

Ph.D. Defense

Analysis and Design of Vector Error Diffusion Systems for Image Halftoning

Niranjana Damera-Venkata

Embedded Signal Processing Laboratory

The University of Texas at Austin

Austin TX 78712-1084

Committee Members

Prof. Ross Baldick

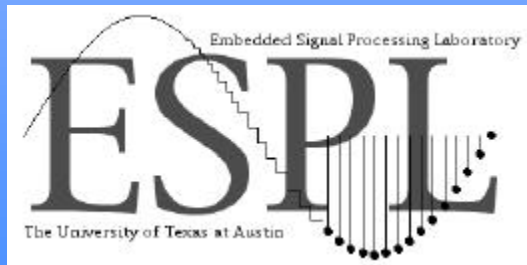
Prof. Alan C. Bovik

Prof. Gustavo de Veciana

Prof. Brian L. Evans (advisor)

Prof. Wilson S. Geisler

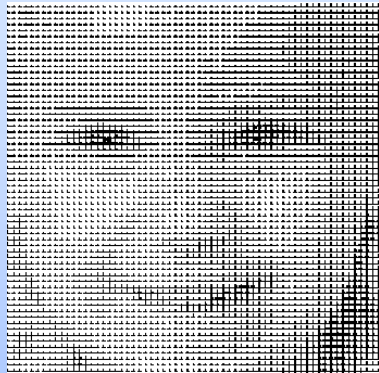
Prof. Joydeep Ghosh



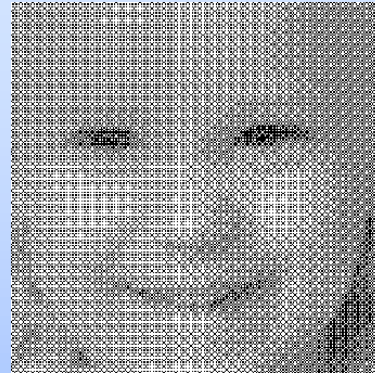
Outline

- **Digital halftoning**
 - Grayscale error diffusion halftoning
 - Color error diffusion halftoning
- ***Contribution #1:* Matrix gain model for color error diffusion**
- ***Contribution #2:* Design of color error diffusion systems**
- ***Contribution #3:* Block error diffusion**
 - Clustered-dot error diffusion halftoning
 - Embedded multiresolution halftoning
- **Contributions**

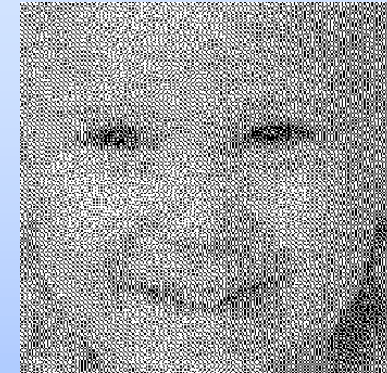
Digital Halftoning



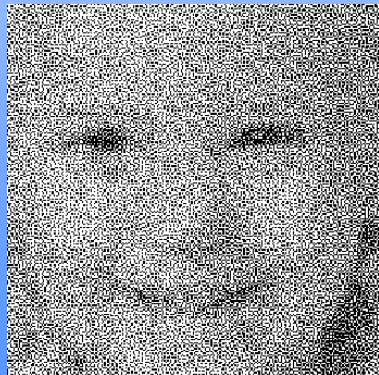
Clutered Dot Dither
AM Halftoning



Dispersed Dot Dither
FM Halftoning



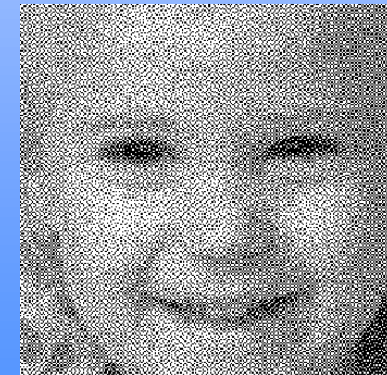
Error Diffusion
FM Halftoning 1975



Blue-noise Mask
FM Halftoning 1993



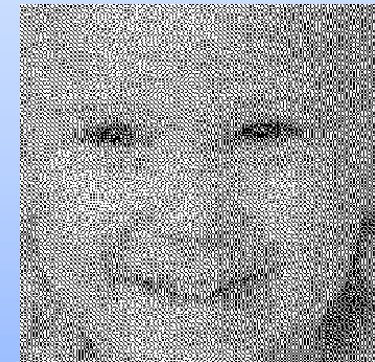
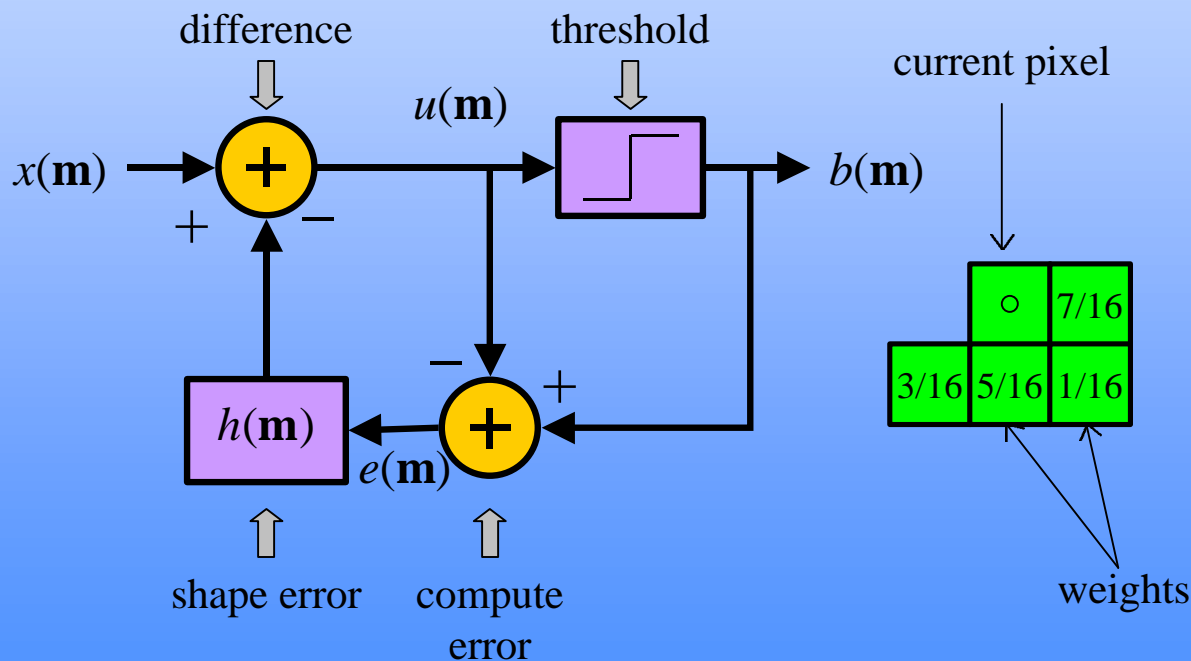
Green-noise Halftoning
AM-FM Halftoning 1992



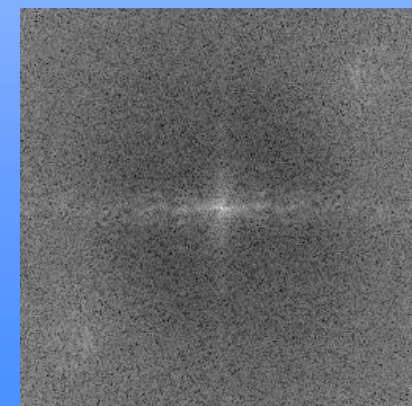
Direct Binary Search
FM Halftoning 1992

Grayscale Error Diffusion

- Shape quantization noise into high frequencies
- Two-dimensional sigma-delta modulation
- Design of error filter is key to high quality



Error Diffusion



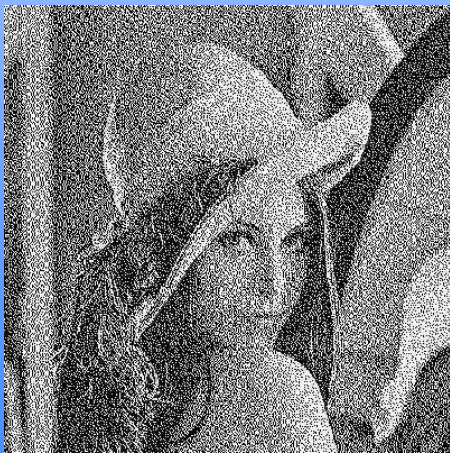
Spectrum

Modeling Grayscale Error Diffusion

- Sharpening is caused by a correlated error image [Knox, 1992]



Floyd-
Steinberg



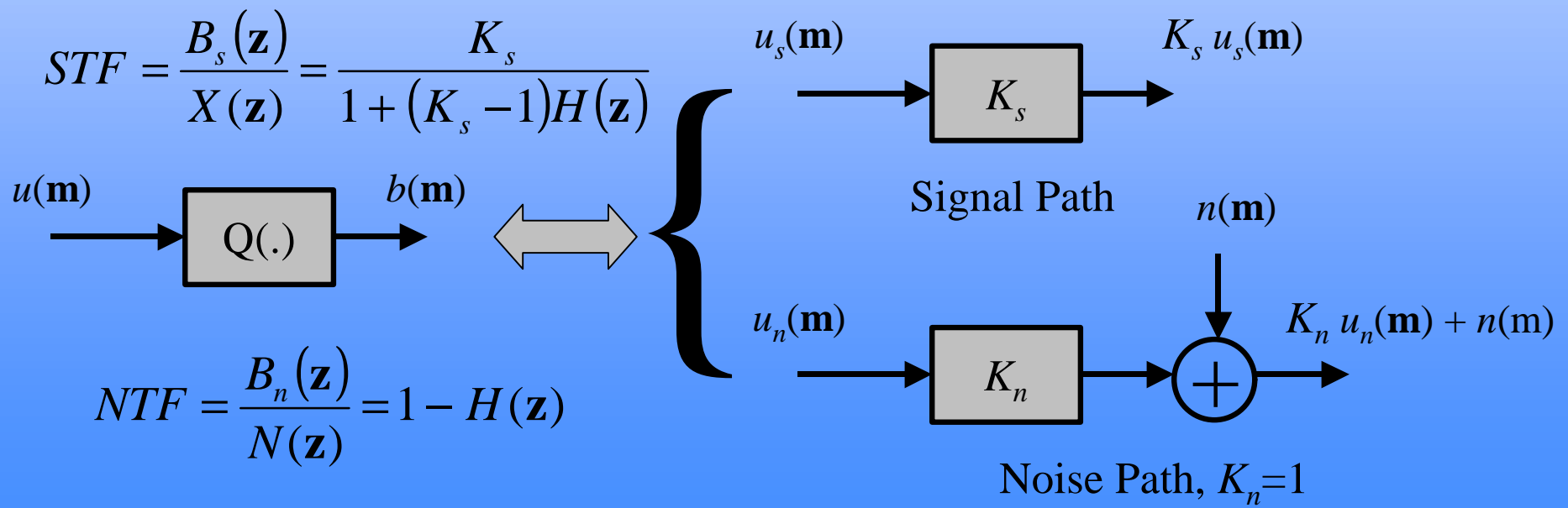
Jarvis

Error images

Halftones

Modeling Grayscale Error Diffusion

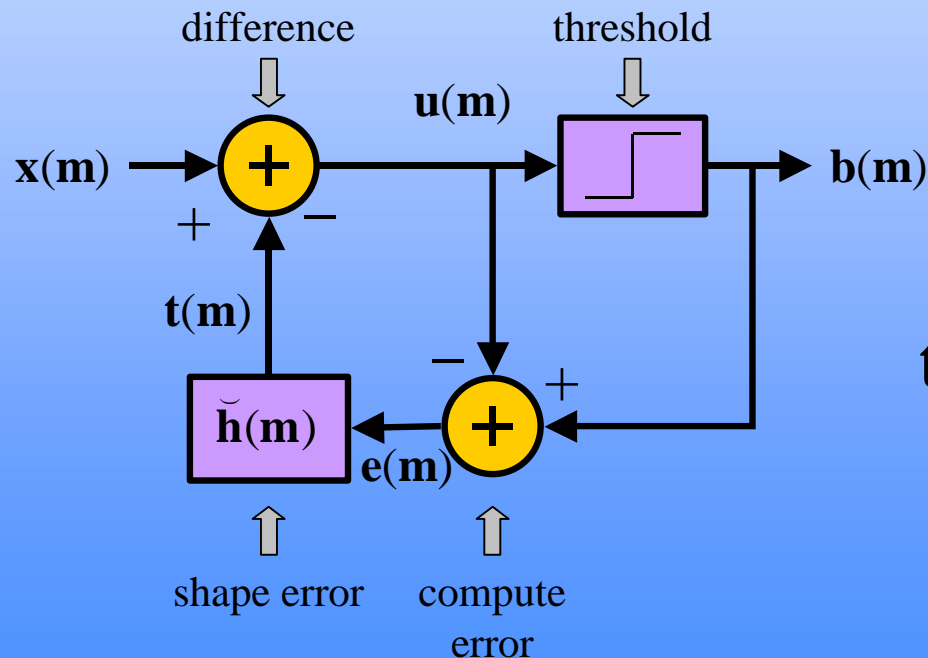
- Apply sigma-delta modulation analysis to two dimensions
 - Linear gain model for quantizer in 1-D [Ardalan and Paulos, 1988]
 - Linear gain model for grayscale image [Kite, Evans, Bovik, 2000]
 - Signal transfer function (STF) and noise transfer function (NTF)
 - $1 - H(z)$ is highpass so $H(z)$ is lowpass



Vector Color Error Diffusion

- Error filter has matrix-valued coefficients
- Algorithm for adapting matrix coefficients

[Akarun, Yardimci, Cetin 1997]



$$t(m) = \sum_{k \in \mathcal{N}} \underbrace{\tilde{h}(k)}_{\text{matrix}} \underbrace{e(m-k)}_{\text{vector}}$$

Color Error Diffusion

- **Open issues**
 - Modeling of color error diffusion in the frequency domain
 - Designing robust fixed matrix-valued error filters
 - Efficient implementation
 - Linear model for the human visual system for color images
- **Contributions**
 - “Matrix gain model” for linearizing color error diffusion
 - Model-based error filter design
 - Parallel implementation of the error filter as a filter bank

Contribution #1:

The Matrix Gain Model

- Replace scalar gain with a matrix

$$\check{\mathbf{K}}_s = \arg \min_{\check{\mathbf{A}}} E \left[\left\| \mathbf{b}(\mathbf{m}) - \check{\mathbf{A}} \mathbf{u}(\mathbf{m}) \right\|^2 \right] = \check{\mathbf{C}}_{bu} \check{\mathbf{C}}_{uu}^{-1}$$

$$\check{\mathbf{K}}_n = \check{\mathbf{I}}$$

$\mathbf{u}(\mathbf{m})$ quantizer input

$\mathbf{b}(\mathbf{m})$ quantizer output

- Noise is uncorrelated with signal component of quantizer input
- Convolution becomes matrix–vector multiplication in frequency domain

$$\mathbf{B}_n(\mathbf{z}) = (\check{\mathbf{I}} - \check{\mathbf{H}}(\mathbf{z})) \mathbf{N}(\mathbf{z})$$

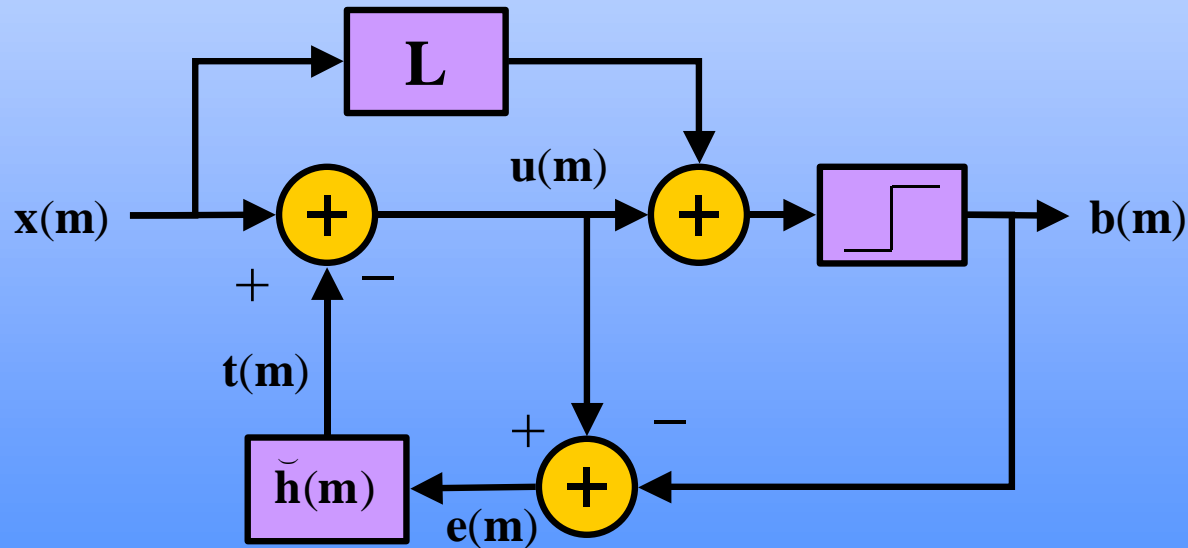
Noise component of output

$$\mathbf{B}_s(\mathbf{z}) = \check{\mathbf{K}} (\check{\mathbf{I}} + \check{\mathbf{H}}(\mathbf{z})(\check{\mathbf{K}} - \check{\mathbf{I}}))^{-1} \mathbf{X}(\mathbf{z})$$

Signal component of output

How to Construct an Undistorted Halftone

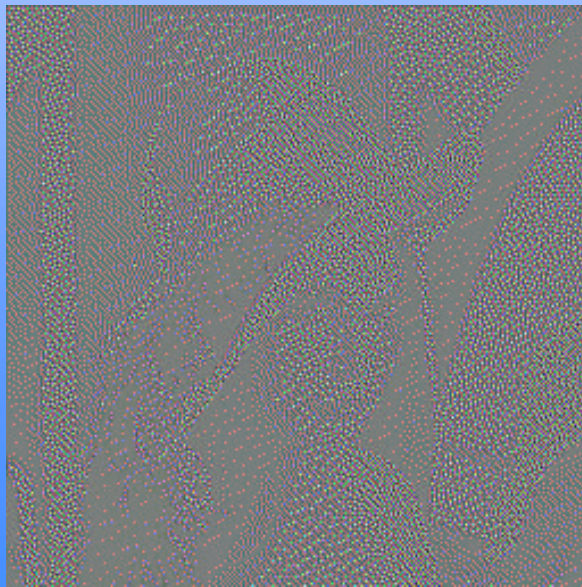
- Pre-filter with inverse of signal transfer function to obtain undistorted halftone $\check{G}(z) = [\check{I} + \check{H}(z)(\check{K} - \check{I})]\check{K}^{-1}$
- Pre-filtering is equivalent to the following when $\check{L} = \check{K}^{-1} - \check{I}$



Modified error diffusion

Validation #1 by Constructing Undistorted Halftone

- **Generate linearly undistorted halftone**
- **Subtract original image from halftone**
- **Since halftone should be “undistorted”, the residual should be uncorrelated with the original**

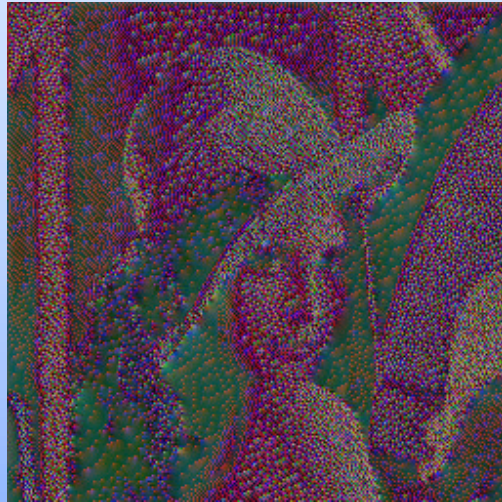


Correlation matrix of residual image (undistorted halftone minus input image) with the input image

$$\tilde{\mathbf{C}}_{\mathbf{rx}} = \begin{pmatrix} 0.0006 & 0.0097 & 0.0020 \\ 0.0013 & 0.0114 & 0.0073 \\ 0.0044 & 0.0024 & 0.0043 \end{pmatrix}$$

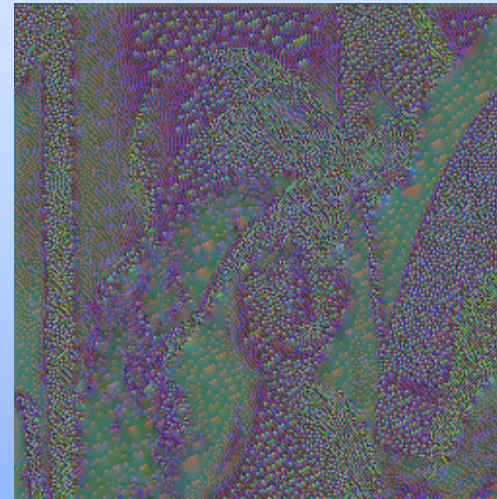
Contribution #1: Matrix Gain Model

Validation #2 by Knox's Conjecture



Correlation matrix for an error image and input image for an error diffused halftone

$$\check{C}_{\text{ex}} = \begin{pmatrix} 0.3664 & 0.0019 & 0.1778 \\ 0.2661 & 0.2348 & 0.1817 \\ 0.2173 & 0.1839 & 0.1816 \end{pmatrix}$$



Correlation matrix for an error image and input image for an undistorted halftone

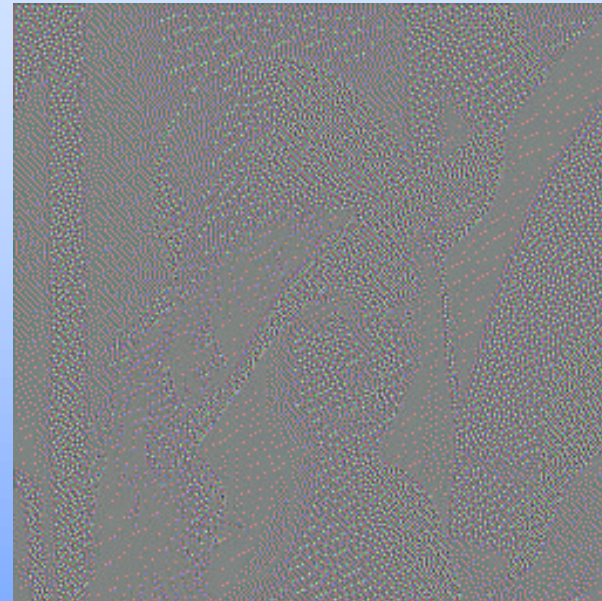
$$\check{C}_{\text{ex}} = \begin{pmatrix} 0.0485 & 0.0839 & 0.0082 \\ 0.0153 & 0.0550 & 0.0229 \\ 0.0160 & 0.0004 & 0.0247 \end{pmatrix}$$

$$\mathbf{E}_s(\mathbf{z}) = 0$$

$$\mathbf{E}_n(\mathbf{z}) = \mathbf{N}(\mathbf{z})$$

Validation #3 by Distorting Original Image

- **Validation by constructing a linearly distorted original**
 - Pass original image through error diffusion with matrix gain substituted for quantizer
 - Subtract resulting color image from color halftone
 - Residual should be shaped uncorrelated noise

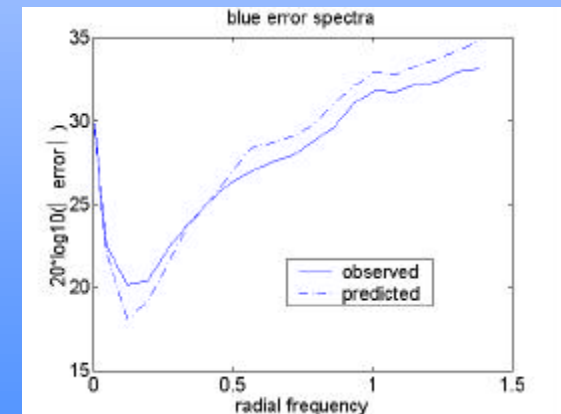
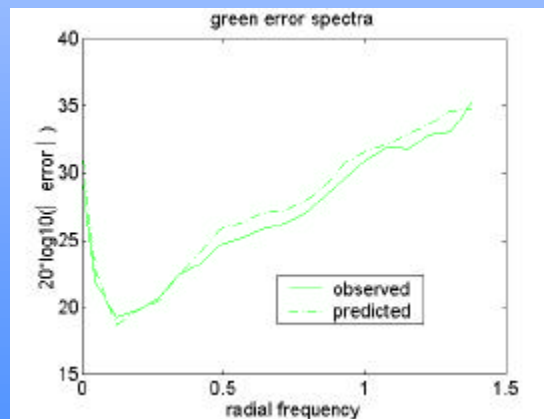
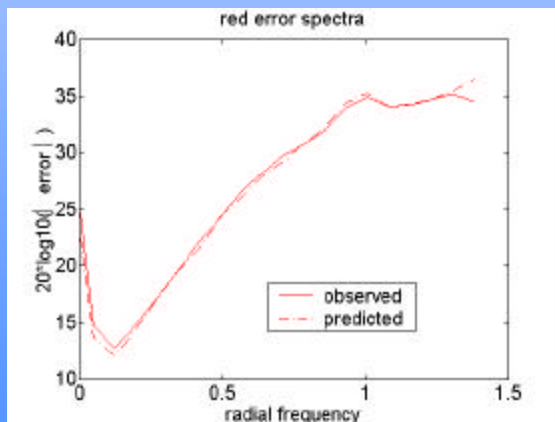


Correlation matrix of residual image (halftone minus distorted input image) with the input image

$$\check{C}_{rx} = \begin{pmatrix} 0.0012 & 0.0140 & 0.0004 \\ 0.0007 & 0.0057 & 0.0126 \\ 0.0015 & 0.0101 & 0.0056 \end{pmatrix}$$

Validation #4 by Noise Shaping

- Noise process is error image for an undistorted halftone
- Use model noise transfer function to compute noise spectrum
- Subtract original image from modeled halftone and compute actual noise spectrum



Designing of the Error Filter

- Eliminate linear distortion filtering before error diffusion
- Optimize error filter $\mathbf{h}(\mathbf{m})$ for noise shaping

$$\min E \left[\left\| \mathbf{b}_n(\mathbf{m}) \right\|^2 \right] = E \left[\left\| \check{\mathbf{v}}(\mathbf{m}) * (\check{\mathbf{I}} - \check{\mathbf{h}}(\mathbf{m})) * \mathbf{n}(\mathbf{m}) \right\|^2 \right]$$

Subject to diffusion constraints

$$\left(\sum_{\mathbf{m}} \check{\mathbf{h}}(\mathbf{m}) \right) \mathbf{1} = \mathbf{1}$$

where

$\check{\mathbf{v}}(\mathbf{m})$ linear model of human visual system
* matrix-valued convolution

Generalized Optimum Solution

- Differentiate scalar objective function for visual noise shaping with respect to matrix-valued coefficients

$$\frac{d\left\{E\left[\|\mathbf{b}_n(\mathbf{m})\|^2\right]\right\}}{d\mathbf{h}(\mathbf{i})} = \mathbf{0} \quad \forall \mathbf{i} \in \mathcal{S}$$

- Write the norm as a trace and then differentiate the trace using identities from linear algebra $\|\mathbf{x}\|^2 = \text{Tr}(\mathbf{x}\mathbf{x}')$

$$\frac{d\left\{\text{Tr}(\tilde{\mathbf{A}}\tilde{\mathbf{X}})\right\}}{d\tilde{\mathbf{X}}} = \tilde{\mathbf{A}}' \qquad \frac{d\left\{\text{Tr}(\tilde{\mathbf{X}}'\tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}})\right\}}{d\tilde{\mathbf{X}}} = \tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}} + \tilde{\mathbf{A}}'\tilde{\mathbf{X}}\tilde{\mathbf{B}}'$$

$$\frac{d\left\{\text{Tr}(\tilde{\mathbf{A}}\tilde{\mathbf{X}}\tilde{\mathbf{B}})\right\}}{d\tilde{\mathbf{X}}} = \tilde{\mathbf{A}}'\tilde{\mathbf{B}}' \qquad \text{Tr}(\tilde{\mathbf{A}}\tilde{\mathbf{B}}) = \text{Tr}(\tilde{\mathbf{B}}\tilde{\mathbf{A}})$$

Generalized Optimum Solution (cont.)

- Differentiating and using linearity of expectation operator give a generalization of the Yule-Walker equations

$$\sum_{\mathbf{k}} \check{\mathbf{v}}'(\mathbf{k}) \check{\mathbf{r}}_{\text{an}}(-\mathbf{i} - \mathbf{k}) = \sum_{\mathbf{p}} \sum_{\mathbf{q}} \sum_{\mathbf{s}} \check{\mathbf{v}}'(\mathbf{s}) \check{\mathbf{v}}(\mathbf{q}) \check{\mathbf{h}}(\mathbf{p}) \check{\mathbf{r}}_{\text{nn}}(-\mathbf{i} - \mathbf{s} + \mathbf{p} + \mathbf{q})$$

where

$$\mathbf{a}(\mathbf{m}) = \check{\mathbf{v}}(\mathbf{m}) * \mathbf{n}(\mathbf{m})$$

- Assuming white noise injection

$$\mathbf{r}_{\text{nn}}(\mathbf{k}) = E[\mathbf{n}(\mathbf{m}) \mathbf{n}'(\mathbf{m} + \mathbf{k})] \approx \mathbf{d}(\mathbf{k})$$

$$\mathbf{r}_{\text{an}}(\mathbf{k}) = E[\mathbf{a}(\mathbf{m}) \mathbf{n}'(\mathbf{m} + \mathbf{k})] \approx \check{\mathbf{v}}(-\mathbf{k})$$

Generalized Optimum Solution (cont.)

- Optimum solution obtained via steepest descent algorithm

$$(\nabla J)_{(\check{\mathbf{h}}(\mathbf{i}))} = -\sum_{\mathbf{k}} \check{\mathbf{v}}'(\mathbf{k}) \check{\mathbf{r}}_{\text{an}}(-\mathbf{i}-\mathbf{k}) + \sum_{\mathbf{p}} \sum_{\mathbf{q}} \sum_{\mathbf{s}} \check{\mathbf{v}}'(\mathbf{s}) \check{\mathbf{v}}(\mathbf{q}) \check{\mathbf{h}}(\mathbf{p}) \check{\mathbf{r}}_{\text{m}}(-\mathbf{i}-\mathbf{s}+\mathbf{p}+\mathbf{q})$$

$$\check{\mathbf{h}}^{(q+1)}(\mathbf{i}) = \mathbf{P} \left(\check{\mathbf{h}}^{(q)}(\mathbf{i}) - \mathbf{a} \left\{ (\nabla J)_{(\check{\mathbf{h}}^{(q)}(\mathbf{i}))} \right\} \right)$$

$$\mathbf{P}(\check{\mathbf{f}}(\mathbf{i})) = \check{\mathbf{f}}(\mathbf{i}) - \left(\frac{1}{3|\mathcal{S}|} \right) \left(\sum_{\mathbf{m} \in \mathcal{S}} \check{\mathbf{f}}(\mathbf{m}) - \check{\mathbf{I}} \right) \mathbf{1}$$

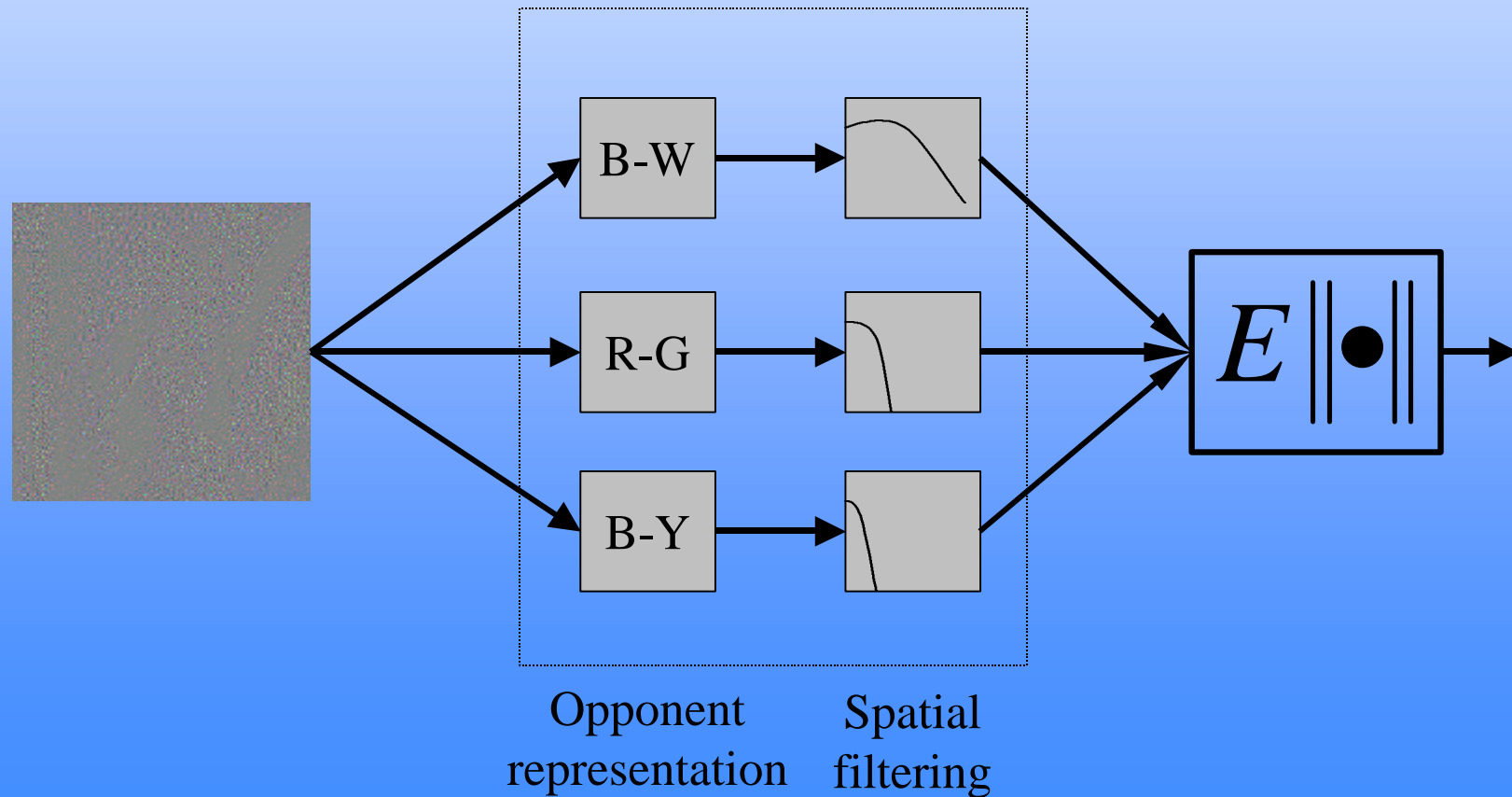
\mathbf{a} - convergence rate parameter

\mathbf{P} - projection operator

q - iteration number

Linear Color Vision Model

- **Pattern-Color separable model** [Poirson and Wandell, 1993]
 - Forms the basis for S-CIELab [Zhang and Wandell, 1996]
 - Pixel-based color transformation

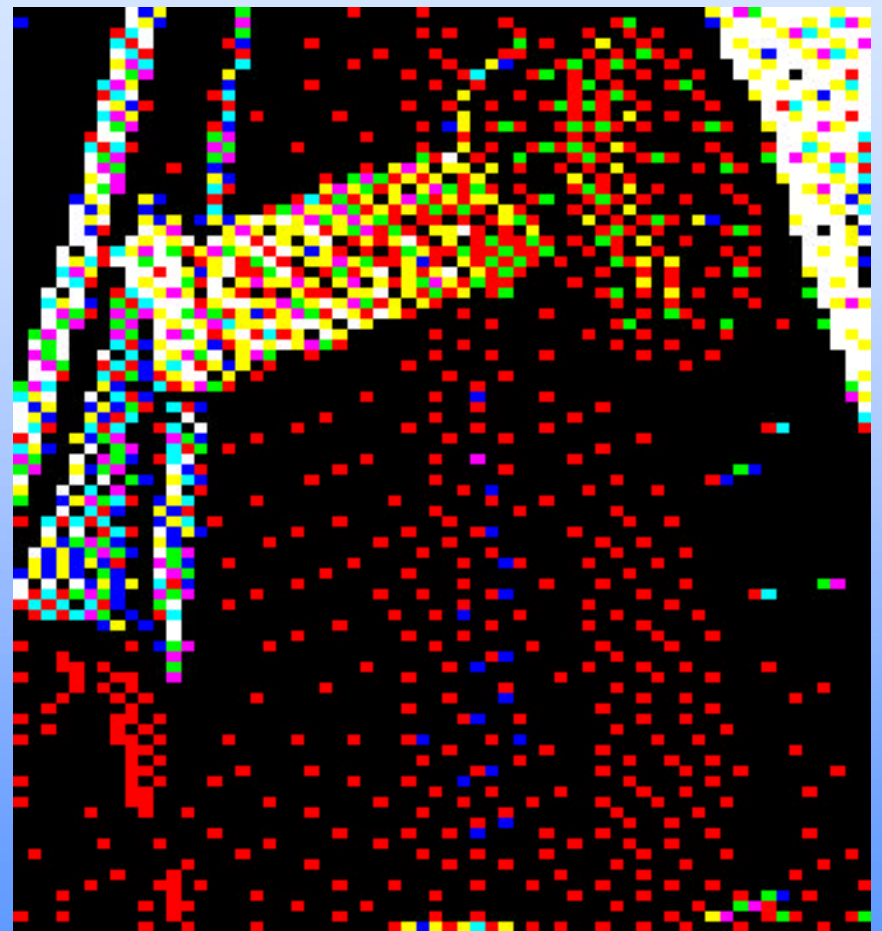


Linear Color Vision Model

- **Undo gamma correction on RGB image**
- **Color separation**
 - Measure power spectral distribution of RGB phosphor excitations
 - Measure absorption rates of long, medium, short (LMS) cones
 - Device dependent transformation \mathbf{C} from RGB to LMS space
 - Transform LMS to opponent representation using \mathbf{O}
 - Color separation may be expressed as $\mathbf{T} = \mathbf{OC}$
- **Spatial filtering is incorporated using matrix filter $\check{\mathbf{d}}(\mathbf{m})$**
- **Linear color vision model**
 $\check{\mathbf{v}}(\mathbf{m}) = \check{\mathbf{d}}(\mathbf{m})\check{\mathbf{T}}$ where $\check{\mathbf{d}}(\mathbf{m})$ is a diagonal matrix



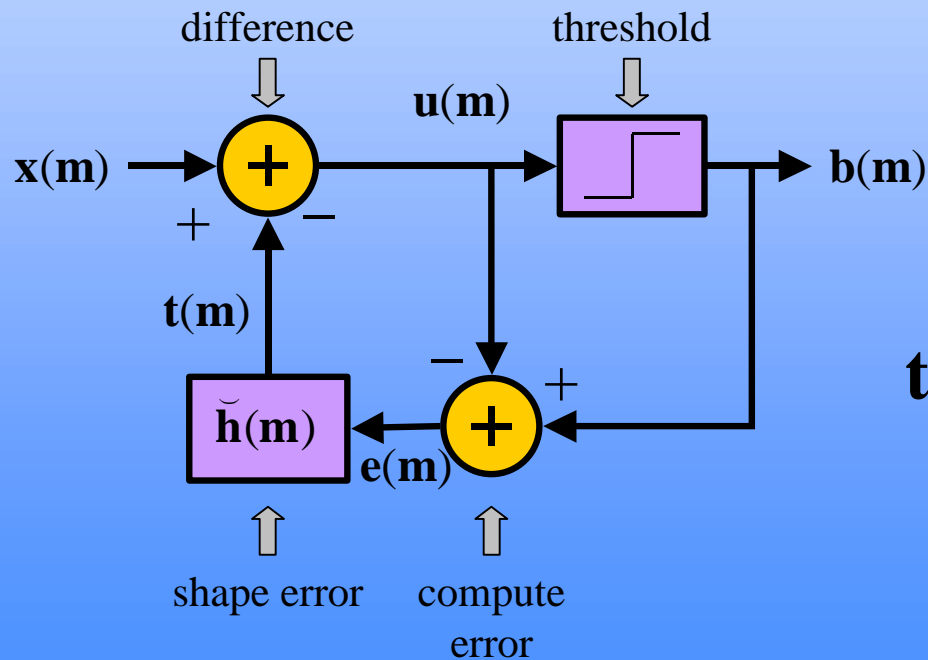
Floyd-Steinberg



Optimum Filter

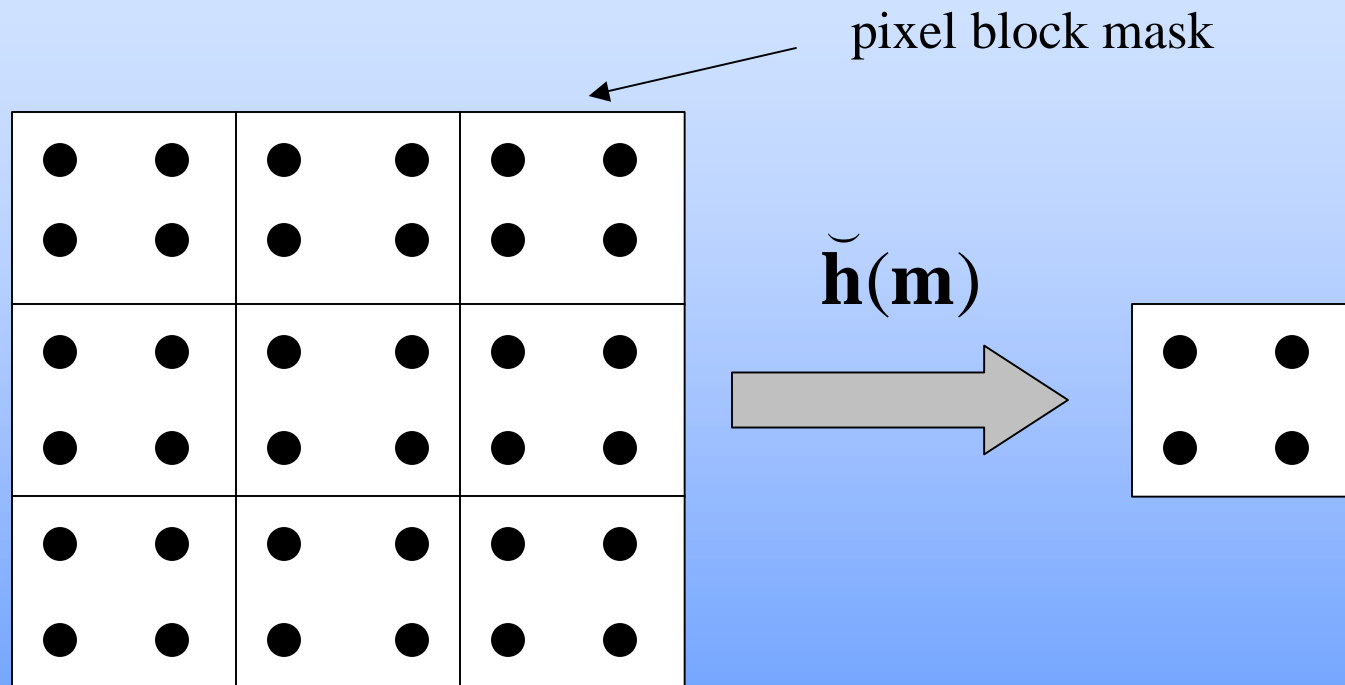
Block Error Diffusion

- Input grayscale image is “blocked”
- Error filter uses all samples from neighboring blocks and diffuses an error block



$$t(m) = \sum_{k \in S} \tilde{h}(k) e(m - k)$$

Block Interpretation of Vector Error Diffusion



- **Four linear combinations of the 36 pixels are required to compute the output pixel block**

Contribution #3: Block Error Diffusion
Block FM Halftoning

- **Why not “block” standard error diffusion output?**
 - Spatial aliasing problem
 - Blurred appearance due to prefiltering
- **Solution**
 - Control dot shape using block error diffusion
 - Extend conventional error diffusion in a natural way
- **Extensions to block error diffusion**
 - AM-FM halftoning
 - Sharpness control
 - Multiresolution halftone embedding
 - Fast parallel implementation

Block FM Halftoning Error Filter Design

- Start with conventional error filter prototype

$$\tilde{\mathbf{a}} = \begin{bmatrix} \frac{1}{16} & \frac{5}{16} & \frac{3}{16} & \frac{7}{16} \end{bmatrix}$$

- Form block error filter as Kronecker product

$$\check{\Gamma} = \tilde{\mathbf{a}} \otimes \check{\mathbf{D}} \quad \check{\mathbf{D}} \text{ diffusion matrix}$$

- Satisfies “lossless” diffusion constraint

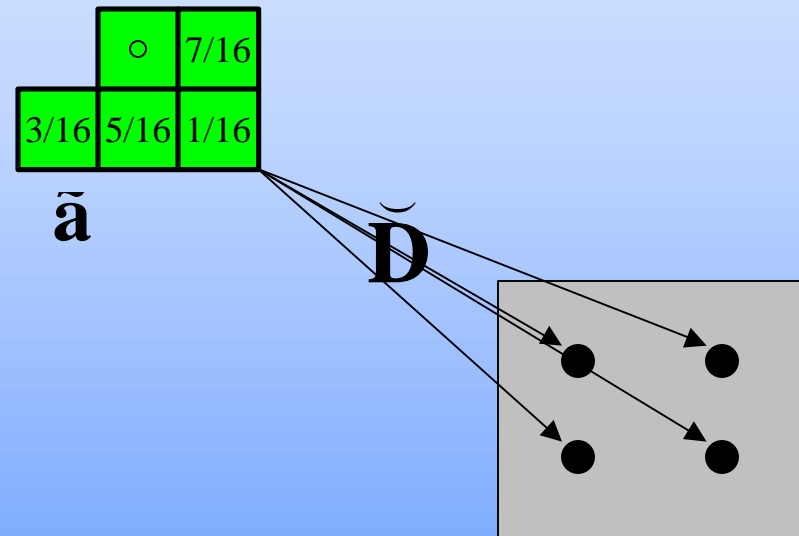
$$\check{\Gamma} \mathbf{1} = \mathbf{1} \quad \check{\Gamma} \geq \check{\mathbf{0}}$$

- Diffusion matrix satisfies

$$\check{\mathbf{D}} \mathbf{1} = \mathbf{1} \quad \check{\mathbf{D}} \geq \check{\mathbf{0}}$$

Block FM Halftoning Error Filter Design

- **FM nature of algorithm controlled by scalar filter prototype**
- **Diffusion matrix decides distribution of error within a block**
- **In-block diffusions are constant for all blocks to preserve isotropy**



Contribution #3: Block Error Diffusion

Block FM Halftoning Results

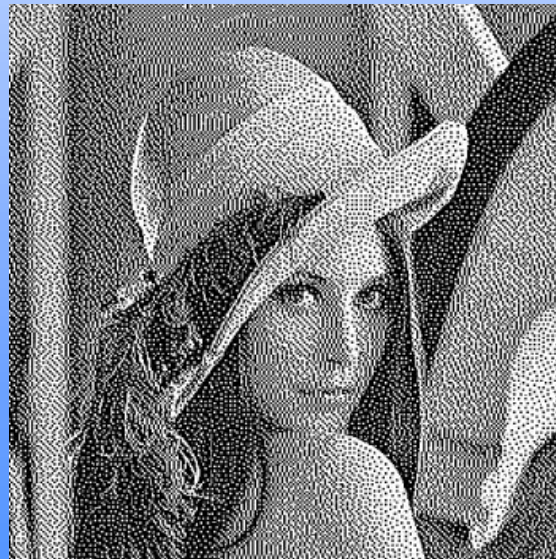
- **Vector error diffusion with diffusion matrix**

$$\check{\mathbf{D}} = \frac{1}{N^2} \begin{bmatrix} \check{\mathbf{1}} \end{bmatrix}$$

N is the block size



Pixel replication



Floyd-Steinberg



Jarvis

Contribution #3: Block Error Diffusion

Block FM Halftoning with Arbitrary Shapes



Plus dots



Cross dots

Embedded Multiresolution Halftoning

- **Only involves designing the diffusion matrix**
 - FM Halftones when downsampled are also FM halftones

LMH	H	MH	H
H	H	H	H
MH	H	MH	H
H	H	H	H

Halftone pixels at Low, Medium and High resolutions

- **Error at a pixel is diffused to the pixels of the same color**

Contribution #3: Block Error Diffusion

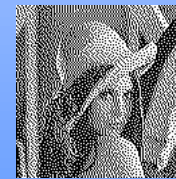
Embedded Halftoning Results



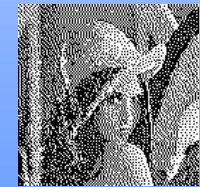
High
resolution
halftone



Medium
resolution
halftone



Low
resolution
halftone



Simple
down-
sampling

Contributions

- **Matrix gain model for vector color error diffusion**
 - Eliminated linear distortion by pre-filtering
 - Validated model in three other ways
- **Model based error filter design for a calibrated device**
- **Block error diffusion**
 - FM halftoning
 - AM-FM halftoning (not presented)
 - Embedded multiresolution halftoning
- **Efficient parallel implementation (not presented)**

Published Halftoning Work Not in Dissertation

N. Damera-Venkata and B. L. Evans, "Adaptive Threshold Modulation for Error Diffusion Halftoning," *IEEE Transactions on Image Processing*, January 2001, to appear.

T. D. Kite, N. Damera-Venkata, B. L. Evans and A. C. Bovik, "A Fast, High Quality Inverse Halftoning Algorithm for Error Diffused Halftoned images," *IEEE Transactions on Image Processing*, vol. 9, no. 9, pp. 1583-1593, September 2000.

N. Damera-Venkata, T. D. Kite, W. S. Geisler, B. L. Evans and A. C. Bovik, "Image Quality Assessment Based on a Degradation Model" *IEEE Transactions on Image Processing*, vol. 9, no. 4, pp. 636-651, April 2000.

N. Damera-Venkata, T. D. Kite, M. Venkataraman, B. L. Evans, "Fast Blind Inverse Halftoning" *IEEE Int. Conf. on Image Processing*, vol. 2, pp. 64-68, Oct. 4-7, 1998.

T. D. Kite, N. Damera-Venkata, B. L. Evans and A. C. Bovik, "A High Quality, Fast Inverse Halftoning Algorithm for Error Diffused Halftoned images," *IEEE Int. Conf. on Image Processing*, vol. 2, pp. 64-68, Oct. 4-7, 1998.

Submitted Halftoning Work in Dissertation

N. Damera-Venkata and B. L. Evans, "Matrix Gain Model for Vector Color Error Diffusion," *IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing*, June 3-5, 2001, to appear.

N. Damera-Venkata and B. L. Evans, "Design and Analysis of Vector Color Error Diffusion Systems," *IEEE Transactions on Image Processing*, submitted.

N. Damera-Venkata and B. L. Evans, "Clustered-dot FM Halftoning Via Block Error Diffusion," *IEEE Transactions on Image Processing*, submitted.

Types of Halftoning Algorithms

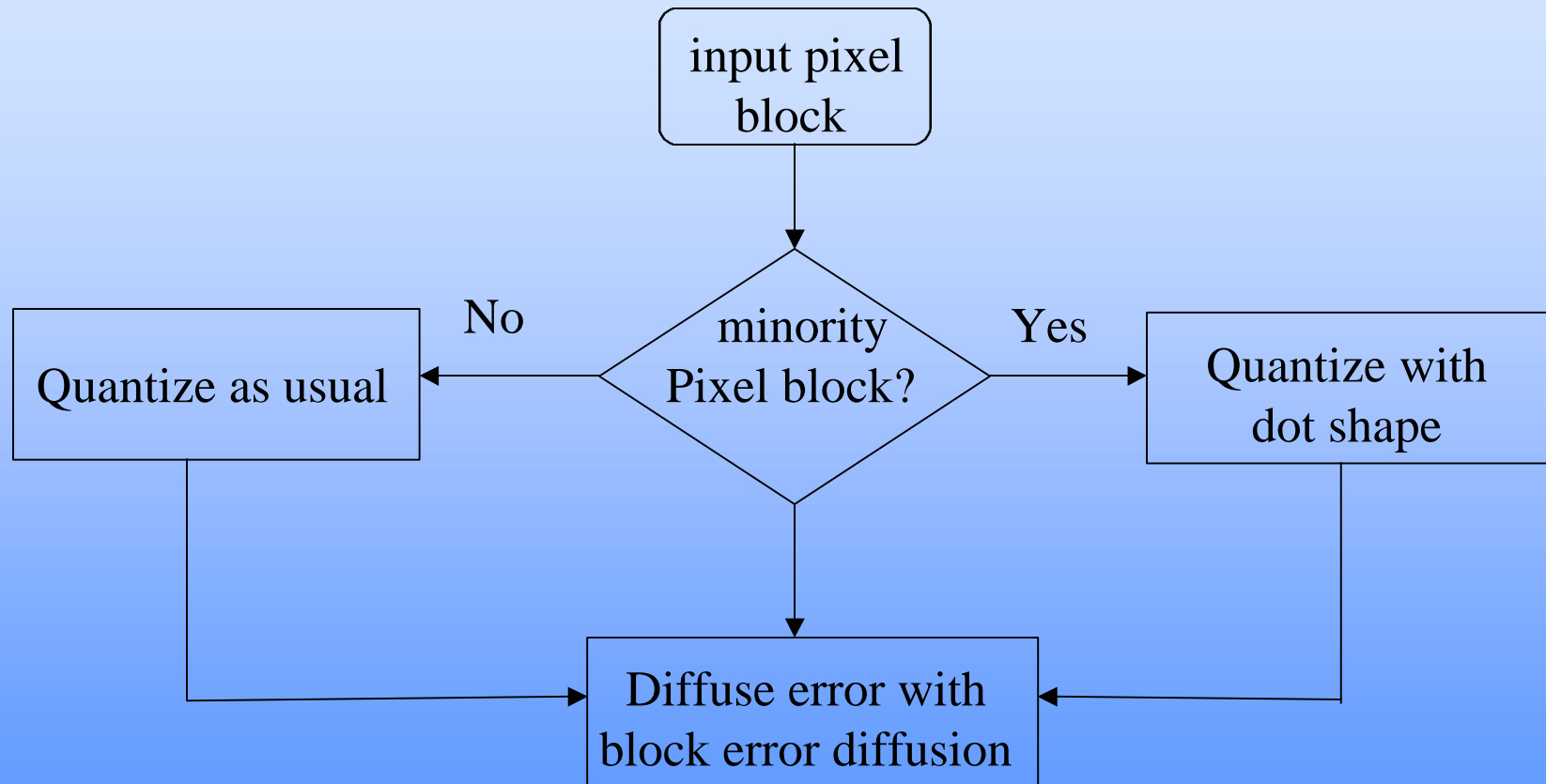
- **AM halftoning**
 - Vary dot size according to underlying graylevel
 - Clustered dot dither is a typical example (laserjet printers)
- **FM halftoning**
 - Vary dot frequency according to underlying graylevel
 - Error diffusion is typical example (inkjet printers)
- **AM-FM halftoning**
 - Vary dot size and frequency
 - Typical example is Levien's "green-noise" algorithm [Levien 1993]

Designing Error Filter in Scalar Error Diffusion

- **Floyd-Steinberg error filter** [Floyd and Steinberg, 1975]
- **Optimize weighted error**
 - Assume error image is white noise [Kolpatzik and Bouman, 1992]
 - Use statistics of error image [Wong and Allebach, 1997]
- **Adaptive methods**
 - Adapt error filter coefficients to minimize *local* weighted mean squared error [Wong, 1996]

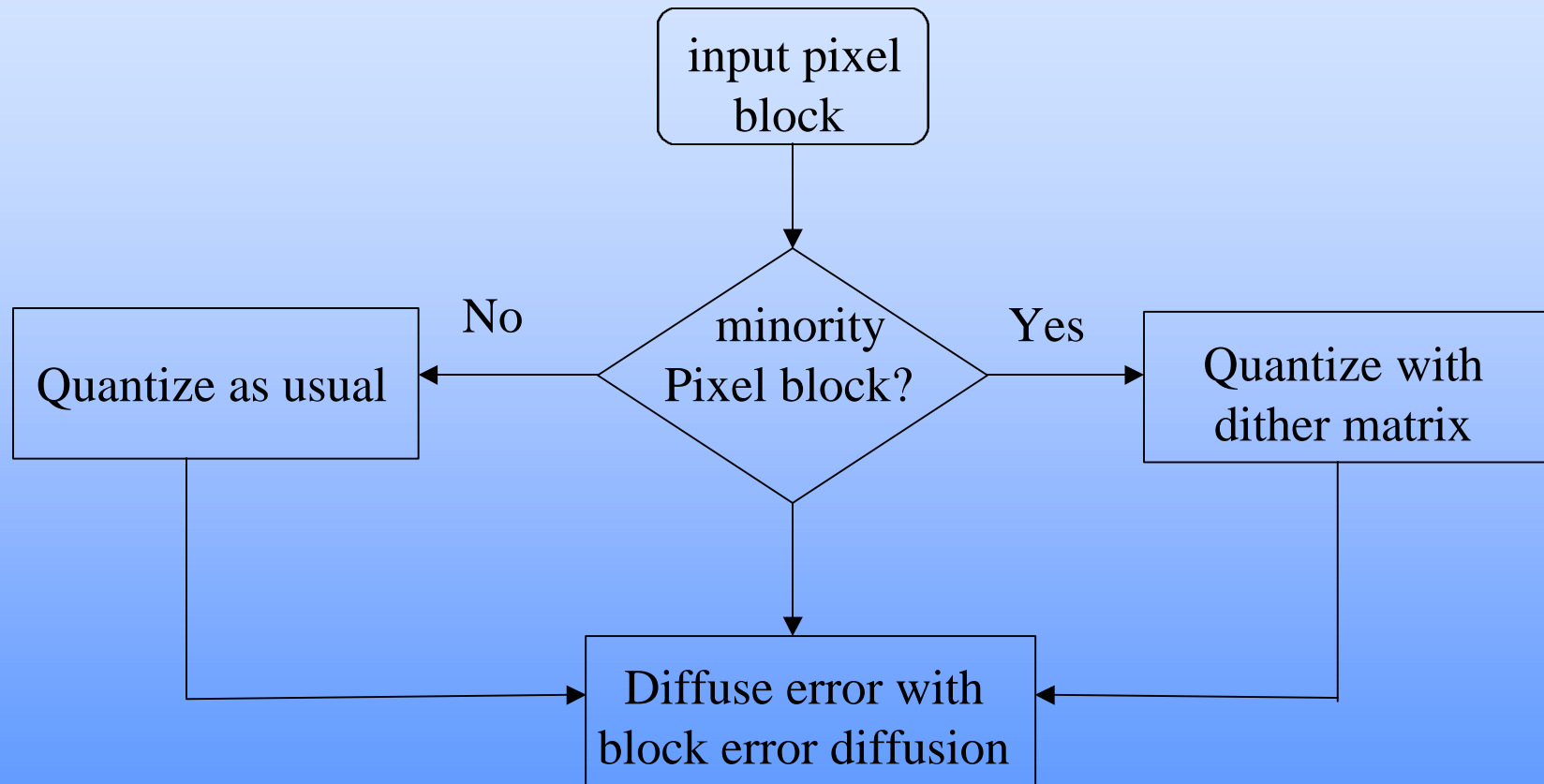
Contribution #3: Block Error Diffusion

FM Halftoning with Arbitrary Dot Shape

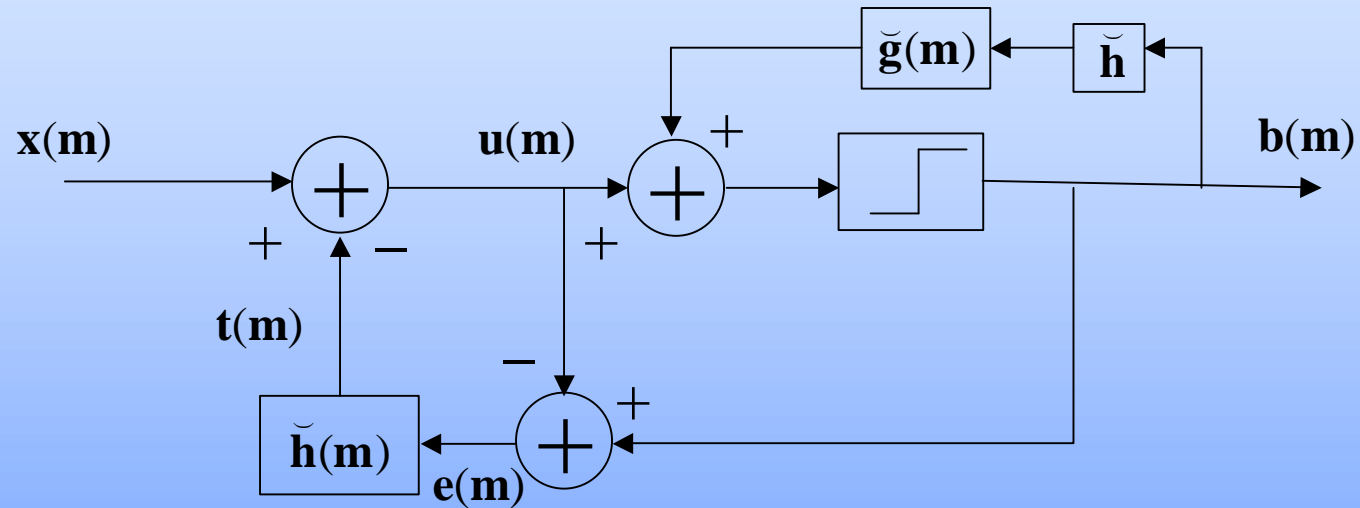


Contribution #3: Block Error Diffusion

AM-FM Halftoning with User-controlled Dot Shape



AM-FM Halftoning with User-controlled Dot Size



Block green noise error diffusion

- Promotes pixel-block clustering into super-pixel blocks

Contribution #3: Block Error Diffusion

AM-FM Halftoning Results

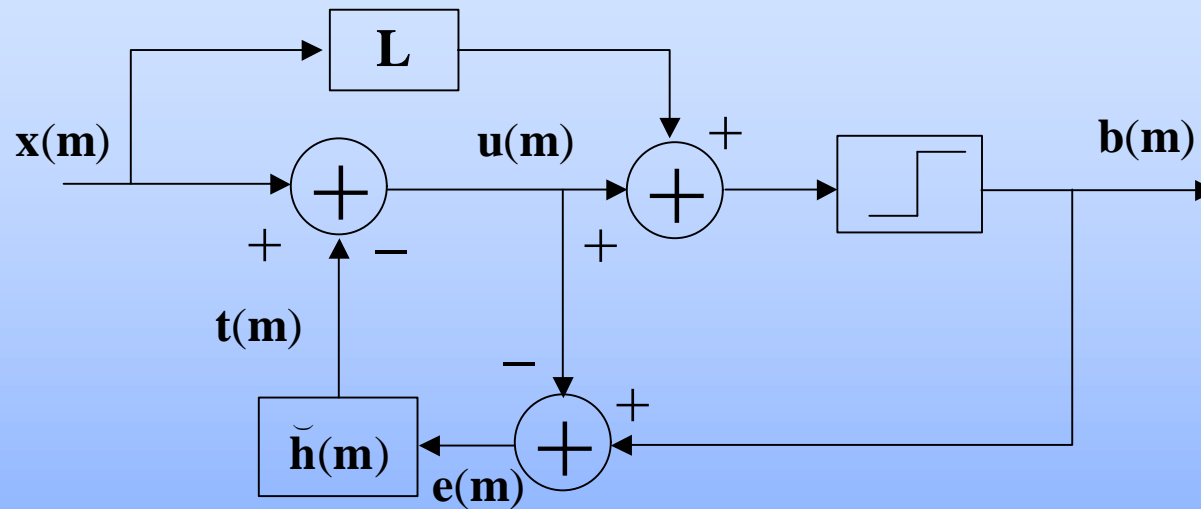


Clustered dot dither
modulation



Output dependent
feedback

Block FM Halftoning with Sharpness Control



Modified error diffusion

- The above block diagram is equivalent to prefiltering with

$$\check{G}_s(z) = [\check{\mathbf{I}} + \check{\mathbf{H}}(z)\{\check{\mathbf{I}} - \check{\mathbf{H}}(z)\}]\check{\mathbf{L}} + \check{\mathbf{I}}$$

Contribution #3: Block Error Diffusion

Block FM Halftoning with Sharpness Control



$$\check{\mathbf{L}} = 0.2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$\check{\mathbf{L}} = 0.6 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Contribution #3: Block Error Diffusion

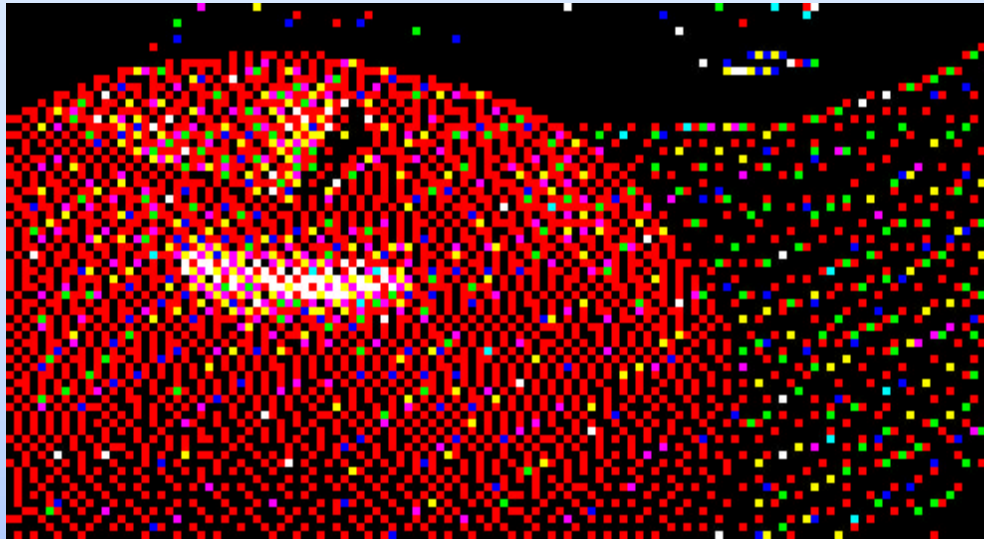
Diffusion Matrix for Embedding

$$\mathbf{l} = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)'$$

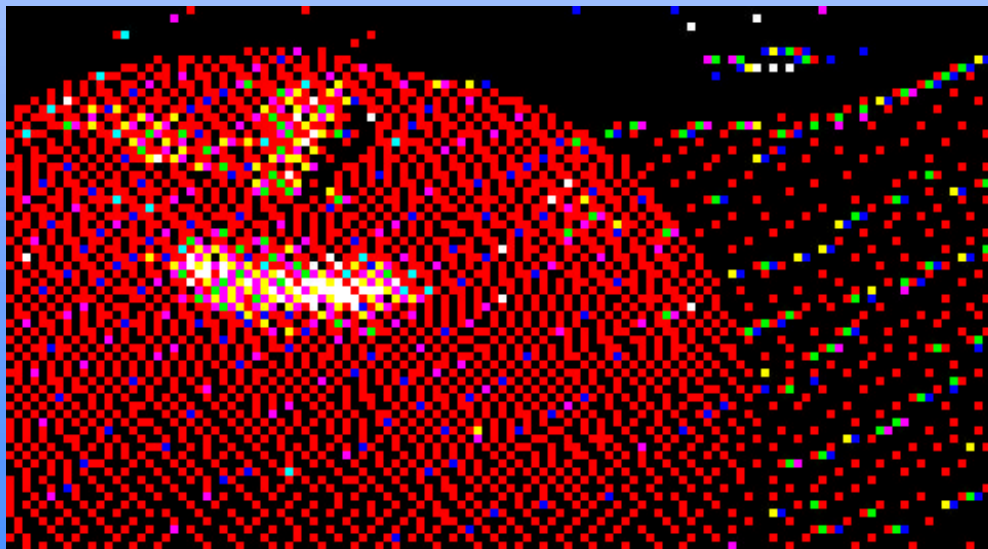
$$\mathbf{m} = \left(0 \ 0 \ \frac{1}{3} \ 0 \ 0 \ 0 \ 0 \ 0 \ \frac{1}{3} \ 0 \ \frac{1}{3} \ 0 \ 0 \ 0 \ 0 \ 0 \right)'$$

$$\mathbf{h} = \left(0 \ \frac{1}{12} \ 0 \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ 0 \ \frac{1}{12} \ 0 \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \ \frac{1}{12} \right)'$$

$$\check{\mathbf{D}} = (\mathbf{l} \ \mathbf{h} \ \mathbf{m} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h} \ \mathbf{m} \ \mathbf{h} \ \mathbf{m} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h} \ \mathbf{h})'$$



Floyd-Steinberg

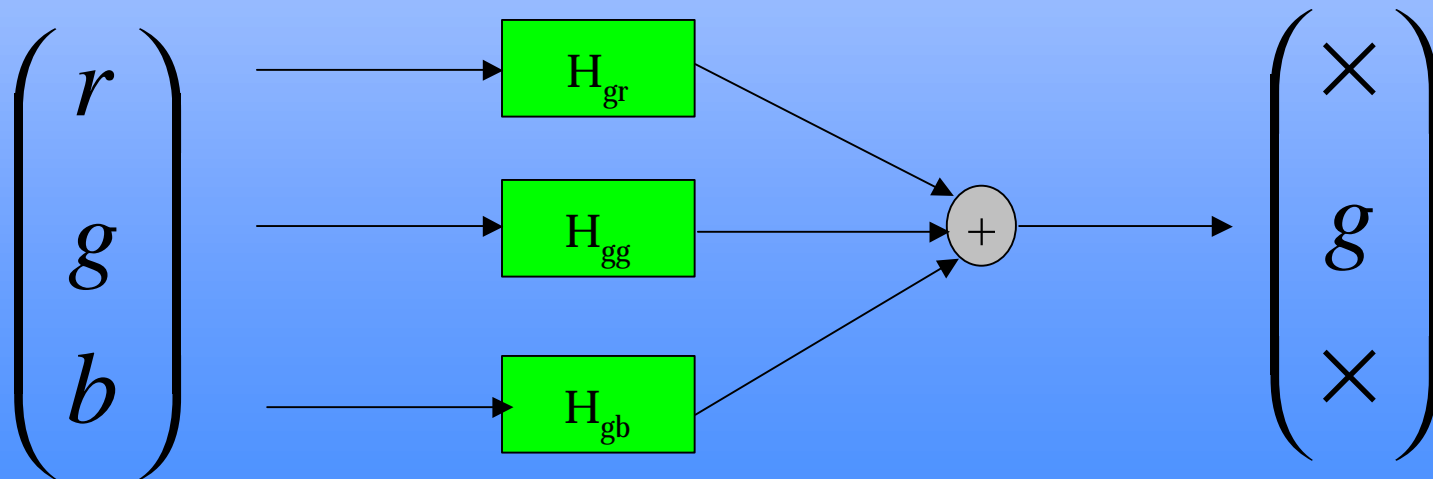


Optimum Filter

Contribution #4:

Implementation of Vector Color Error Diffusion

$$\check{\mathbf{H}}(\mathbf{z}) = \begin{pmatrix} H_{rr}(\mathbf{z}) & H_{rg}(\mathbf{z}) & H_{rb}(\mathbf{z}) \\ H_{gr}(\mathbf{z}) & H_{gg}(\mathbf{z}) & H_{gb}(\mathbf{z}) \\ H_{br}(\mathbf{z}) & H_{bg}(\mathbf{z}) & H_{bb}(\mathbf{z}) \end{pmatrix}$$



Contribution #4:

Implementation of Block Error Diffusion

