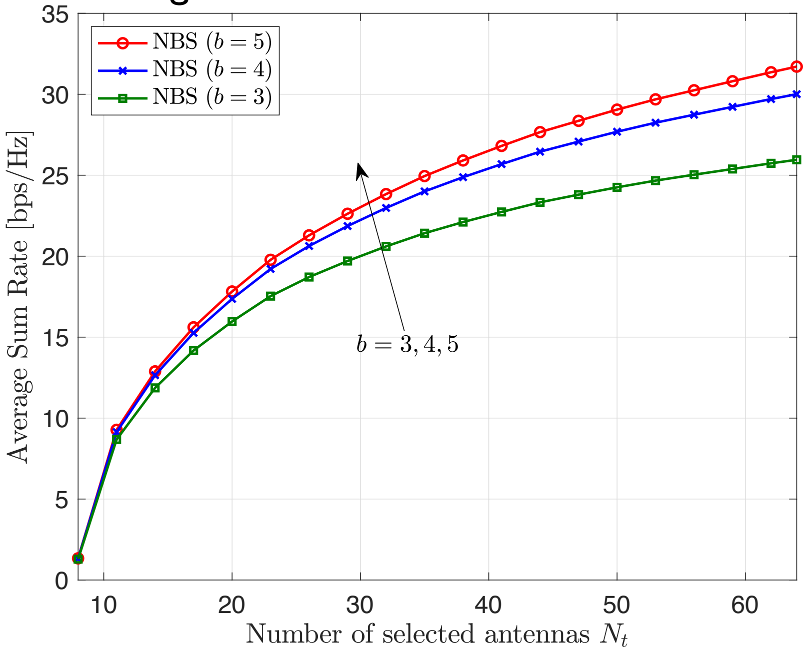
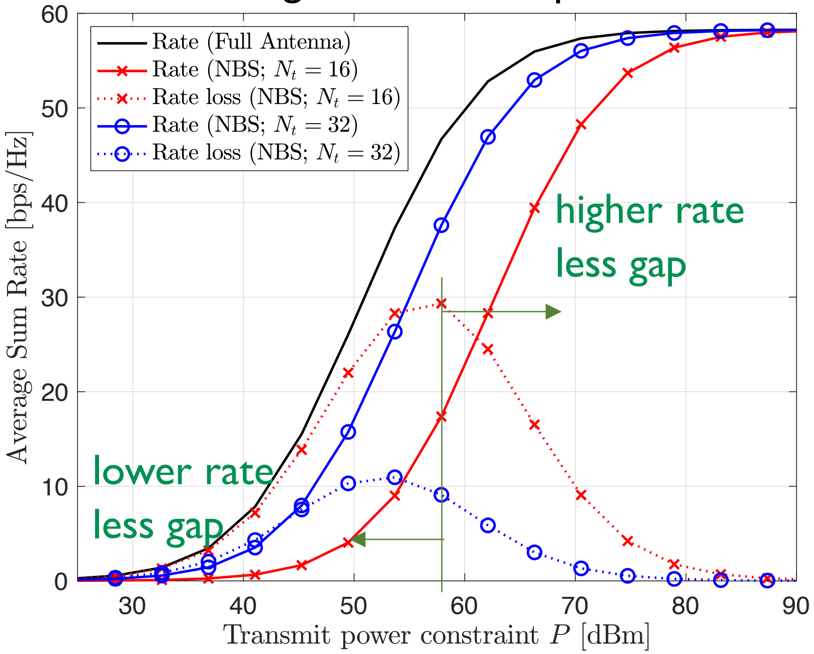


Fig. 8: Rate vs. Tx power



- Validations
  - Non-decreasing property w.r.t number of selected antennas N_t
  - Presence of maximum rate loss and rate convergence

More antennas vs. more tx power

SUMMARY

Uplink BS antenna selection

✓ Main assumption

BS selects rx antenna subset depending on channels

✓ Main results

Algorithm

$$J = \underset{j}{\mathrm{argmax}} \frac{c_{j,n}}{d_j}$$

Performance lower bound

$$\mathcal{R}(\mathcal{K}_{\mathrm{qfas}}) \geq \left(1 - \frac{1}{e}\right)\mathcal{R}(\mathcal{K}^\star)$$

Extension to wideband OFDM system

$$\mathcal{K}^\star_{\mathrm{ofdm}} = \underset{\mathcal{K}\subseteq\mathcal{S}:|\mathcal{K}|=N_r\geq N_{\mathrm{MS}}}{\mathrm{argmax}} \sum_{n=1}^{N_{\mathrm{sc}}} \mathcal{R}_n(\mathcal{K})$$

✓ Takeaway Message

Quantization error needs to be considered

Greedy choice is suboptimal but efficient

Downlink BS antenna selection

✓ Main assumption

BS selects tx antenna subset depending on channels

✓ Main results

Equivalent problem to perfect quantization

Monotonicity

$$\mathcal{R}(\mathcal{T}_{\mathrm{opt1}};b) < \mathcal{R}(\mathcal{T}_{\mathrm{opt2}};b)$$
$$\mathcal{R}(\mathcal{T}_1;b) < \mathcal{R}(\mathcal{T}_2;b)$$

Max loss

$$P_D^{\max} = \sqrt{\frac{\mathrm{tr}((\mathbf{H}_{\mathcal{T}_2}\mathbf{H}_{\mathcal{T}_2}^H)^{-1})\mathrm{tr}((\mathbf{H}_{\mathcal{T}_1}\mathbf{H}_{\mathcal{T}_1}^H)^{-1})}{1-\alpha_b}}$$

Extension to wideband OFDM system

✓ Takeaway Message

More antennas always provide higher rate

Tx power can fully compensate reduced # antennas

Contribution 4

TWO-STAGE ANALOG COMBINING IN HYBRID BEAMFORMING SYSTEMS WITH LOW-RESOLUTION ADCS

Related publications:

[1]. Jinseok Choi, Gilwon Lee, and Brian L. Evans, "Two-Stage Analog Combining in Hybrid Beamforming Systems with Low-Resolution ADCs", *IEEE Transactions on Signal Processing*, vol. 67, no. 9, pp. 2410-2425, May 1, 2019

[2]. Jinseok Choi, Gilwon Lee, and Brian L. Evans, "A Hybrid Combining Receiver with Two-Stage Analog Combiner and Low-Resolution ADCs", *IEEE Int. Conf. on Communications*, 2019, pp. 1-6. doi: 10.1109/ICC.2019.8761780

Motivation

- Optimal analog combiner design
  - If feasible, is SVD* analog combiner optimal?

    \# In perfect quantization system: Yes!

    : collects all of the channel gains onto reduced number of RF chains

    \# In coarse quantization system: Maybe not..

    : too large signal power experiences large quantization error

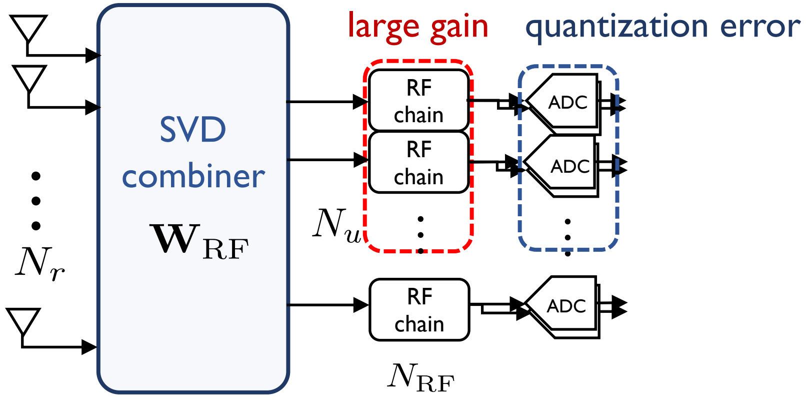


Fig. 9: Signal power on each RF chain

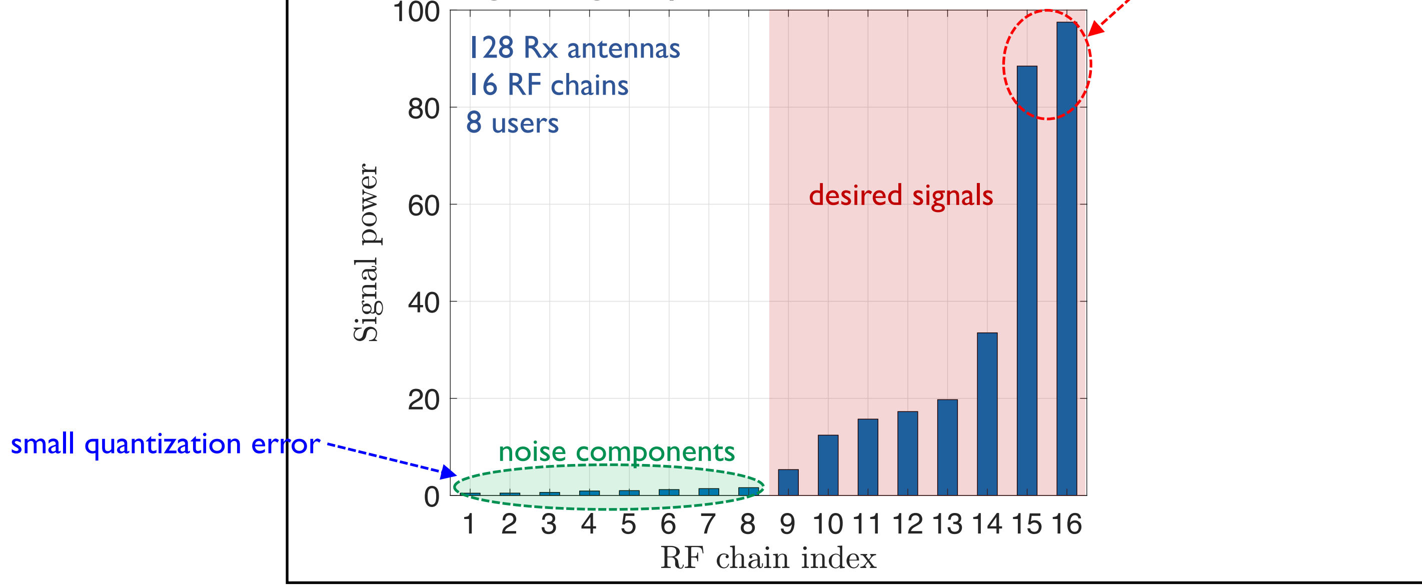
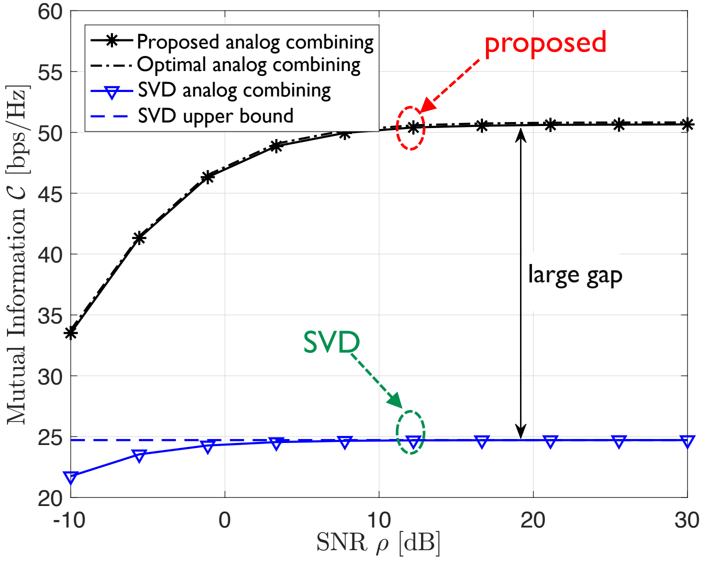


Fig. 10: Mutual information vs. SNR



*SVD: singular value decomposition

System Model & Problem Formulation

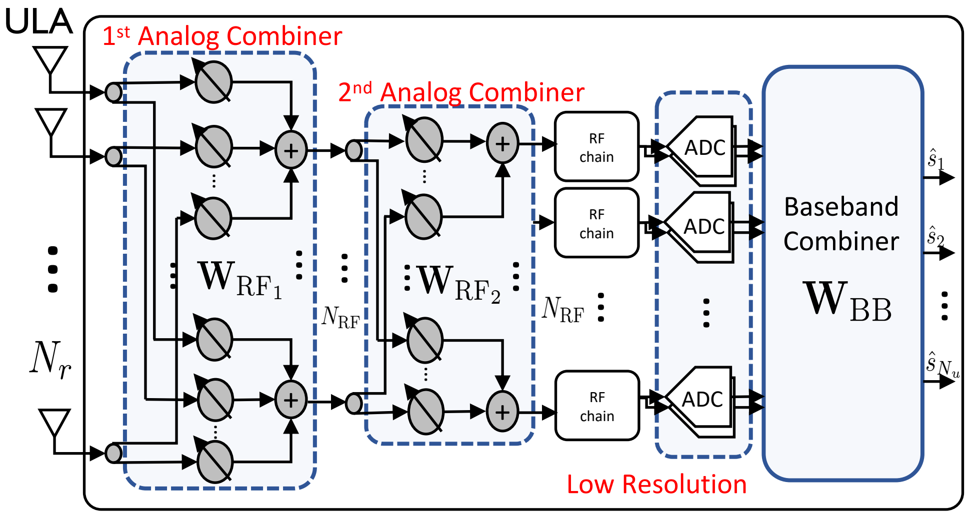
- Proposed two-stage analog combining architecture
  - Single cell environment
  - Phase shifter-based two-stage analog combining
  - Uniform linear array (ULA)
  - Serve $N_u \leq N_{RF}$ users with single antenna
  - MmWave narrowband channel [Akdeniz&Rappaport14]

$$\mathbf{h}_k = \sqrt{\frac{N_r}{L_k}} \sum_{\ell=1}^{L_k} g_{\ell,k} \mathbf{a}(\theta_{\ell,k})$$

channel path gain

angle of arrival

array response vector (ARV)



- Maximizing mutual information
  - Mutual information

$$\mathcal{C}(\mathbf{W}_{\mathrm{RF}}) = \log_2 \left| \mathbf{I}_{N_{\mathrm{RF}}} + \rho\alpha_b^2 \left(\alpha_b^2 \mathbf{W}_{\mathrm{RF}}^H \mathbf{W}_{\mathrm{RF}} + \mathbf{R}_{\mathbf{qq}}\right)^{-1} \mathbf{W}_{\mathrm{RF}}^H \mathbf{H}\mathbf{H}^H \mathbf{W}_{\mathrm{RF}} \right|$$

Covariance matrix of quantization noise

$$\mathbf{R}_{\mathbf{qq}} = \alpha_b \beta_b \mathrm{diag}\{\rho \mathbf{W}_{\mathrm{RF}}^H \mathbf{H}\mathbf{H}^H \mathbf{W}_{\mathrm{RF}} + \mathbf{W}_{\mathrm{RF}}^H \mathbf{W}_{\mathrm{RF}}\}$$

Analog combiner

$$\mathbf{W}_{\mathrm{RF}} = \mathbf{W}_{\mathrm{RF}_1} \mathbf{W}_{\mathrm{RF}_2}$$

: key role in changing quantization distribution

  - Relaxation: no constant modulus constraint on elements of analog combiner matrix
  - Relaxed maximum mutual-information problem

$$\mathcal{P}1: \ \mathbf{W}_{\mathrm{RF}}^{\mathrm{opt}} = \arg\max_{\mathbf{W}_{\mathrm{RF}}} \ \mathcal{C}(\mathbf{W}_{\mathrm{RF}}), \ \text{s.t.} \ \mathbf{W}_{\mathrm{RF}}^H \mathbf{W}_{\mathrm{RF}} = \mathbf{I}.$$

Optimal Scaling Law & Two-Stage Solution

- Optimal scaling law with respect to number of RF chains N_{RF}

Theorem 2 (Optimal scaling law)

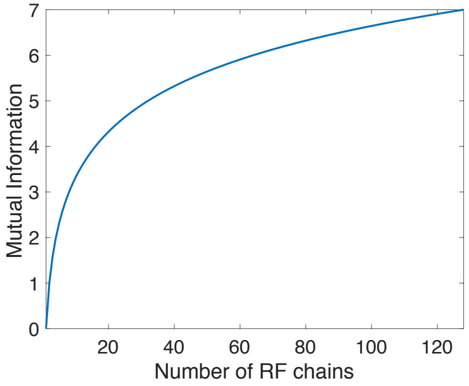
Optimal solution to $\mathcal{P}1$ achieves following scaling law w.r.t. number of RF chains:

$$\mathcal{C}(\mathbf{W}_{\mathrm{RF}}^{\mathrm{opt}}) \sim N_u \log_2 N_{\mathrm{RF}}$$

Optimal scaling law can be also achieved by using following two-stage combiners:

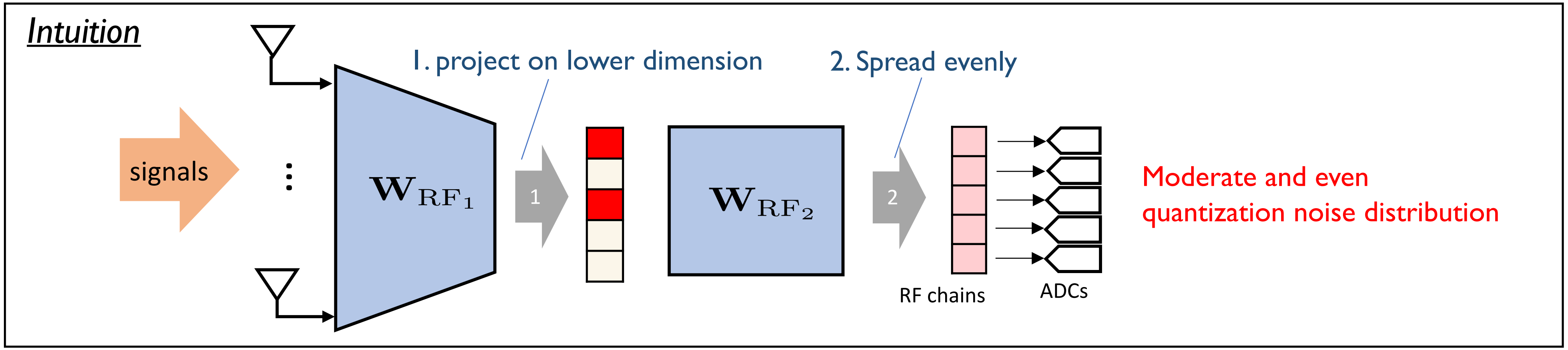
(i) $\mathbf{W}_{\mathrm{RF}_1}^{\star} = [\mathbf{U}_{1:N_u} \mathbf{U}_{\perp}]$: SVD combiner

(ii) $\mathbf{W}_{\mathrm{RF}_2}^{\star}$: any N_{RF} x N_{RF} unitary matrix with constant modulus



$$\mathcal{C}(\mathbf{W}_{\mathrm{RF}}^{\mathrm{opt}}) \sim N_u \log_2 N_{\mathrm{RF}}$$

$$\mathcal{C}(\mathbf{W}_{\mathrm{RF}_1}^{\star}\mathbf{W}_{\mathrm{RF}_2}^{\star}) \sim N_u \log_2 N_{\mathrm{RF}}$$



Bounded MI for Conventional Optimal Solution

- Mutual information achieved by SVD analog combining

Corollary 6 (Upper bound for SVD analog combining)

MI for conventional optimal solution $\mathbf{W}_{\rm RF}^{\rm cv} = [\mathbf{U}_{1:N_u}\mathbf{U}_{\perp}]$ for perfect quantization systems is bounded by

$$\mathcal{C}(\mathbf{W}_{\rm RF}^{\rm cv}) < \mathcal{C}_{\rm svd}^{\rm ub} = N_u \log_2\left(1 + \frac{\alpha_b}{1-\alpha_b}\right) \qquad \text{as } \rho \to \infty$$

Proof

matrix of channel eigenvalues

$$\mathcal{C}(\mathbf{W}_{\rm RF}^{\rm cv}) = \log_2\left|\mathbf{I} + \frac{\alpha_b}{\beta_b}{\rm diag}^{-1}\left\{\mathbf{\Lambda}_{N_{\rm RF}} + \frac{1}{\beta_b\rho}\mathbf{I}\right\}\mathbf{\Lambda}_{N_{\rm RF}}\right|$$

$$= \sum_{i=1}^{N_u}\log_2\left(1 + \frac{\alpha_b\lambda_i}{\beta_b\lambda_i + 1/\rho}\right) \overset{(a)}{<} N_u\log_2\left(1+\frac{\alpha_b}{\beta_b}\right)$$

Quantization error term

: collects all channel gains: N_u eigenvalues

: increases effective quantization noise: large gains on a few ADCs

Second analog combiner $\mathbf{W}_{\rm RF2}$ in Theorem 2 addresses quantization noise enhancement

Maximum Mutual Information – Special Case

- Maximizing MI for special case: homogeneous channel singular values

Theorem 3 (Maximum Mutual Information)

Two-stage analog combining solution in Theorem 2, $\mathbf{W}_{\rm RF}^{\star} = \mathbf{W}_{{\rm RF}_1}^{\star}\mathbf{W}_{{\rm RF}_2}^{\star}$, is solution for:

$$\mathbf{W}_{\rm RF}^{\star} = \arg\max_{\mathbf{W}_{\rm RF}} \mathcal{C}(\mathbf{W}_{\rm RF})$$
$$\text{s.t. } \mathbf{W}_{\rm RF}^{H}\mathbf{W}_{\rm RF} = \mathbf{I}_{N_{\rm RF}} \text{ and } \lambda_1 = \cdots = \lambda_{N_u} = \lambda$$

and achieves maximum mutual information:

$$\mathcal{C}_{\rm opt} \triangleq \mathcal{C}(\mathbf{W}_{\rm RF}^{\star}) = N_u \log_2\left(1 + \frac{\alpha_b \lambda N_{\rm RF}}{\lambda N_u(1-\alpha_b) + N_{\rm RF}/\rho}\right)$$

: achieves optimal scaling

Proof sketch

(a) Show $\bar{\mathbf{\Lambda}} = {\rm diag}\{\bar{\lambda}_1, \cdots, \bar{\lambda}_m, 0, \cdots, 0\}$ is upper bounded by $\lambda\mathbf{I}$

(b) Then, $\|[\mathbf{G}_{\rm sub}]_{j,:}\|^2$ is maximized for any given $\overline{\mathbf{W}}_{\rm RF}$ when $\bar{\lambda}_i$ achieves λ for all $i = 1, \cdots, m$

(c) Find upper bound of $C(\mathbf{W}_{\rm RF})$ by using Jensen's inequality with (b)

(d) Show upper bound of $C(\mathbf{W}_{\rm RF})$ can be achieved with $\mathbf{W}_{\rm RF}^{\star} = \mathbf{W}_{{\rm RF}_1}^{\star}\mathbf{W}_{{\rm RF}_2}^{\star}$ by replacing $\lambda_i = \lambda$ in Proof of Theorem 2

Two-Stage Analog Combining Algorithm

- Implementation of two-stage analog combiner under practical constraints
  - Key constraints: (1) Constant modulus condition on elements of analog combining matrix
    (2) Finite resolution of phase shifters

Algorithm 1: ARV-based TSAC

1 **Initialization**: set $\mathbf{W}_{\mathrm{RF}_1}$ = empty matrix, $\mathbf{H}_{\mathrm{rm}} = \mathbf{H}$, and $\mathcal{V} = \{\vartheta_1, \ldots, \vartheta_{|\mathcal{V}|}\}$ where $\vartheta_n = \frac{2n}{|\mathcal{V}|} - 1$

:AoA codebook

1st analog combiner

2 **for** $i = 1 : N_{\mathrm{RF}}$ **do**

(a) $\mathbf{a}(\vartheta^\star) = \operatorname{argmax}_{\vartheta \in \mathcal{V}} \|\mathbf{a}(\vartheta)^H \mathbf{H}_{\mathrm{rm}}\|^2$

(b) $\mathbf{W}_{\mathrm{RF}_1} = \left[\, \mathbf{W}_{\mathrm{RF}_1} \mid \mathbf{a}(\vartheta^\star) \,\right]$: capture max channel gain

(c) $\mathbf{H}_{\mathrm{rm}} = \mathcal{P}^{\perp}_{\mathbf{a}(\vartheta^\star)} \mathbf{H}_{\mathrm{rm}}$, where $\mathcal{P}^{\perp}_{\mathbf{a}(\vartheta)} = \mathbf{I} - \mathbf{a}(\vartheta)\mathbf{a}(\vartheta)^H$

(d) $\mathcal{V} = \mathcal{V} \setminus \{\vartheta^\star\}$: null space projection (for orthogonality)

3 **end**

2nd analog combiner

4 Set $\mathbf{W}_{\mathrm{RF}_2} = \mathbf{W}_{\mathrm{DFT}}$ where $\mathbf{W}_{\mathrm{DFT}}$ is a normalized $N_{\mathrm{RF}} \times N_{\mathrm{RF}}$ DFT matrix.

- First analog combiner
  - Closely meets first condition in Theorem 2
    : left eigenvectors (channel gain aggregation)

    ARVs collect most sparse beam-domain channel gain

- Second analog combiner
  - Perfectly meets second condition in Theorem 2
    : unitary with constant modulus (spreading)

    DFT matrix or Hadamard matrix can be used
  - Low cost; negligible power consumption once configured
    : independent to channel condition

    implemented with fixed phase shifters

Two-stage analog combining architecture provides favorable structure for implementation

*ARV: array response vector *TSAC: two-stage analog combiner

Performance Analysis

- Ergodic rate of ARV-TSAC method with maximum ratio combining (MRC)
  - Two-stage analog combining

Theorem 4 (Ergodic rate of two-stage combining)

For MRC digital combining, ergodic rate of ARV-TSAC is approximated as

$$\bar{\mathcal{R}}^{\mathrm{mrc}} \approx N_u \log_2\left(1 + \frac{\rho\alpha_b N_{\mathrm{RF}}(1+1/L)}{\kappa + \rho(N_u - 1) + 2\rho(1-\alpha_b)}\right)$$

where $\kappa = N_{\mathrm{RF}}/N_r$: achieves optimal scaling

  - One-stage analog combining

Corollary 7 (Ergodic rate of one-stage combining)

For MRC digital combining, ergodic rate of one-stage analog combining is approximated as

$$\bar{\mathcal{R}}^{\mathrm{mrc}}_{\mathrm{one}} \approx N_u \log_2\left(1 + \frac{\rho\alpha_b N_{\mathrm{RF}}(1+1/L)}{\kappa + \rho(N_u - 1) + 2\rho(1-\alpha_b) N_{\mathrm{RF}}/L}\right)$$

where $\kappa = N_{\mathrm{RF}}/N_r$: cannot achieve optimal scaling

Two-stage analog combining achieves optimal scaling law with linear receiver

SIMULATION RESULTS I

- Simulated algorithms

Two-stage
- ARV-TSAC: proposed two-stage analog combining — Infeasible
- SVD+DFT: $\mathbf{W}_{\mathrm{RF}_1} = \mathbf{U}_{1:N_{\mathrm{RF}}}$ and $\mathbf{W}_{\mathrm{RF}_2} = \mathbf{W}_{\mathrm{DFT}}$ (Theorem 2)

One-stage
- ARV: $\mathbf{W}_{\mathrm{RF}} = \mathbf{W}_{\mathrm{RF}_1}$ designed from ARV-TSAC
- Greedy-MI: greedy maximization based on ARVs
- SVD: $\mathbf{W}_{\mathrm{RF}_1} = \mathbf{U}_{1:N_{\mathrm{RF}}}$ Infeasible

Fig. 11	Fig. 12	Fig. 13
$N_r = 256$	$N_r = 128$	$N_r = 128$
$N_u = 8$	$N_u = 8$	$N_u = 4$
# paths = 4	# paths = 3	# paths = 3
$b = 2$	$b = 2$	$N_{RF} = 16$
SNR = 0 dB	$N_{RF} = 43, 64$	SNR = 10 dB

Fig. 11: Mutual information vs. # RF chains

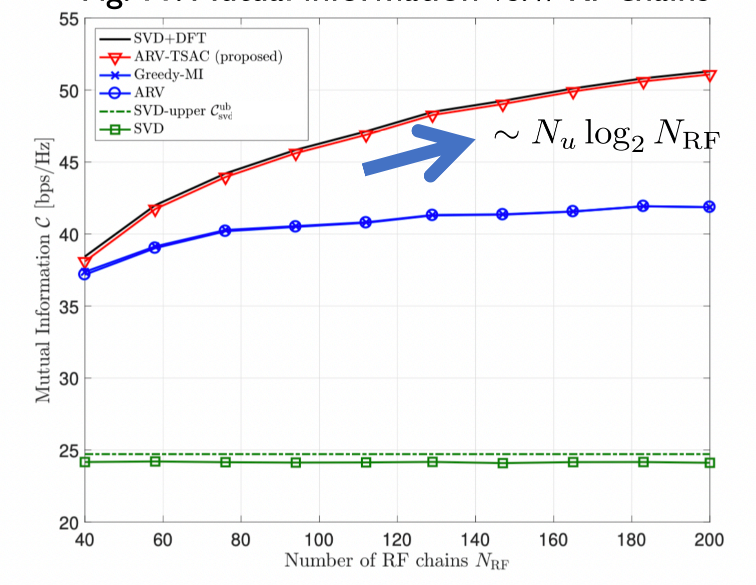


Fig. 12: Mutual information vs. SNR

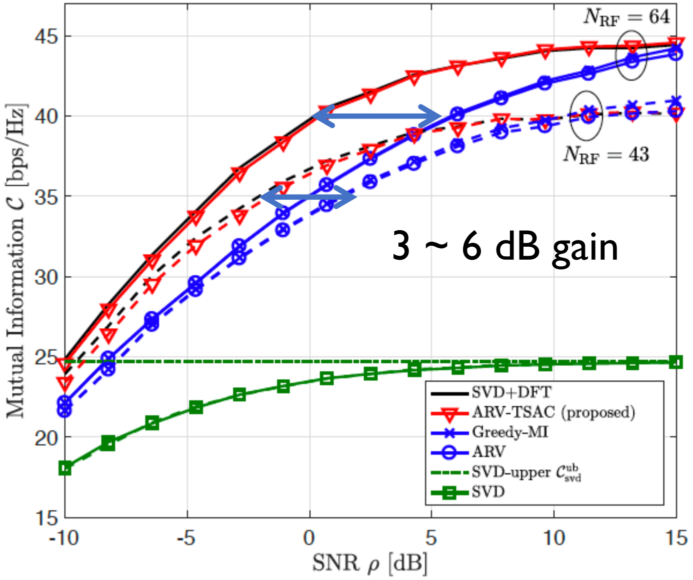
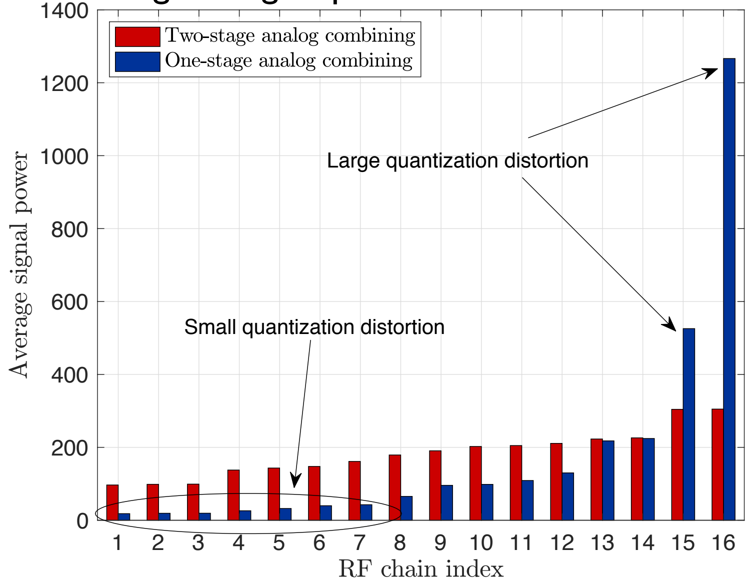


Fig. 13: Signal power on each RF chain



SIMULATION RESULTS 2

- Linear digital equalizer: maximum ratio combiner
  - (Figure 14) N_{RF}/N_r = 1/3, N_u = 8, b = 2, # paths = 3, and SNR = 0 dB
  - (Figure 15) N_r = 128, N_{RF} = 43, N_u = 8, b = 2, # paths = 3

Fig. 14: Rate vs. # RF chains

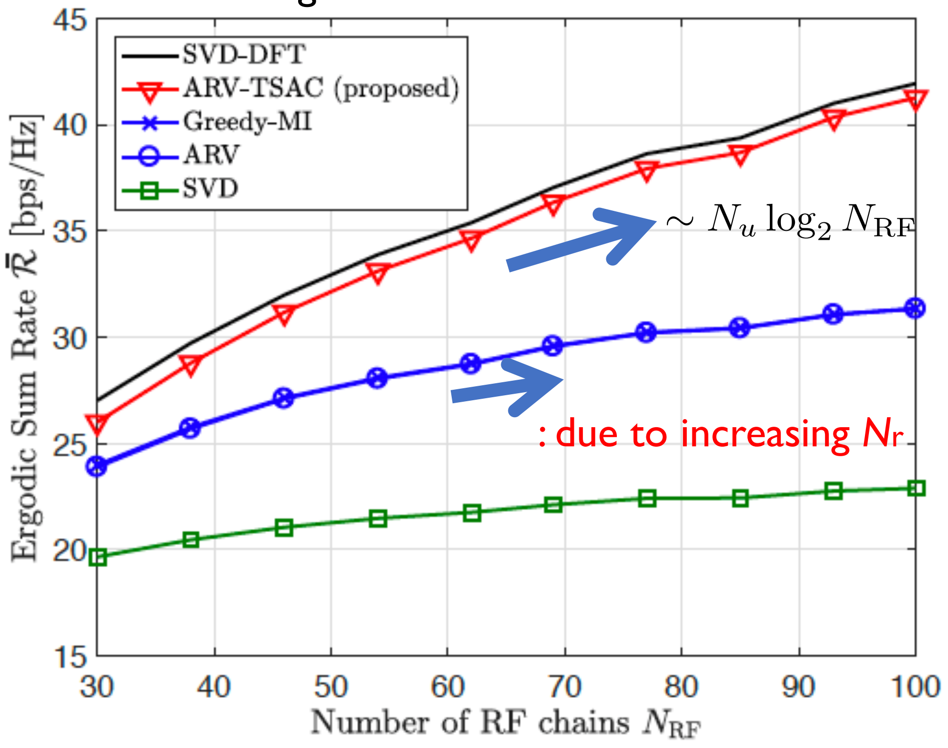
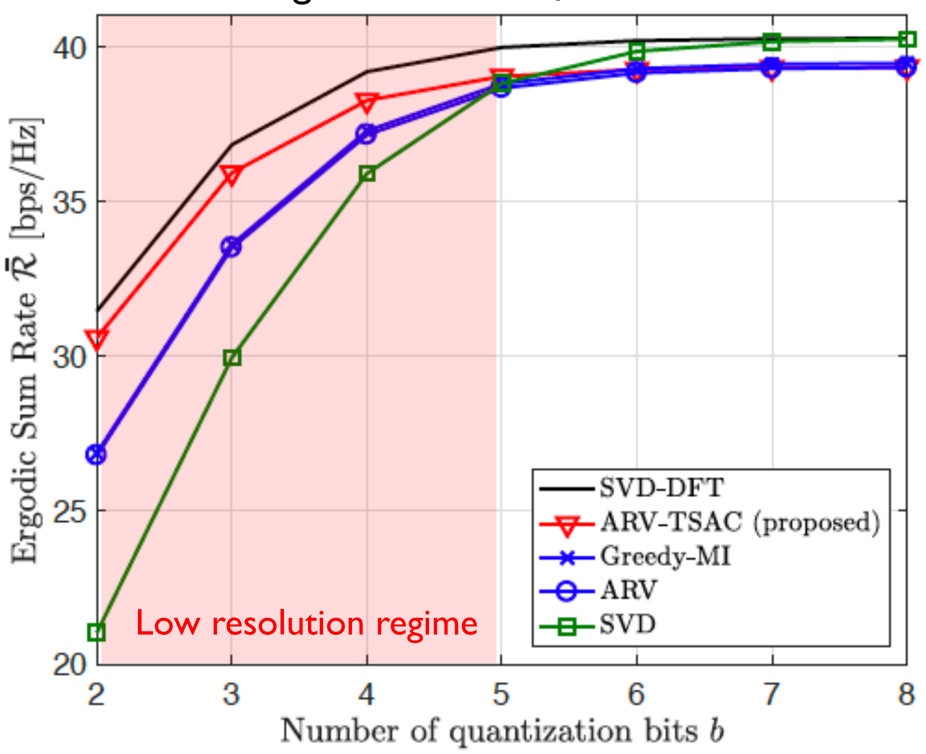


Fig. 15: Rate vs. Quantization bits



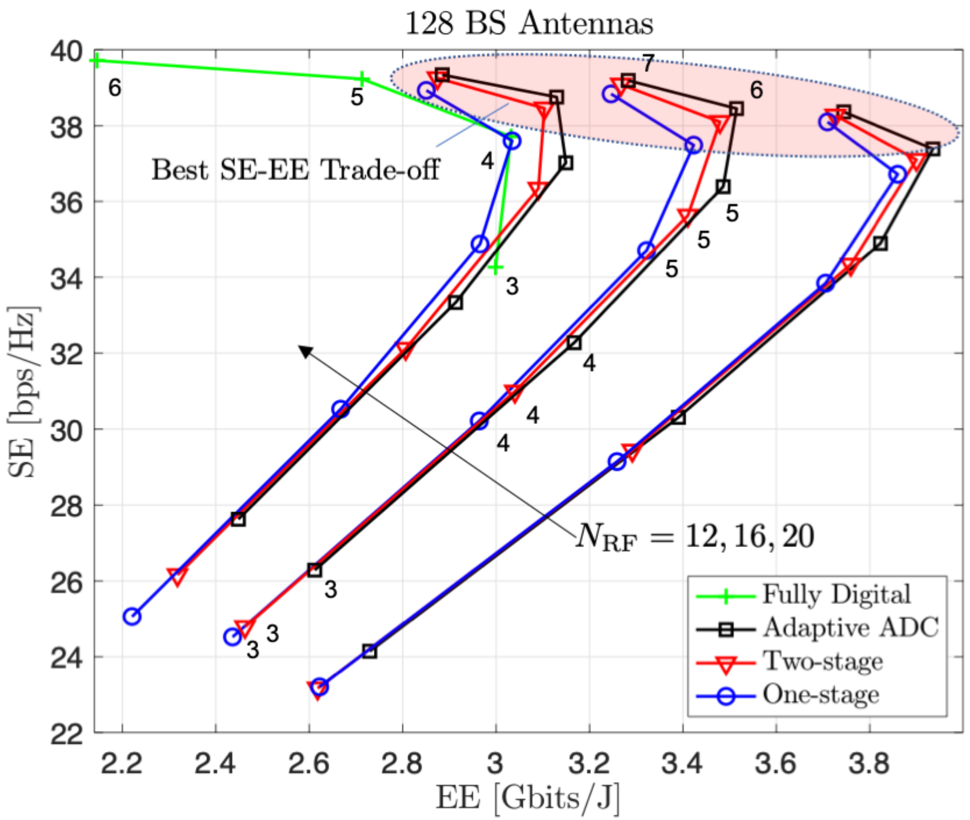
Two-stage analog combining is effective in low-resolution ADC regime

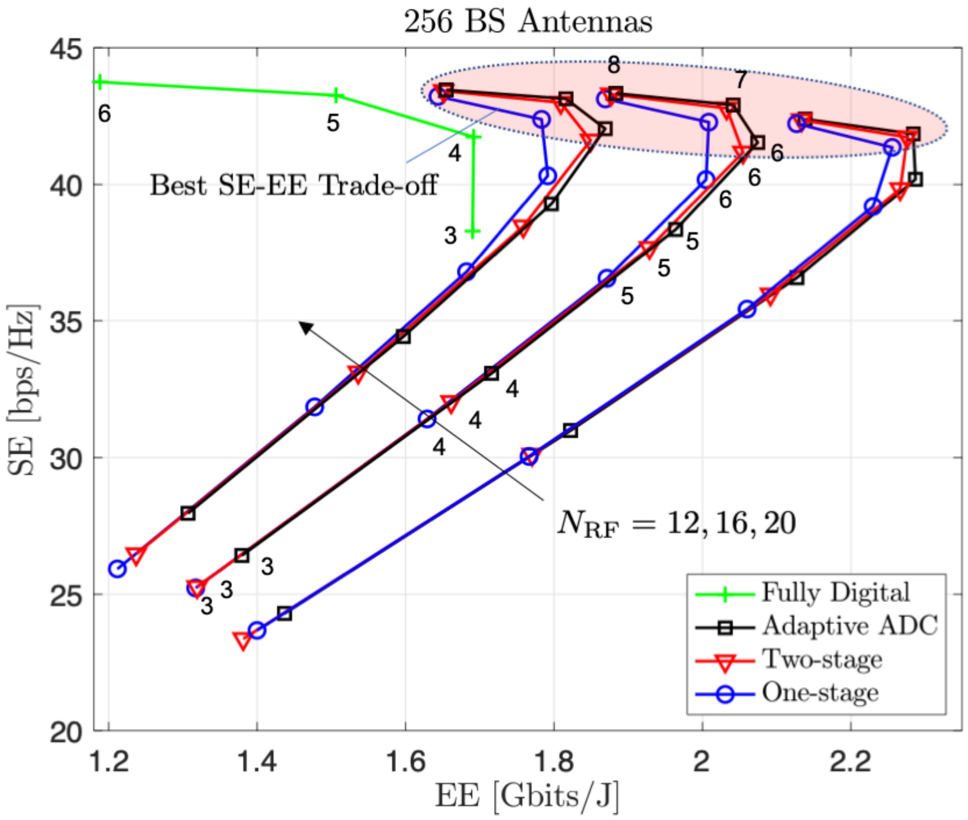
SPECTRAL VS ENERGY EFFICIENCY TRADE-OFF

- Simulated algorithms in low-resolution ADC system
  - Two-stage analog combining receiver (contribution 4)
  - Resolution-adaptive ADC receiver (contribution 2)
  - Conventional one-stage analog combining receiver
  - Fully digital receiver

- System parameters

Bandwidth	1 GHz	# users	4
SNR	10 dB	# channel paths	2

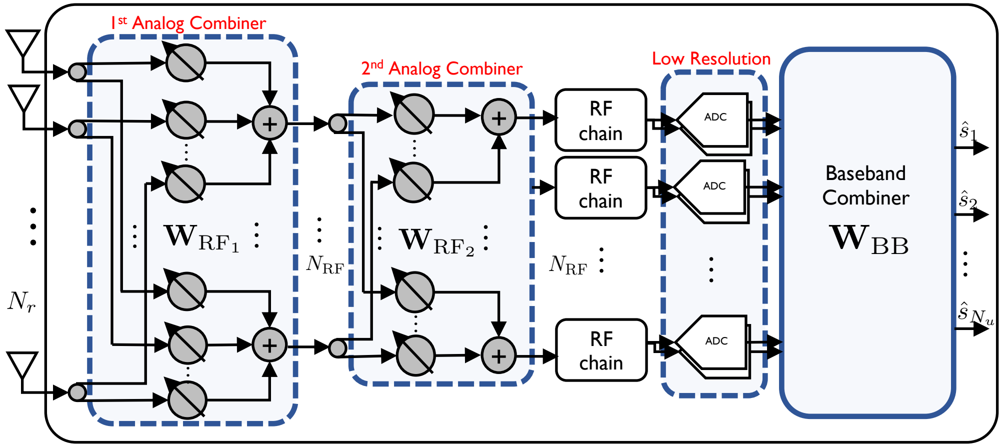




*SE: spectral efficiency *EE: energy efficiency

SUMMARY

Proposed two-stage analog combining receiver



Optimality

Two-stage combining solution

$(i)\ \mathbf{W}^{\star}_{\mathrm{RF}_1} = [\mathbf{U}_{1:N_u}\mathbf{U}_{\perp}]\ (ii)\ \mathbf{W}^{\star}_{\mathrm{RF}_2}$: unitary

Optimal scaling law

Maximum mutual information

Algorithm

Sparse channel assumption

ARV-TSAC algorithm

$\tilde{\mathbf{W}}^{\star}_{\mathrm{RF}} = \mathbf{W}_{\mathrm{AoA}}\mathbf{W}_{\mathrm{DFT}}$

Ergodic rate analysis with MRC receiver

Second analog combiner is essential in reducing effective quantization error

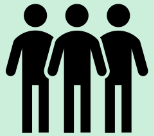
SUMMARY AND CONCLUSION

- Considered system: hybrid beamforming with low-resolution ADC system for high energy efficiency

Optimizations for mitigating quantization error

MAC Layer Perspective

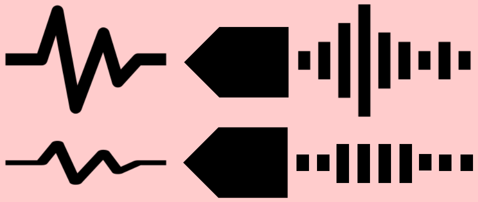
User scheduling



optimize set of users

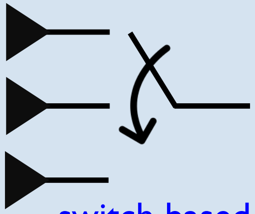
PHY Layer Perspective

Adaptive ADC



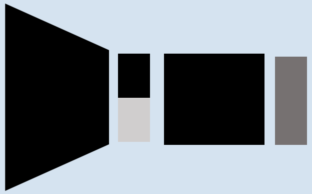
optimize ADC resolution

Antenna selection



- switch based -

Two-stage combining



- phase shifter based -

optimize analog combiner

Advanced architectures and techniques at different parts of wireless systems can significantly increase spectral and energy efficiency

FUTURE WORK

- Channel estimation in two-stage analog combining system
  - Use estimation techniques for hybrid system after multiplying inverse matrix of second combiner
    : Less performance degradation thanks to even distribution of quantization error (QE)
  - Develop new estimation technique by estimating quantization noise level
    : Maybe easier to estimate quantization error thanks to even distribution of QE under same total QE

- Extension of receiver design work into wideband communications
  - Base station antenna selection: similar results both in narrowband and wideband OFDM
    : It is not proved for two-stage analog combining system and resolution-adaptive ADC system

- Cooperation of multiple base stations under limited total power consumption
  - Optimization of ADC resolution over multiple BSs in multiple cells
    : It can be jointly optimized with user transmit power

PUBLICATIONS – JOURNAL ARTICLES

- **Jinseok Choi,** Junmo Sung, Narayan Prasad, Xiao-Feng Qi, Brian L. Evans, and Alan Gatherer, "Base Station Antenna Selection for Low-Resolution ADC Systems", *IEEE Transactions on Communications* (under revision).
- Faris B. Mismar, **Jinseok Choi**, and Brian L. Evans, "A Framework for Automated Cellular Network Tuning with Reinforcement Learning", *IEEE Transactions on Communications* accepted for publication.
- **Jinseok Choi,** Gilwon Lee, and Brian L. Evans, "User Scheduling for Millimeter Wave Hybrid Beamforming Systems with Low-Resolution ADCs", *IEEE Transactions on Wireless Communications*, vol. 18, no. 4, pp. 2401-2414, Apr. 2019.
- **Jinseok Choi,** Gilwon Lee, and Brian L. Evans, "Two-Stage Analog Combining in Hybrid Beamforming Systems with Low-Resolution ADCs", *IEEE Transactions on Signal Processing*, vol. 67, no. 9, pp. 2410-2425, May 1, 2019.
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- **Jinseok Choi**, Brian L. Evans, and Alan Gatherer, "Resolution-Adaptive Hybrid MIMO Architectures for Millimeter Wave Communications", *IEEE Transactions on Signal Processing,* vol. 65, no. 23, pp. 6201-6216, Dec. 2017.
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- **Jinseok Choi**, Yunseong Cho, Brian L. Evans, and Alan Gatherer, "Robust Learning-Base ML Detection for Massive MIMO Systems with One-Bit Quantized Signals", *IEEE Global Communications Conf*. Dec. 9-13, 2019, Waikoloa, HI, USA.
- **Jinseok Choi**, Gilwon Lee, and Brian L. Evans, "A Hybrid Beamforming Receiver with Two-Stage Analog Combiner and Low-Resolution ADCs", *IEEE Int. Conf. on Communications*, 2019, May 2019, Shanghai, China.
- **Jinseok Choi**, Brian L. Evans, and Alan Gatherer, "Antenna Selection for Large-Scale MIMO Systems with Low-Resolution ADCs", *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing,* Apr. 15-20, 2018, Calgary, Alberta, Canada, accepted.
- Junmo Sung, **Jinseok Choi**, and Brian L. Evans, "Narrowband Channel Estimation for Hybrid Beamforming Millimeter Wave Communication Systems with One-bit Quantization", *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing,* Apr. 15-20, 2018.
- **Jinseok Choi**, and Brian L. Evans, "User Scheduling for Millimeter Wave MIMO Communications with Low-Resolution ADCs", *Proc. IEEE Int. Conf. on Communications,* May 20-24, 2018, Kansas City, MO, USA.
- **Jinseok Choi,** Junmo Sung, Brian L. Evans, and Alan Gatherer, "ADC Bit Optimization for Spectrum- and Energy-Efficient Millimeter Wave Communications", *Proc. IEEE Global Communications Conf.,* Dec. 4-8, 2017, Singapore.
- **Jinseok Choi,** Brian L. Evans, and Alan Gatherer, "ADC Bit Allocation under a Power Constraint for MmWave Massive MIMO Communication Receivers", *Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Mar. 5-9, 2017, New Orleans, LA, USA.
- **Jinseok Choi**, Brian L. Evans, and Alan Gatherer, "Space-Time Fronthaul Compression of Complex Baseband Uplink LTE Signals", *Proc. IEEE Int. Conf. on Communications*, May 23-27, 2016, Kuala Lumpur, Malaysia.

SOFTWARE RELEASES

Available at http://users.ece.utexas.edu/~bevans/projects/mimo/software.html

- **Jinseok Choi** and Brian L. Evans, "Two-Stage Analog Beamforming", MATLAB code to accompany a paper entitled "Two-Stage Analog Combining in Hybrid Beamforming Systems with Low-Resolution ADCs" in the *IEEE Transactions on Signal Processing*, vol. 67, no. 9, May 1, 2019, pp. 2410-2425, DOI 10.1109/TSP.2019.2904931. Version 1.0 (July 27, 2019)
- **Jinseok Choi** and Brian L. Evans, "Antenna Selection for Large-Scale MIMO Systems with Low-Resolution ADCs", MATLAB code to accompany a paper of the same title in the 2018 *IEEE International Conference on Acoustics, Speech and Signal Processing*. Version 1.0 (October 27, 2017).
- **Jinseok Choi** and Brian L. Evans, "User Scheduling Algorithms for Millimeter Wave MIMO Systems", MATLAB code to accompany a paper of the same title in the 2018 *IEEE International Conference on Communications*. Version 1.0 (October 13, 2017).
- **Jinseok Choi** and Brian L. Evans, "Resolution-Adaptive Hybrid MIMO Architectures for Millimeter Wave Communications", MATLAB code to accompany a paper of the same title in the *IEEE Transactions on Signal Processing*, vol. 65, no. 23, pp. 6201-6216, Dec. 2017, DOI 10.1109/TSP.2017.2745440. Software release is version 1.0 (Nov. 15, 2018).
- **Jinseok Choi** and Brian L. Evans, "Space-Time Baseband LTE Compression Software", copyright © 2016 by The University of Texas. This MATLAB release implements algorithms to compress uplink baseband cellular LTE signals received by an antenna array. Software release accompanies the paper "Space-Time Fronthaul Compression of Complex Baseband Uplink LTE Signals" in the 2016 *IEEE International Conference on Communications*. Version 1.0 (April 4, 2016).

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- [Nemhauser78] G. L. Nemhauser, L. A. Wolsey, and M. L. Fisher, “An analysis of approximations for maximizing submodular set functions–I,” *Math. Programming*, vol. 14, no. 1, pp. 265–294, 1978
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OUTLINE

Background

- Millimeter wave communications with a large number of antennas
- Motivation of my PhD dissertation

Contributions

1 User scheduling in low-resolution ADC systems
2 Resolution-adaptive ADC receiver architecture
3 Base station antenna selection in low-resolution ADC systems
4 Two-stage analog combining receiver architecture in low-resolution ADC systems

Conclusion & Future work

- Future work
- Summary

Millimeter Wave Spectrum

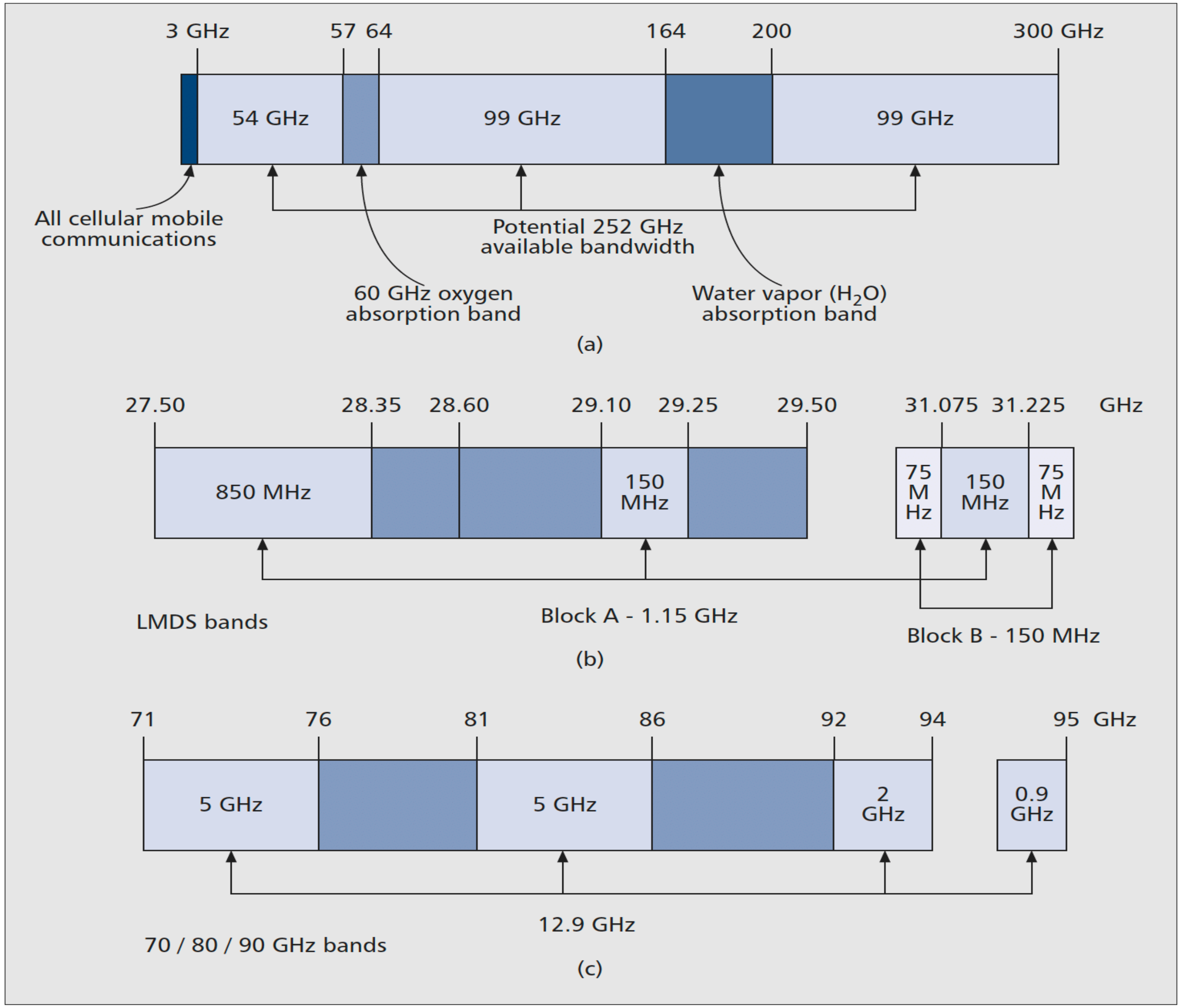
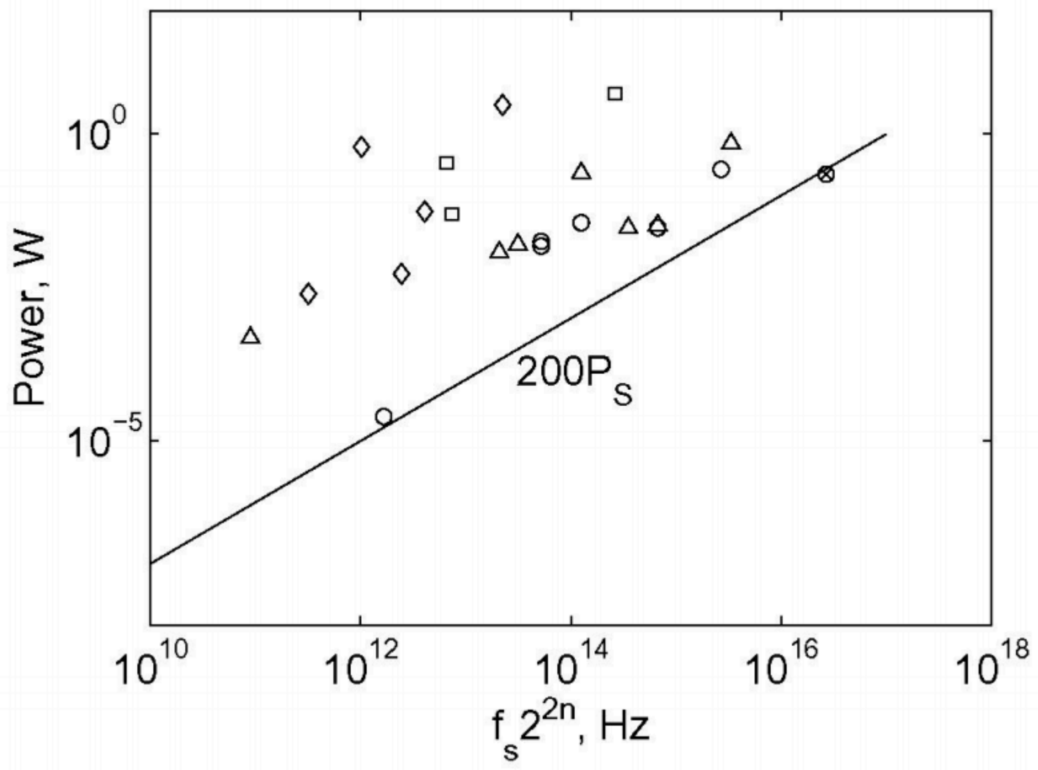


Figure 1. *Millimeter-wave spectrum.*

[Pi&Khan11] Pi, Zhouyue, and Farooq Khan. "An introduction to millimeter-wave mobile broadband systems." *IEEE Comm. Mag.* 49.6 (2011).

ADC Power Consumption



n: quantization bits
fs: sampling rate

Svensson, Christer, Stefan Andersson, and Peter Bogner. "On the power consumption of analog to digital converters." *2006 NORCHIP*. IEEE, 2006.

SYSTEM ASSUMPTIONS

3GPP TR 36.931 V12.0.0 (2014-09)

Table 5.3.3-1: Macro system assumptions

Parameters	Assumptions	
Carrier frequency	2000 MHz	
System bandwidth	10 MHz(aggressor), 10 MHz(victim)	
Cellular layout	Hexagonal grid, 19 cell sites, with BTS in the corner of the cell , 65-degree sectored beam.	
Wrap around	Employed	
Inter-site distance	750 m	
Traffic model	Full buffer	
UE distribution	UEs dropped with uniform density within the macro coverage area, Indoor UEs ratio is a parameter depending on the simulation scenario.	
Path loss model	L = 128.1 + 37.6 log10 (R), R in kilometers	
Lognormal shadowing	Log Normal Fading with 10 dB standard deviation	
LTE BS Antenna gain after cable loss	15 dBi	
UE Antenna gain	0 dBi	
Outdoor wall penetration loss	10 dB	
White noise power density	-174 dBm/Hz	
BS noise figure	5 dB	
UE noise figure	9 dB	
Maximum BS TX power	46dBm	
Maximum UE TX power	23dBm	
Minimum UE TX power	-30dBm	
MCL	70 dB	
Scheduling algorithm	Round Robin	
RB width	180 kHz, total 50 RBs	
RB numbers per user	Downlink:1	Uplink:16

CONTRIBUTION 1

USER SCHEDULING

New User Scheduling Criteria

- Solution of $\mathcal{P}2$: structural scheduling criteria
  - For total # of channel paths $\leq N_{RF}$

$\mathcal{L}_k$: index set of nonzero elements

Theorem A-1

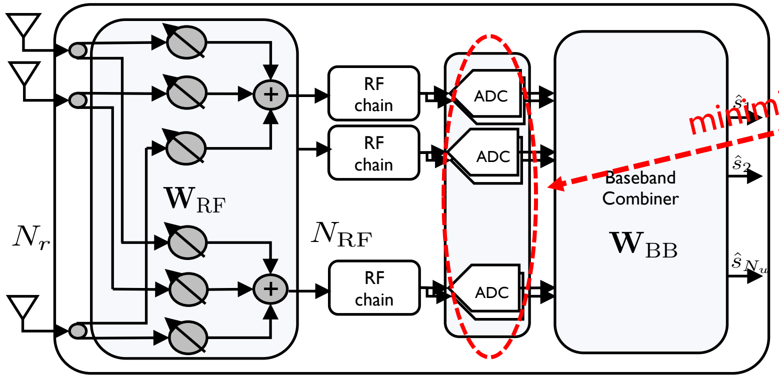
I. Unique *AoAs at receiver for channel paths of each scheduled user:

$$\mathcal{L}_{\mathcal{S}(k)} \cap \mathcal{L}_{\mathcal{S}(k')} = \emptyset \text{ if } k \neq k'.$$

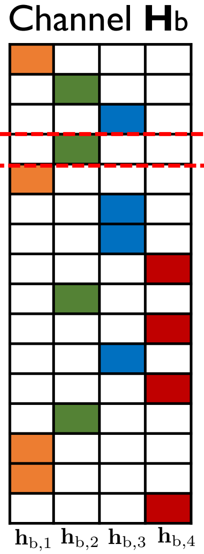
II. Equal power spread across beamspace complex gains within each channel:

$$|h_{\mathrm{b},i,\mathcal{S}(k)}| = \sqrt{\gamma_{\mathcal{S}(k)}/L_{\mathcal{S}(k)}} \text{ for } i \in \mathcal{L}_{\mathcal{S}(k)}.$$

*Angle of arrivals



minimize error here



Aggregated channel gain at ADC

- Unique AoAs
- Equal power spread

$$\text{minimize } \|[\mathbf{H}_\mathrm{b}]_{i,:}\|^2$$

PROOF OF THEOREM A-1

- Case: total # of channel paths $\leq N_{RF}$
  - Two-stage maximization approach

$$r_k(\mathbf{H}_\mathrm{b}) = \log_2\left(1 + \frac{\alpha\rho}{\rho(1-\alpha)\mathbf{w}_{\mathrm{zf},k}^H \mathrm{diag}\left(\mathbf{H}_\mathrm{b}\mathbf{H}_\mathrm{b}^H\right)\mathbf{w}_{\mathrm{zf},k} + \|\mathbf{w}_{\mathrm{zf},k}\|^2}\right).$$

I. Minimize $\|\mathbf{w}_{\mathrm{zf},k}\|^2$

: channels have to be orthogonal

II. Maximize under orthogonal condition

$$r_k(\mathbf{H}_\mathrm{b}|\mathbf{h}_{\mathrm{b},k} \perp \mathbf{h}_{\mathrm{b},k'}) \overset{(a)}{=} \log_2\left(1 + \frac{\alpha\rho\|\mathbf{h}_{\mathrm{b},k}\|^4}{\rho(1-\alpha)\mathbf{h}_{\mathrm{b},k}^H \mathrm{diag}\left(\mathbf{H}_\mathrm{b}\mathbf{H}_\mathrm{b}^H\right)\mathbf{h}_{\mathrm{b},k} + \|\mathbf{h}_{\mathrm{b},k}\|^2}\right)$$

$$= \log_2\left(1 + \frac{\alpha\rho\gamma^2}{\rho(1-\alpha)\sum_{i\in\mathcal{L}_k}|h_{\mathrm{b},i,k}|^2\left(|h_{\mathrm{b},i,k}|^2 + \boxed{\sum_{u\neq k}|h_{\mathrm{b},i,u}|^2}\right) + \gamma_k}\right)$$

Non-overlap of channel gains

KKT condition: equal power spread

$$\overset{(b)}{\leq} \log_2\left(1 + \frac{\alpha\rho\gamma^2}{\rho(1-\alpha)\sum_{i\in\mathcal{L}_k}|h_{\mathrm{b},i,k}|^4 + \gamma_k}\right)$$

$$\overset{(c)}{\leq} \log_2\left(1 + \frac{\alpha\rho}{\rho(1-\alpha)/L_k + 1/\gamma_k}\right)$$: max rate for single user case

(a-c): sufficient conditions for maximizing sum rate

Partial CSI-Based User Scheduling

- Alternative to instantaneous full CSI
  - *Angles of arrival (AoA): slowly-varying channel characteristics
    : reduces burden of estimating instantaneous full CSI at every channel coherence time
- Chordal distance-based user scheduling
  - Key idea

measure separation between channel subspaces

Channel Vector

$$\mathbf{h}_k = \sqrt{\frac{N_r}{L_k}} \sum_{\ell=1}^{L} g_{k,\ell} \mathbf{a}(\phi_{k,\ell})$$

$$\mathbf{A}_k = [\mathbf{a}(\phi_{k,1}), \ldots, \mathbf{a}(\phi_{k,|\mathcal{V}_k|})]$$

Chordal Distance

$$d_{\text{cd}}(k, k') = \sqrt{L_{min} - \text{tr}\left(\mathbf{Q}_k^H \mathbf{Q}_{k'} \mathbf{Q}_{k'}^H \mathbf{Q}_k\right)}$$

$$\mathbf{Q}_k = \text{column basis of } \mathbf{A}_k$$

  - Steps

1. RF chain angle filtering

Filter users:
no AoAs in reduced range of angles of RF chains

2. Chordal distance filtering

Filter users:

$$d_{\text{cd}}(\mathcal{S}_{\text{cd}}(i-1), k)/\sqrt{L_{min}} < d_{\text{th}}$$

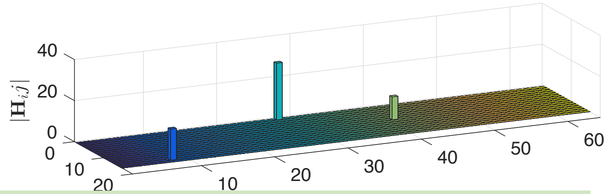
3. Max # of AoAs (+ max. chordal distance)

Select user with max. # of AoAs.

If many: $\mathcal{S}_{\text{cd}}(i) = \arg\max_{k \in \mathcal{U}} d_{\text{cd}}(\mathcal{S}_{\text{cd}}(i-1), k)$

Performance Analysis – Partial CSI

- Ergodic sum rate analysis for single path
  - Exact AoA alignment



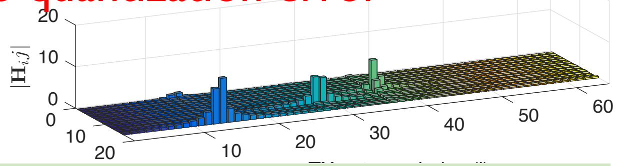
Proposition 1

$$\bar{\mathcal{R}}_1 = \frac{N_u}{\ln 2}\left(\underbrace{e^{\frac{1}{p_u N_r}}\Gamma\left(0, \frac{1}{p_u N_r}\right)} - \underbrace{e^{\frac{1}{p_u(1-\alpha)N_r}}\Gamma\left(0, \frac{1}{p_u(1-\alpha)N_r}\right)}\right)$$

Ergodic rate without quantization

rate loss due to quantization error

  - Arbitrary AoA



Proposition 2

$$\bar{\mathcal{R}}_2^{lb} \approx \frac{N_u}{\ln 2}\left(e^{\frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u\alpha N_r + p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}}\Gamma\left(0, \frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u\alpha N_r + p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}\right)\right.$$
$$\left. - e^{\frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}}\Gamma\left(0, \frac{1+p_u(1-\alpha)(N_u-1)N_r^2\mathcal{F}_2(N_r)}{p_u(1-\alpha)N_r^2\mathcal{F}_1(N_r)}\right)\right)$$

Remark

As $b \to \infty$, both converge to $\frac{N_u}{\ln 2}e^{\frac{1}{p_u N_r}}\Gamma\left(0, \frac{1}{p_u N_r}\right)$

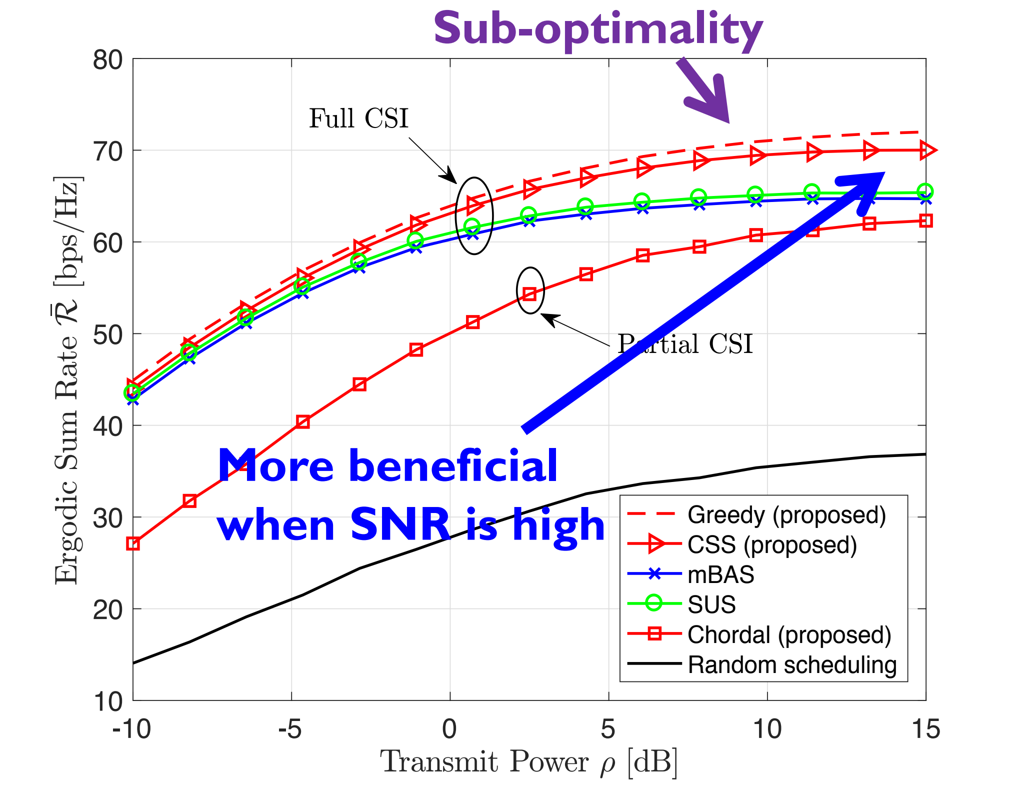
$\Gamma(a, z)$: incomplete gamma function

SIMULATION – PERFORMANCE VALIDATION

Greedy: schedules user who provides maximum sum rate (sub-optimal performance with prohibitively high complexity)

Sum Rate vs. Transmit power

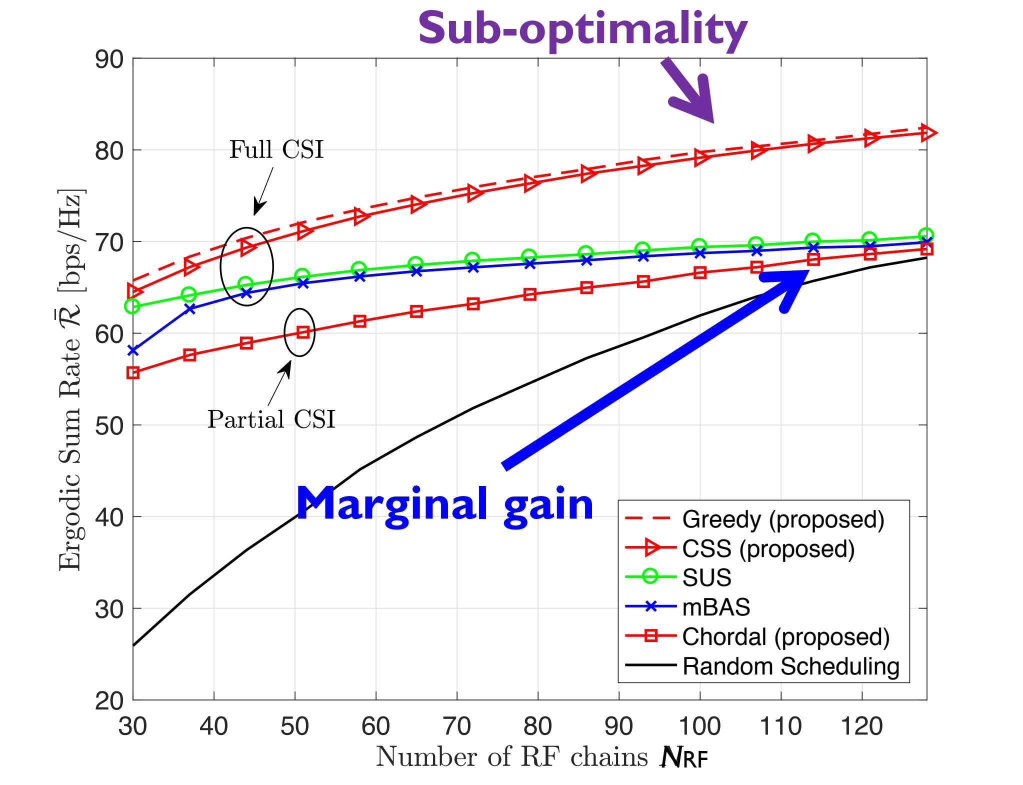
Figure 1 (40 RF chains)



Why? Quantization error dominates thermal noise
: CSS is effective under coarse quantization

Sum Rate vs. # of RF chains

Figure 2 (6 dB Tx power)



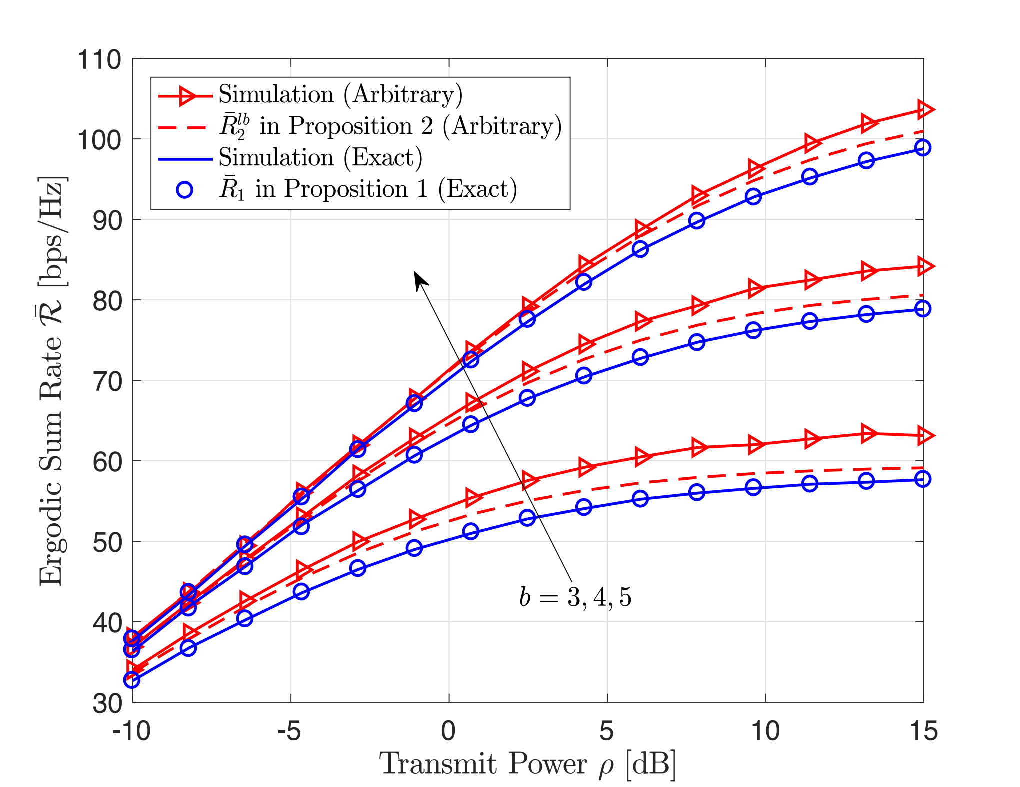
Why? As N_{RF} increases, channels become more orthogonal
: quantization error becomes major bottleneck

Settings 128 antennas, 200 candidate users, 12 scheduled users, 3 ADC bits, 3 average channel paths

SIMULATION – ANALYSIS VALIDATION

Sum Rate vs. Transmit power

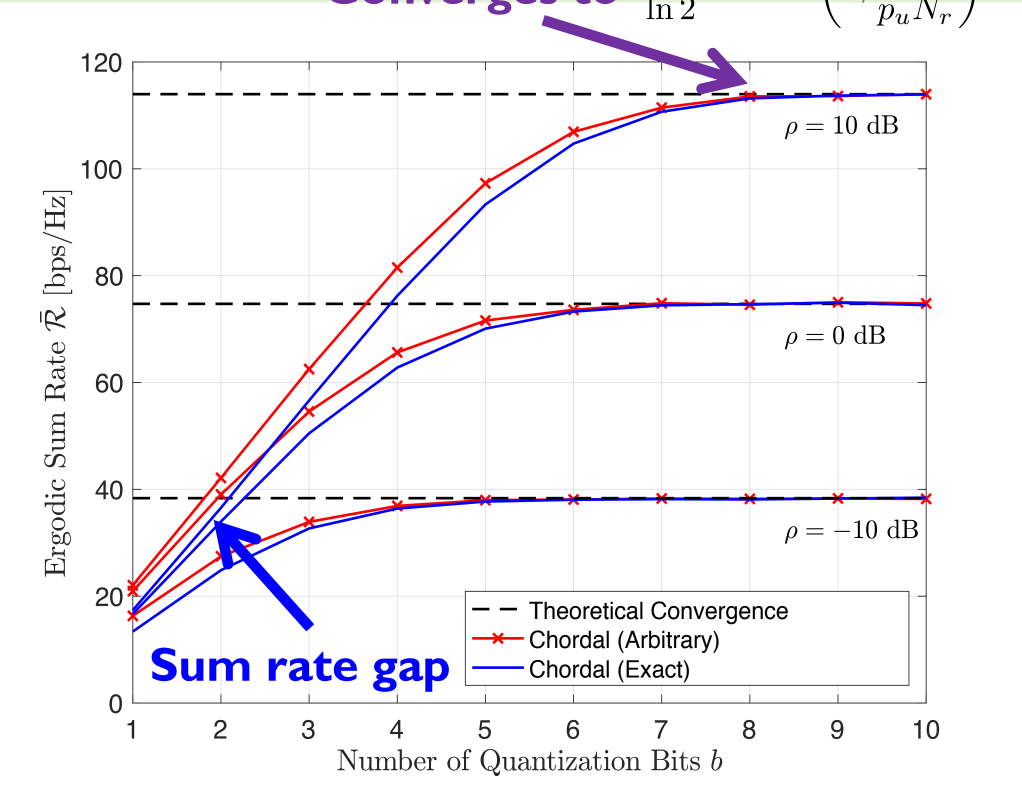
Figure 3



Sum Rate vs. # of ADC bits

Figure 4

Converges to $\frac{N_u}{\ln 2} e^{\frac{1}{p_u N_r}} \Gamma\left(0, \frac{1}{p_u N_r}\right)$



Why? Channel leakage reduces quantization error : equal power spread in Theorem 1

Settings	128 antennas, 128 RF chains, 200 candidate users, 12 scheduled users, single channel path

CONTRIBUTION 2

ADAPTIVE ADC

Extension to Receiver Power Constraint

- Joint binary search algorithm

  : solves total power constrained MMSQE bit allocation problem

$$\mathcal{P}2: \quad \mathbf{b}_2^\star = \arg \min_{\mathbf{b}\in\mathbb{R}^{N_{\rm RF}}} \sum_{i=1}^{N_{\rm RF}} \mathcal{E}_{x_i}(b_i) \quad \text{s.t.} \quad P_{\rm tot} \le \boxed{p}$$

Total receiver power constraint

- Challenges in total receiver power

$$P_{\rm tot} = N_r P_{\rm LNA} + \boxed{N_{\rm act}}(N_r P_{\rm PS} + P_{\rm RFchain}) + 2\sum_{i=1}^{N_{\rm RF}}\left(\boxed{P_{\rm ADC}(b_i)} + \boxed{P_{\rm SW}(b_i)}\right) + P_{\rm BB}$$

$$P_{\rm ADC}(b) = cf_s 2^b$$
$$P_{\rm SW}(b) = c_{sw}|2^b - 2^{b_p}|$$

# of active RF chains

: function of ADC bits (0-bit: inactive)

Resolution-switching power consumption

: function of previous bits and current bits

- Steps

Joint binary search for $N_{\rm act}$ & b

*Channel gains are sorted in descending order

1. Offline P_{SW} estimation

Training and modeling avg. P_{SW} as function of power constraint

2. BA solution for fixed $N_{\rm act}$

$$b_i^{\rm s} = \log_2 \frac{\tilde p}{2c\, f_s} + \log_2\left(\frac{\|[\mathbf{H}_{\rm b}]_{i,:}\|^{\frac{2}{3}}}{\sum_{j=1}^{M_s}\|[\mathbf{H}_{\rm b}]_{j,:}\|^{\frac{2}{3}}}\right)$$

3. Total MSQE computation & comparison

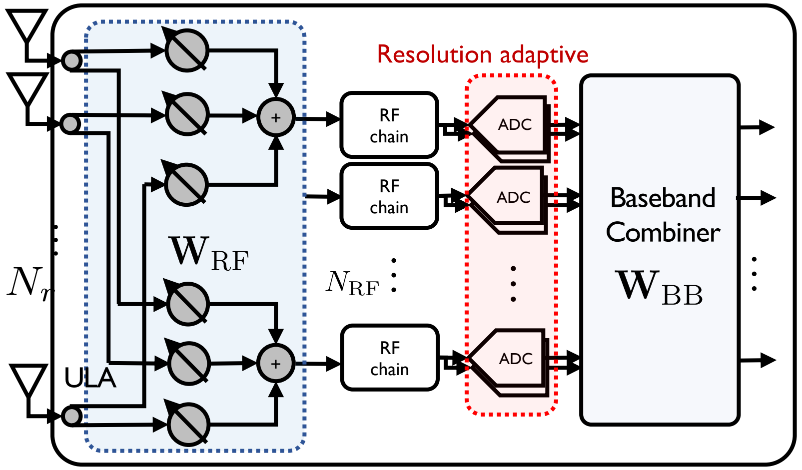
Case: $(N_{\rm act} - 1)$ $(N_{\rm act})$ $(N_{\rm act} + 1)$

MSQE: $\sum_{i=1}^{N_{act}-1}\mathcal{E}_{x_i}(b_i^l)$ vs $\sum_{i=1}^{N_{act}}\mathcal{E}_{x_i}(b_i)$ vs $\sum_{i=1}^{N_{act}+1}\mathcal{E}_{x_i}(b_i^r)$

Go to smaller half

SUMMARY

Proposed resolution-adaptive ADC receiver



Near optimal bit-allocation

High SNR: minimizes mean squared quantization error

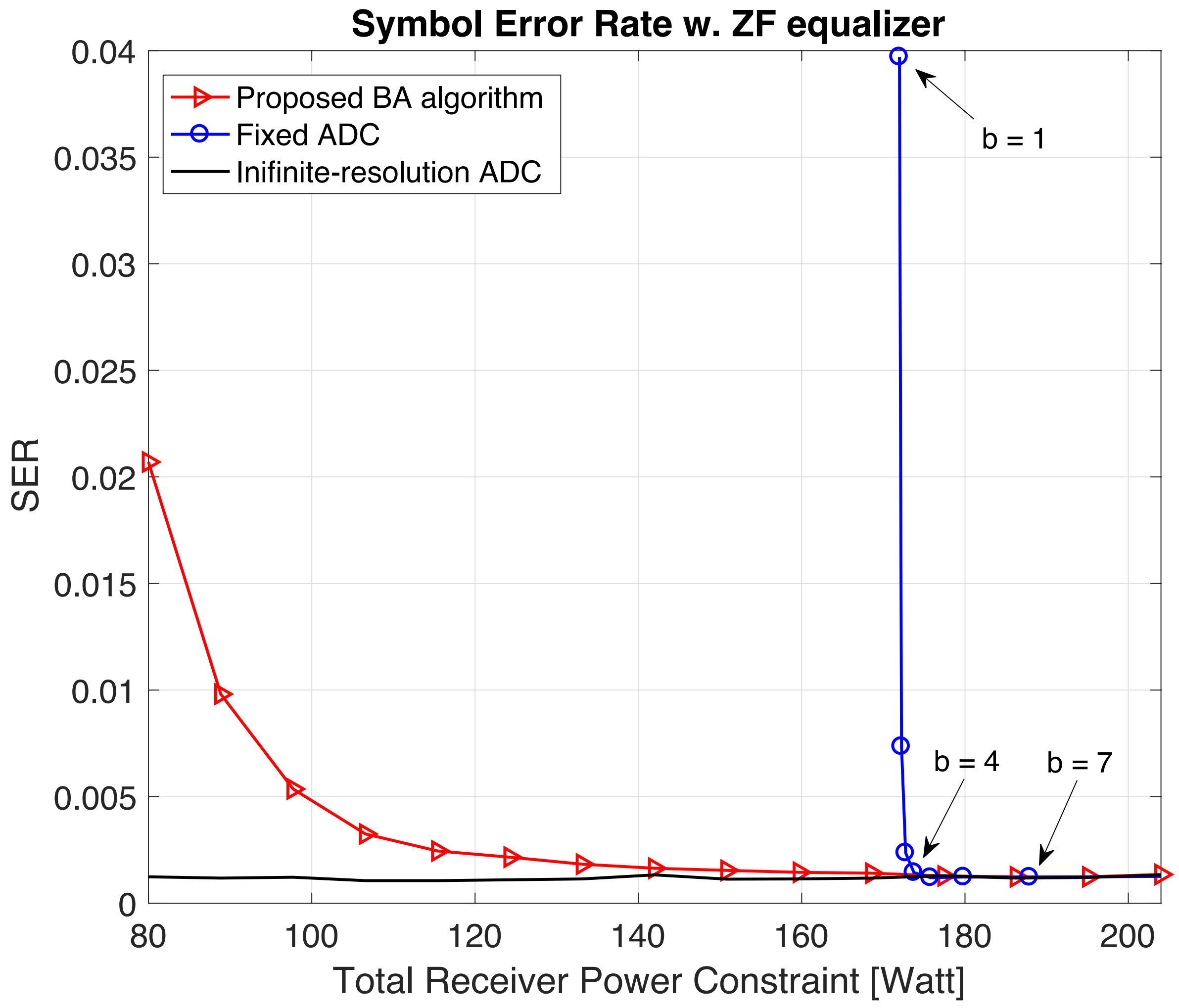
Low SNR: maximizes generalized mutual information

Worst case analysis

Lower bound of Ergodic rate for proposed receiver architecture

Selective bit allocation works for sparse channel with limited power consumption

SYMBOL ERROR RATE (SER)



Proposed Method

- Achieves SER comparable to infinite-resolution case at around Ptot = 120 W
- 30 % total receiver power saving from 4~5-bit ADC system
- 80 % total receiver power saving from *infinite-bit (b = 12) ADC system

*Power consumption of infinite-bit ADC system = 689 W

CONTRIBUTION 3

ANTENNA SELECTION

Greedy MI-Maximizing Antenna Selection

- Decomposition of mutual information

  - Mutual information

quantization noise covariance

$$\mathbf{R}_{\mathbf{qq}} = \alpha(1-\alpha)\,\mathrm{diag}(\rho\mathbf{H}_{\mathcal{K}}\mathbf{H}_{\mathcal{K}}^H + \mathbf{I})$$

$$C(\mathbf{H}_{\mathcal{K}}) = \log_2\left|\mathbf{I} + \rho\alpha^2\left(\alpha^2\mathbf{I} + \mathbf{R}_{\mathbf{qq}}\right)^{-1}\mathbf{H}_{\mathcal{K}}\mathbf{H}_{\mathcal{K}}^H\right|$$

$$= \log_2\left|\mathbf{I} + \rho\alpha\mathbf{D}_{\mathcal{K}}^{-1}\mathbf{H}_{\mathcal{K}}\mathbf{H}_{\mathcal{K}}^H\right| \quad \text{where} \quad \mathbf{D}_{\mathcal{K}} = \mathrm{diag}(1 + \rho(1-\alpha)\|\mathbf{f}_{\mathcal{K}(i)}\|^2)$$

  - At (n+1)th selection stage

$$C(\mathbf{H}_{n+1}) = \log_2\left|\mathbf{I} + \rho\alpha\mathbf{D}_{n+1}^{-1}\mathbf{H}_{n+1}\mathbf{H}_{n+1}^H\right|$$

$$= \log_2\left|\mathbf{I} + \rho\alpha\mathbf{H}_{n+1}^H\mathbf{D}_{n+1}^{-1}\mathbf{H}_{n+1}\right|$$

(n+1) th row of $\mathbf{H}_{n+1}$

$$= \log_2\left|\mathbf{I} + \rho\alpha\left(\mathbf{H}_n^H\mathbf{D}_n^{-1}\mathbf{H}_n + \frac{1}{d_{\mathcal{K}(n+1)}}\mathbf{f}_{\mathcal{K}(n+1)}\mathbf{f}_{\mathcal{K}(n+1)}^H\right)\right|$$

*Diagonal entry of **D** corresponding to $\mathbf{f}_{\mathcal{K}(n+1)}$*

(a)

$$= C(\mathbf{H}_n) + \log_2\left(1 + \frac{\rho\alpha}{d_{\mathcal{K}(n+1)}}c_{\mathcal{K}(n+1),n}\right) \quad \text{where} \quad c_{\mathcal{K}(n+1),n} = \mathbf{f}_{\mathcal{K}(n+1)}^H\left(\mathbf{I} + \rho\alpha\mathbf{H}_n^H\mathbf{D}_n^{-1}\mathbf{H}_n\right)^{-1}\mathbf{f}_{\mathcal{K}(n+1)}$$

Here, (a) comes from matrix determinant lemma $|\mathbf{A} + \mathbf{u}\mathbf{v}^H| = |\mathbf{A}|(1 + \mathbf{v}^H\mathbf{A}^{-1}\mathbf{u})$

Complexity of QFAS Algorithm

- Complexity analysis
  - Complexity for step 5: $O(KN_u^2)$
    - K *iterations* x *Inner product* $\boldsymbol{Qf}_J$
  - Complexity for step 6: $O(KN_rN_u)$
    - K iterations

      x N_r updates

      x Inner product $\boldsymbol{f}_j^H\boldsymbol{a}$
  - Large antenna arrays ($N_r \gg N_u$)

Overall complexity becomes O(K N_r N_u)

- Proposed algorithm

Quantization-Aware Fast Antenna Selection (QAFAS)	
1) Initialize: $\mathcal{T} = \{1, \ldots, N_r\}$ and $\mathbf{Q} = \mathbf{I}$. 2) Initialize antenna gain and compute penalty: $c_j = \|\mathbf{f}_j\|^2$ and $d_j = 1 + \rho(1-\alpha)\|\mathbf{f}_j\|^2$ for $j \in \mathcal{T}$. 3) Select antenna : $J = \mathrm{argmax}_{j\in\mathcal{T}}\, c_j/d_j$. 4) Update candidate set: $\mathcal{T} = \mathcal{T} \setminus \{J\}$. 5) Compute: $\mathbf{a} = \left(c_J + \frac{d_J}{\rho\alpha}\right)^{-\frac{1}{2}}\mathbf{Q}\mathbf{f}_J$ and $\mathbf{Q} = \mathbf{Q} - \mathbf{a}\mathbf{a}^H$. 6) Update $c_j = c_j - \|\mathbf{f}_j^H\mathbf{a}\|^2$ for $j \in \mathcal{T}$. 7) Go to step 3 and repeat until select K antennas.	
Complexity	$\boxed{O(K\,N_r\,N_u)}$

same as FAS [Gharavi-Alkhansari04]

CONTRIBUTION 4

TWO-STAGE ANALOG COMBINING

Proof of Theorem 2 - Scaling Law

- Rewrite mutual information

$$\mathcal{C}(\mathbf{W}_{\rm RF}) = \log_2\left|\mathbf{I}_{N_{\rm RF}}+\rho\alpha_b^2\left(\alpha_b^2\mathbf{W}_{\rm RF}^H\mathbf{W}_{\rm RF}+\mathbf{R}_{\mathbf{qq}}\right)^{-1}\mathbf{W}_{\rm RF}^H\mathbf{H}\mathbf{H}^H\mathbf{W}_{\rm RF}\right|$$

$$=\log_2\left|\mathbf{I}+\frac{\alpha_b}{\beta_b}{\rm diag}^{-1}\left\{\overline{\mathbf{W}}_{\rm RF}^H\bar{\mathbf{\Lambda}}\overline{\mathbf{W}}_{\rm RF}+\frac{1}{\beta_b\rho}\mathbf{I}\right\}\overline{\mathbf{W}}_{\rm RF}^H\bar{\mathbf{\Lambda}}\overline{\mathbf{W}}_{\rm RF}\right|.$$

- Define $\mathbf{G} = \overline{\mathbf{W}}_{\rm RF}^H\bar{\mathbf{\Lambda}}^{1/2} = [\mathbf{G}_{\rm sub}\ \mathbf{0}]$ and rewrite mutual information

$$\mathcal{C}(\mathbf{W}_{\rm RF}) = \log_2\left|\mathbf{I}_m + \frac{\alpha_b}{\beta_b}\mathbf{G}_{\rm sub}^H{\rm diag}^{-1}\left\{\|[\mathbf{G}_{\rm sub}]_{i,:}\|^2+\frac{1}{\beta_b\rho}\right\}\mathbf{G}_{\rm sub}\right|$$

$$=\log_2\left|\mathbf{I}_m + \frac{\alpha_b}{\beta_b}\tilde{\mathbf{G}}_{\rm sub}^H\tilde{\mathbf{G}}_{\rm sub}\right|$$

$$=\sum_{i=1}^m\log_2\left(1+\frac{\alpha_b}{\beta_b}\lambda_i\{\tilde{\mathbf{G}}_{\rm sub}^H\tilde{\mathbf{G}}_{\rm sub}\}\right)$$

$$\stackrel{(a)}{\le} m\log_2\left(1+\frac{\alpha_b}{\beta_b m}\sum_{i=1}^m\lambda_i\{\tilde{\mathbf{G}}_{\rm sub}^H\tilde{\mathbf{G}}_{\rm sub}\}\right)$$

Jensen's inequality

$$\stackrel{(b)}{=} m\log_2\left(1+\frac{\alpha_b}{\beta_b m}\sum_{i=1}^{N_{\rm RF}}\frac{\|[\mathbf{G}_{\rm sub}]_{i,:}\|^2}{\|[\mathbf{G}_{\rm sub}]_{i,:}\|^2+\frac{1}{\beta_b\rho}}\right)$$

$$\sum_{i=1}^m\lambda_i\{\tilde{\mathbf{G}}_{\rm sub}^H\tilde{\mathbf{G}}_{\rm sub}\}={\rm Tr}\{\tilde{\mathbf{G}}_{\rm sub}^H\tilde{\mathbf{G}}_{\rm sub}\}=\sum_{i=1}^{N_{\rm RF}}\frac{\|[\mathbf{G}_{\rm sub}]_{i,:}\|^2}{\|[\mathbf{G}_{\rm sub}]_{i,:}\|^2+\frac{1}{\beta_b\rho}}$$

$$< m\log_2\left(1+\frac{\alpha_b N_{\rm RF}}{\beta_b m}\right) \le N_u\log_2\left(1+\frac{\alpha_b N_{\rm RF}}{\beta_b N_u}\right) \sim N_u\log_2(N_{\rm RF})$$

$$\mathbf{W}_{\rm RF} = [\mathbf{U}_{||}\ \mathbf{U}_{\perp}]\bar{\mathbf{W}}_{\rm RF},$$

$$\mathbf{W}_{\rm RF}^H\mathbf{H}\mathbf{H}^H\mathbf{W}_{\rm RF}$$
$$= \bar{\mathbf{W}}_{\rm RF}^H[\mathbf{U}_{||}\ \mathbf{U}_{\perp}]^H\mathbf{U}\mathbf{\Lambda}\mathbf{U}^H[\mathbf{U}_{||}\ \mathbf{U}_{\perp}]\bar{\mathbf{W}}_{\rm RF}$$
$$= \bar{\mathbf{W}}_{\rm RF}^H\underbrace{\begin{bmatrix}\mathbf{U}_{||}^H\mathbf{U}_{1:N_u}\mathbf{\Lambda}_{N_u}\mathbf{U}_{1:N_u}^H\mathbf{U}_{||} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}}_{\triangleq\mathbf{Q}}\bar{\mathbf{W}}_{\rm RF}$$

$$\mathbf{Q} = \mathbf{U}_{\mathbf{Q}}\bar{\mathbf{\Lambda}}\mathbf{U}_{\mathbf{Q}}^H$$

$$\bar{\mathbf{W}}_{\rm RF} = \mathbf{U}_{\mathbf{Q}}\overline{\mathbf{W}}_{\rm RF}$$

$\overline{\mathbf{W}}_{\rm RF}$: $N_{\rm RF}\times N_{\rm RF}$ unitary matrix

$\bar{\mathbf{\Lambda}} = {\rm diag}\{\bar\lambda_1,\cdots,\bar\lambda_m,0,\cdots,0\}$

$1\le m\le N_u$

PROOF OF THEOREM 2 – TWO-STAGE SOLUTION

- With two-stage solution in Theorem 2: $\mathbf{W}_{\rm RF}^{\star} = \mathbf{W}_{\rm RF_1}^{\star}\mathbf{W}_{\rm RF_2}^{\star}$

(i) $\mathbf{W}_{\rm RF_1}^{\star} = [\mathbf{U}_{1:N_u}\mathbf{U}_{\perp}]$

(ii) $\mathbf{W}_{\rm RF_2}^{\star}$: $N_{\rm RF}$ x $N_{\rm RF}$ unitary matrix with constant modulus

$$\mathcal{C}(\mathbf{W}_{\rm RF}^{\star}) = \log_2\left|\mathbf{I}_{N_{\rm RF}} + \frac{\alpha_b}{\beta_b}{\rm diag}^{-1}\left\{\mathbf{W}_{\rm RF}^{\star H}\mathbf{H}\mathbf{H}^H\mathbf{W}_{\rm RF}^{\star} + \tfrac{1}{\beta_b\rho}\mathbf{I}_{N_{\rm RF}}\right\}\mathbf{W}_{\rm RF}^{\star H}\mathbf{H}\mathbf{H}^H\mathbf{W}_{\rm RF}^{\star}\right|$$

Condition (i) and (ii)

$$\stackrel{(a)}{=} \log_2\left|\mathbf{I}+\frac{\alpha_b}{\beta_b}\left(\frac{\sum_{i=1}^{N_u}\lambda_i}{N_{\rm RF}}+\frac{1}{\beta_b\rho}\right)^{-1}\mathbf{W}_{\rm RF_2}^{\star H}\mathbf{\Lambda}_{N_{\rm RF}}\mathbf{W}_{\rm RF_2}^{\star}\right|$$

$$= \sum_{k=1}^{N_u}\log_2\left(1+\frac{\alpha_b\rho N_{\rm RF}\lambda_k}{N_{\rm RF}+(1-\alpha_b)\rho\sum_{i=1}^{N_u}\lambda_i}\right)$$

$$= \sum_{k=1}^{N_u}\log_2\left(1+\frac{\alpha_b\rho N_{\rm RF}\lambda_k/N_r}{\kappa+(1-\alpha_b)\rho\sum_{i=1}^{N_u}\lambda_i/N_r}\right)$$

$$\stackrel{(b)}{\sim} N_u\log_2 N_{\rm RF}, \text{ as } N_{\rm RF}\to\infty.$$

Recall channel model: $\mathbf{h}_k = \sqrt{\frac{N_r}{L_k}}\sum_{\ell=1}^{L_k} g_{\ell,k}\mathbf{a}(\theta_{\ell,k})$

As $N_{\rm RF}\to\infty$ i.e., $N_r\to\infty$, we have:

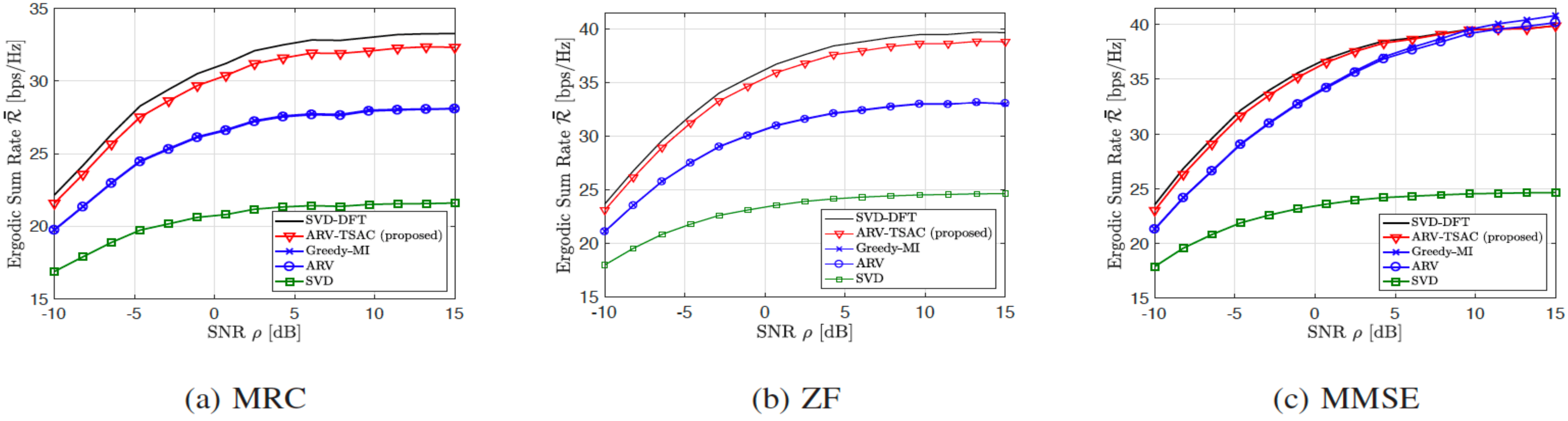
$$\frac{1}{N_r}\mathbf{H}^H\mathbf{H}\to{\rm diag}\{\frac{1}{L_1}\sum_{\ell=1}^{L_1}|g_{\ell,1}|^2,\cdots,\frac{1}{L_{N_u}}\sum_{\ell=1}^{L_{N_u}}|g_{\ell,N_u}|^2\}$$

$\frac{\lambda_i}{N_r}\to\frac{1}{L_i}\sum_{\ell=1}^{L_i}|g_{\ell,i}|^2<\infty$ and $\kappa = N_{\rm RF}/N_r < 1$

SIMULATION RESULTS 3

- Linear digital equalizers
  - N_r = 128, N_{RF} = 43, N_u = 8, b = 2, and # paths = 3

Fig. 14: Rate vs. SNR



- Proposed algorithm provides higher rates than one-stage algorithms
- Proposed algorithm achieves MMSE performance with ZF whereas one-stage algorithms cannot achieve it – one-stage algorithms suffer from large quantization errors

Simple equalizer can achieve high rate with two-stage analog combining

Analog Beamforming in OFDM with Low-Resolution ADCs

- Mutual Information for subcarrier *n*

$$\mathcal{R}_n = \log_2 \left| \mathbf{I}_{N_r} + \rho\alpha_b^2(\alpha_b^2\mathbf{I}_{N_r} + \mathbf{R}_{\mathbf{q}_n\mathbf{q}_n})^{-1}\mathbf{W}_{\mathrm{RF}}^H\mathbf{G}_n\mathbf{G}_n^H\mathbf{W}_{\mathrm{RF}} \right|$$

Quantization noise covariance matrix

$$\begin{aligned}\mathbf{R}_{\mathbf{q}_n\mathbf{q}_n} &= \alpha_b(1-\alpha_b)\mathrm{diag}\{\mathbb{E}[\mathbf{r}_n\mathbf{r}_n^H]\} \\ &= \alpha_b(1-\alpha_b)\mathrm{diag}\{\rho\mathbf{W}_{\mathrm{RF}}^H\mathbf{B}\mathbf{B}^H\mathbf{W}_{\mathrm{RF}} + \mathbf{I}_{N_r}\}\end{aligned}$$

$$\mathbf{B} = [\mathbf{H}_0, \mathbf{0}, \cdots, \mathbf{0}, \mathbf{H}_{L-1}, \cdots, \mathbf{H}_1]$$

Quantization noise covariance matrix

$$\mathbf{G}_n = \sum_{\ell=0}^{L-1}\mathbf{H}_\ell e^{-\frac{j2\pi(n-1)\ell}{N_{\mathrm{sc}}}}$$

Maximize gain by capturing frequency domain channel gain

VS

Minimize quantization error by manipulating time domain delay channels