EE382M.20: System-on-Chip (SoC) Design

Lecture 12 – Operation Scheduling

Source: G. De Micheli, Integrated Systems Center, EPFL
"Synthesis and Optimization of Digital Circuits", McGraw Hill, 2001.
Additional sources:
Notes by Kia Bazargan, http://www.ece.umn.edu/users/kia/Courses/EE5301
Notes by Rajesh Gupta, UCSD, http://www.cecs.uci.edu/~rgupta/ics280.html

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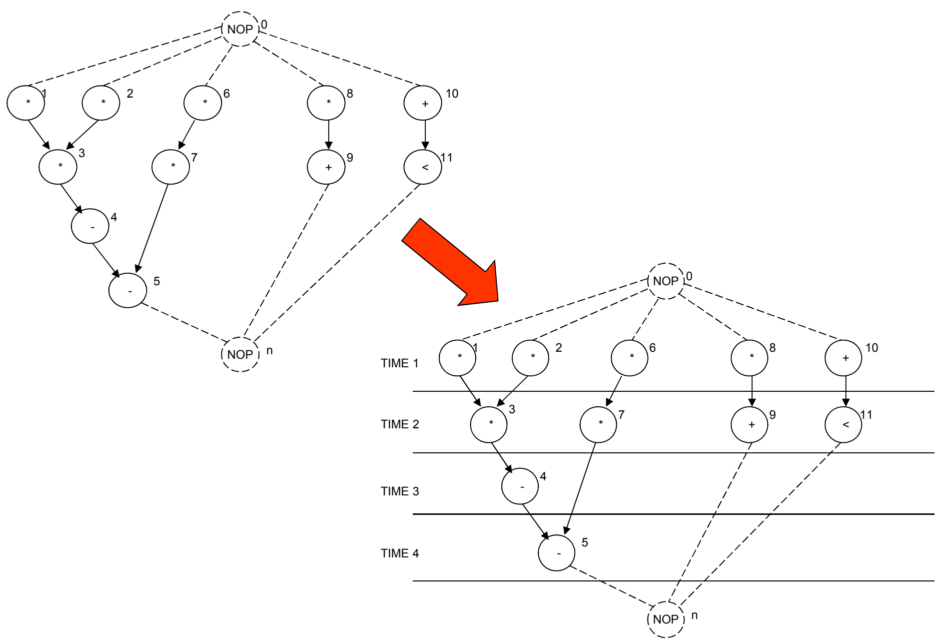
Lecture 12: Outline

- **The scheduling problem**
  - Case analysis
- **Unconstrained scheduling**
  - ASAP and ALAP schedules
- **Resource constrained (RC) scheduling**
  - List scheduling
- **Time constrained (TC) scheduling**
  - Force-directed scheduling
- **Advanced scheduling problems**
  - Chaining
  - Pipelining

Scheduling

- **Circuit model:**
  - Sequencing graph
  - Cycle-time is given
  - Operation delays expressed in cycles
- **Scheduling:**
  - Determine the start times for the operations
  - Satisfying all the sequencing (timing and resource) constraint
- **Goal:**
  - Determine *area/latency* trade-off

Example



Operation Scheduling

- **Input:**
  - Sequencing graph G(V, E), with n vertices
  - Cycle time τ
  - Operation delays $D = \{d_i: i=0..n\}$
- **Output:**
  - Schedule ϕ determines start time t_i of operation v_i.
  - Latency $\lambda = t_n - t_0$.
- **Goal: determine area / latency tradeoff**
- **Classes:**
  - Non-hierarchical and unconstrained
  - Latency constrained
  - Resource constrained
  - Hierarchical

Simplest Method

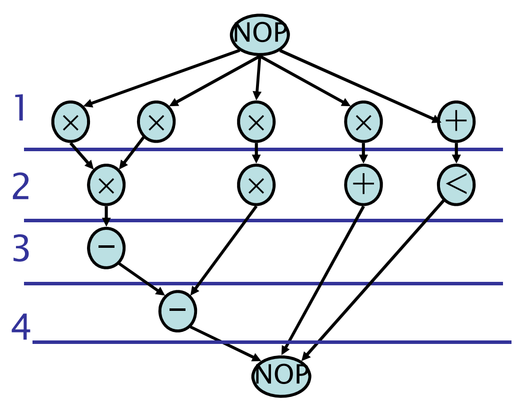
- **All operations have bounded delays**
- **All delays are in cycles:**
  - Cycle-time is given
- **No constraints – no bounds on area**
- **Goal:**
  - Minimize latency

Min Latency Unconstrained Scheduling

- **Simplest case: no constraints, find min latency**
- **Given set of vertices V, delays D and a partial order > on operations E,**
- **find an integer labeling of operations $\phi: V \rightarrow Z^+$ such that:**
  - $t_i = \phi(v_i)$
  - $t_i \geq t_j + d_j \quad \forall (v_j, v_i) \in E$
  - and $\lambda = t_n - t_0$ is minimum
- **Solvable in polynomial time**
  - Bounds on latency for resource constrained problems
  - ASAP algorithm used: topological order

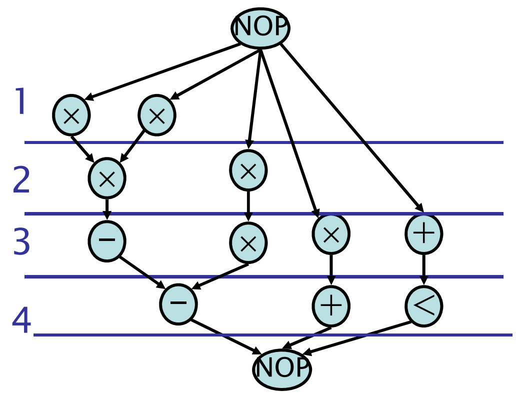
ASAP Schedules

- **Schedule v_0 at t_0=0**
- **While (v_n not scheduled)**
  - Select v_i with all scheduled predecessors
  - Schedule v_i at $t_i = \max \{t_j+d_j\}$, v_j being a predecessor of v_i
- **Return t_n**



ALAP Schedules

- **Schedule v_n at t_n=l**
- **While (v_0 not scheduled)**
  - Select v_i with all scheduled successors
  - Schedule v_i at $t_i = \min \{t_j - d_j\}$, v_j being a succecessor of v_i



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Remarks

- **ALAP solves a latency-constrained problem**
  - Latency bound can be set to latency computed by ASAP algorithm
- **Mobility**
  - Defined for each operation
  - Difference between ALAP and ASAP schedule
  - ➢ Slack on the start time

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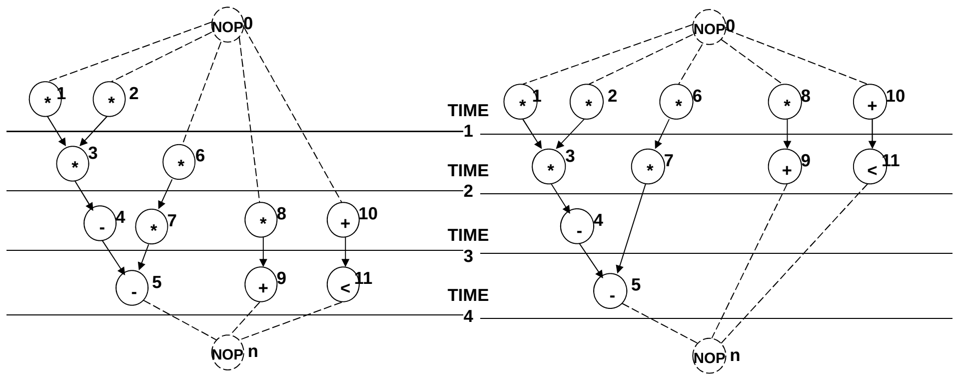
Example

- **Operations with zero mobility:**
  - $\{ v_1, v_2, v_3, v_4, v_5 \}$
  - Critical path
- **Operations with mobility one:**
  - $\{ v_6, v_7 \}$
- **Operations with mobility two:**
  - $\{ v_8, v_9, v_{10}, v_{11} \}$



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Lecture 12: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- **Resource constrained (RC) scheduling**
  - Exact formulations
    - ILP
    - Hu's algorithm
  - Heuristic methods
    - List scheduling
- Time constrained (TC) scheduling
- Advanced scheduling problems

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Scheduling under Resource Constraints

- **Classical scheduling problem**
  - Fix area bound – minimize latency (ML-RCS)
    - Minimum latency resource constrained scheduling
  - The amount of available resources affects the achievable latency
- **Dual problem:**
  - Fix latency bound – minimize resources (MR-LCS)
    - Minimum resources latency constrained scheduling
- **Assumption:**
  - All delays bounded and known

ML-RCS

- **Given**
  - a set of ops V with integer delays D
  - a partial order on the operations E
  - upper bounds $\{ a_k;\ k = 1, 2, \ldots, n_{res} \}$ on resource usage
- **Find an integer labeling $\phi : V \rightarrow Z^+$ such that:**
  - $t_i = \phi(v_i)$,
  - $t_i \geq t_j + d_j$ for all i,j s.t. $(v_j, v_i) \in E$,
  - $| \{ v_i \,|\, T(v_i) = k$ and $t_i \leq l < t_j + d_j \} | \leq a_k$
    - for all types $k = 1,2,\ldots,n_{res}$ and steps l
  - ➢ and t_n is minimum

➢ **Intractable problem**

ILP Formulation

- **Binary decision variables**
  - $X = \{ x_{il}, \quad i = 1,2,\ldots. n; \; l = 1,2,\ldots, \overline{\lambda} + 1\}$
  - x_{il} is **TRUE** only when operation v_i starts in step l of the schedule (i.e. $l = t_i$)
  - $\overline{\lambda}$ is an upper bound on latency
- **Start time of operation v_i : $\Sigma_l \, l \cdot x_{il}$**

ILP Constraints

- **Operations start only once**

  $\Sigma \, x_{il} = 1 \quad i = 1, 2, \ldots, n$
- **Sequencing relations must be satisfied**

  $t_i \geq t_j + d_j \rightarrow \quad t_i - t_j - d_j \geq 0$ for all $(v_j, v_i) \in E$

  $\Sigma \, l \cdot x_{il} - \Sigma \, l \cdot x_{jl} - d_j \geq 0$ for all $(v_j, v_i) \in E$
- **Resource bounds must be satisfied**

  Simple case (unit delay)

  $\sum_{l, \; i:T(v_i)=k} x_{il} \leq a_k \quad k = 1,2,\ldots n_{res}$; for all l

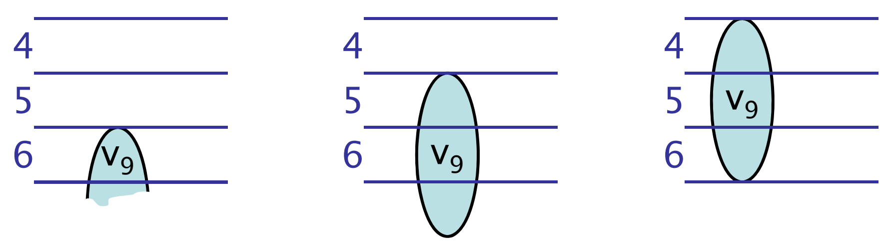
Start Time vs. Execution Time

- **For each operation v_i, only one start time**
- **If d_i=1, then the following questions are the same:**
  - Does operation v_i start at step l?
  - Is operation v_i running at step l?
- **But if $d_i > 1$, the two questions should be formulated as:**
  - Does operation v_i start at step l?
    - Does $x_{il} = 1$ hold?
  - Is operation v_i running at step l?
    - Does the following hold?

$$\sum_{m=l-d_i+1}^{l} x_{im} \stackrel{?}{=} 1$$

Operation v_i Still Running at Step l ?

- **Is v_9 running at step 6?**
  - Is $x_{9,6} + x_{9,5} + x_{9,4} = 1$?

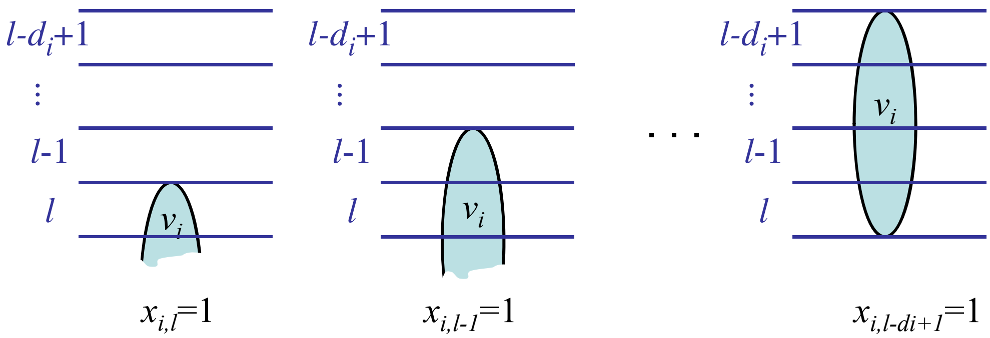


$x_{9,6}=1$ $x_{9,5}=1$ $x_{9,4}=1$

- **Note:**
  - Only one (if any) of the above three cases can happen
  - To meet resource constraints, we have to ask the same question for ALL steps, and ALL operations of that type

Operation v_i Still Running at Step l ?

- **Is v_i running at step l ?**
  - Is $x_{i,l} + x_{i,l-1} + \ldots + x_{i,l-di+1} = 1$?



ILP Formulation of ML-RCS

- **Constraints:**
  - Unique start times: $\sum_l x_{il} = 1, \quad i = 0,1,\ldots,n$
  - Sequencing (dependency) relations must be satisfied

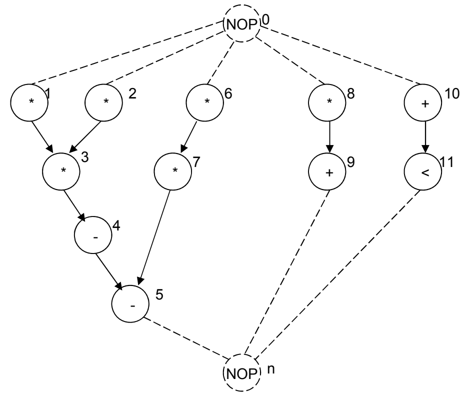
    $$t_i \geq t_j + d_j \ \forall (v_j, v_i) \in E \Rightarrow \sum_l l.x_{il} \geq \sum_l l.x_{jl} + d_j$$

  - Resource constraints

    $$\sum_{i:T(v_i)=k} \ \sum_{m=l-d_i+1}^{l} x_{im} \leq a_k, \quad k = 1,\ldots,n_{res}, \quad l = 1,\ldots,\bar{\lambda}+1$$

- **Objective:** **min** $c^T t$
  - $\boldsymbol{t}$ = start times vector, $\boldsymbol{c}$ = cost weight (e.g., [0 0 ... 1])
  - When $\boldsymbol{c}$ = [0 0 ... 1], $\boldsymbol{c}^T\boldsymbol{t} = \sum_l l.x_{nl}$

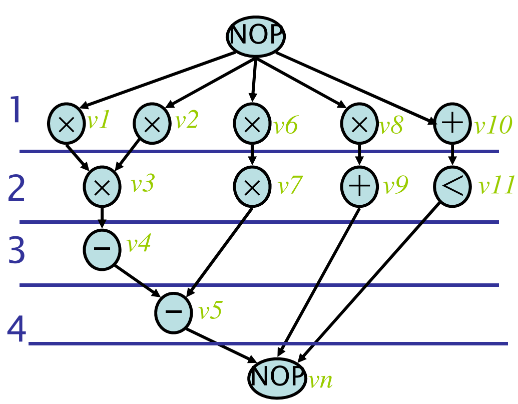
ILP Example

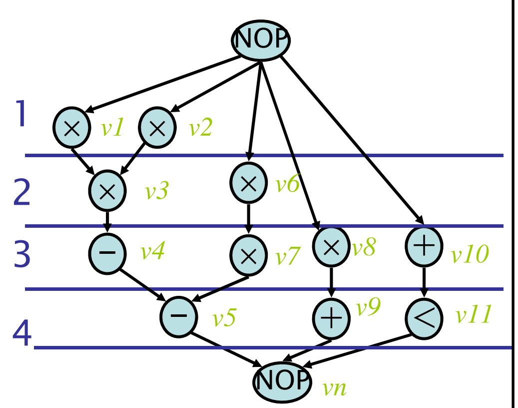


- **Resource constraints**
  - 2 ALUs; 2 Multipliers
  - $a_1 = 2$; $a_2 = 2$
- **Single-cycle operation**
  - $d_i = 1$ for all i

ILP Example

- **Assume** $\overline{\lambda} = 4$
- **First, perform ASAP and ALAP**
  - (we can write the ILP without ASAP and ALAP, but using ASAP and ALAP will simplify the inequalities)





ILP Example: Unique Start Times

- **Without using ASAP and ALAP values:**

$$x_{1,1}+x_{1,2}+x_{1,3}+x_{1,4}=1$$
$$x_{2,1}+x_{2,2}+x_{2,3}+x_{2,4}=1$$
$$\ldots$$
$$\ldots$$
$$\ldots$$
$$x_{11,1}+x_{11,2}+x_{11,3}+x_{11,4}=1$$

- **Using ASAP and ALAP:**

$$x_{1,1}=1$$
$$x_{2,1}=1$$
$$x_{3,2}=1$$
$$x_{4,3}=1$$
$$x_{5,4}=1$$
$$x_{6,1}+x_{6,2}=1$$
$$x_{7,2}+x_{7,3}=1$$
$$x_{8,1}+x_{8,2}+x_{8,3}=1$$
$$x_{9,2}+x_{9,3}+x_{9,4}=1$$
$$\ldots.$$

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ILP Example: Dependency Constraints

- **Using ASAP and ALAP, the non-trivial inequalities are: (assuming unit delay for + and *)**

$$2x_{7,2}+3x_{7,3}-x_{6,1}-2x_{6,2}-1\geq 0$$
$$2x_{9,2}+3x_{9,3}+4x_{9,4}-x_{8,1}-2x_{8,2}-3x_{8,3}-1\geq 0$$
$$2x_{11,2}+3x_{11,3}+4x_{11,4}-x_{10,1}-2x_{10,2}-3x_{10,3}-1\geq 0$$
$$4x_{5,4}-2x_{7,2}-3x_{7,3}-1\geq 0$$
$$5x_{n,5}-2x_{9,2}-3x_{9,3}-4x_{9,4}-1\geq 0$$
$$5x_{n,5}-2x_{11,2}-3x_{11,3}-4x_{11,4}-1\geq 0$$

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ILP Example: Resource Constraints

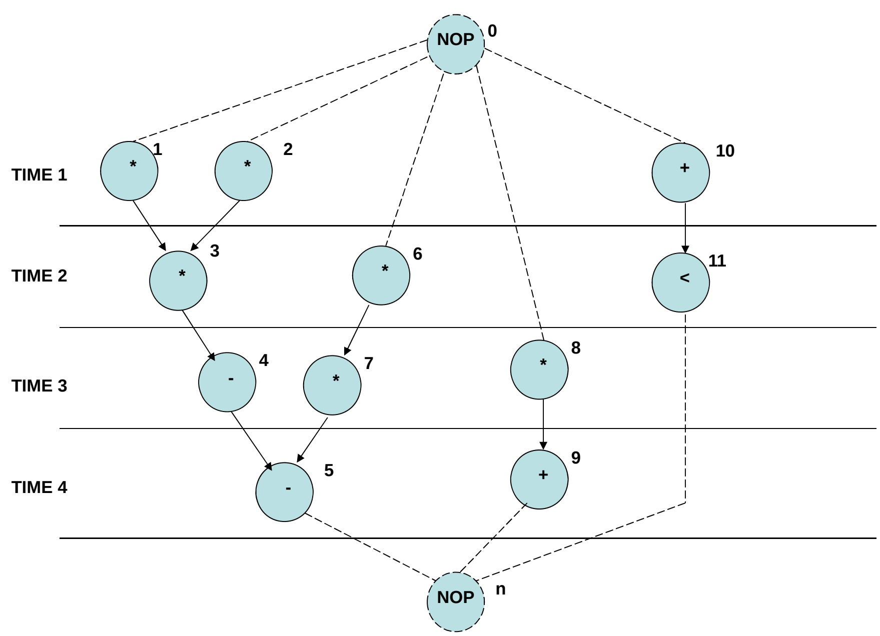
- **Resource constraints (assuming 2 adders and 2 multipliers)**

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$
$$x_{3,2} + x_{6,2} + x_{7,2} + x_{8,2} \leq 2$$
$$x_{7,3} + x_{8,3} \leq 2$$
$$x_{10,1} \leq 2$$
$$x_{9,2} + x_{10,2} + x_{11,2} \leq 2$$
$$x_{4,3} + x_{9,3} + x_{10,3} + x_{11,3} \leq 2$$
$$x_{5,4} + x_{9,4} + x_{11,4} \leq 2$$

- **Objective:**
  - Since λ=4 and sink has no mobility, any feasible solution is optimum, but we can use the following anyway:

$$Min \quad x_{n,1} + 2.x_{n,2} + 3.x_{n,3} + 4.x_{n,4}$$

ILP Example: Solution

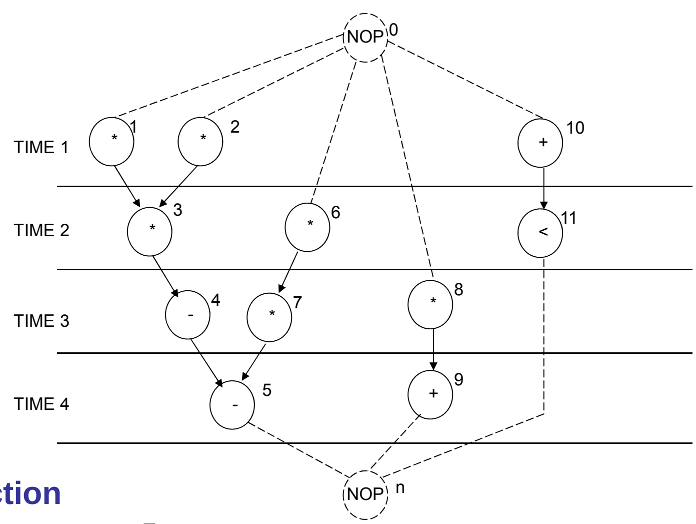


MR-LCS Dual ILP formulation

- **Minimize resource usage under latency constraint**
- **Additional constraint**
  - Latency bound must be satisfied
  - $\Sigma_l \, l \, x_{nl} \leq \lambda + 1$

➢ **Resource usage is unknown in the constraints**
  - Resource usage is the objective to minimize

MR-LCS ILP Example



- **Cost function**
  - Multiplier area = 5
  - ALU area = 1
  - Objective function: $5a_1 + a_2$

ILP Solving

- **Use standard ILP packages**
- **Transform into LP problem**
- **Advantages**
  - Exact method
  - Others constraints can be incorporated
- **Disadvantages**
  - Works well up to few thousand variables

Hu's Algorithm

- **Simple case of the scheduling problem**
  - Operations of unit delay
  - Operations (and resources) of the same type
- **Hu's algorithm**
  - Greedy, polynomial *and* optimal (exact)
    - Computes lower bound on number of resources for given latency
      OR
      Computes lower bound on latency subject to resource constraints
- **Basic idea**
  - Label operations based on their distances from the sink
  - Try to schedule nodes with higher labels first
    (i.e., most "critical" operations have priority)

Hu's Algorithm with $\bar{a}$ Resources

- **Label operations with distance to sink**
- **Set step $l = 1$**
- **Repeat until all ops are scheduled**
  - U = unscheduled vertices in V
    - Predecessors have been scheduled (or no predecessors)
  - Select $S \subseteq U$ resources with
    - $|S| \leq \bar{a}$
    - Maximal labels
  - Schedule the S operations at step l
  - Increment step $l = l + 1$

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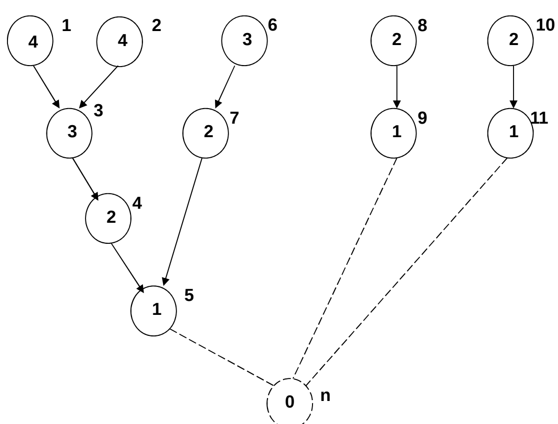
Hu's Algorithm Example



- **Assumptions**
  - One resource type only
  - All operations have unit delay
- **Labels**
  - Distance to sink

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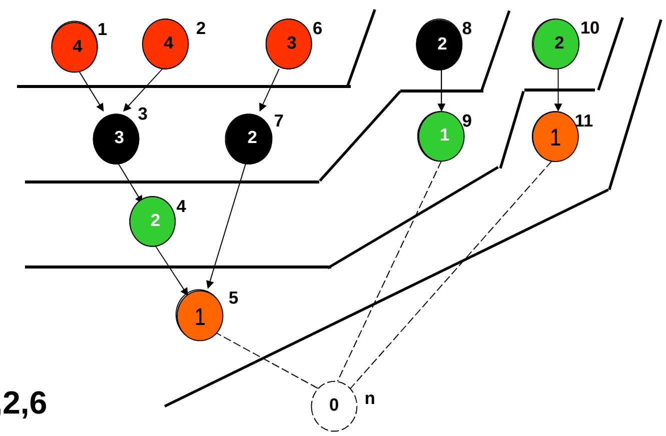
Hu's Algorithm Example

$\bar{a} = 3$



Step 1: Op 1,2,6

Step 2: Op 3,7,8

Step 3: Op 4,9,10

Step 4: Op 5,11

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List Scheduling

- **Heuristic method for:**
  - Min *latency* subject to *resource bound* (ML-RCS)
  - Min *resource* subject to *latency bound* (MR-LCS)
- **Greedy strategy (like Hu's)**
  - Does not guarantee optimality (unlike Hu's)
- **General graphs (unlike Hu's)**
  - Resource constraints on different resource types
  - Operations of arbitrary delay
- **Priority list heuristics**
  - Priority decided by criticality (similar to Hu's)
  - Longest path to sink, longest path to timing constraint
  - $O(n)$ time complexity

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List Scheduling for Minimum Latency

LIST_L($G(V, E), a$**) {**

  $l = 1$;

  repeat **{**

    for each **resource type** $k = 1, 2, \ldots, n_{res}$ **{**

      Determine ready operations $U_{l,k}$;

      Determine unfinished operations $T_{l,k}$;

      Select $S_k \subseteq U_{l,k}$ **vertices, s.t.** $|S_k| + |T_{l,k}| \leq a_k$;

      Schedule the S_k **operations at step** l;

      }

    $l = l + 1$;

  }

  until (v_n **is scheduled) ;**

  return (t);

}

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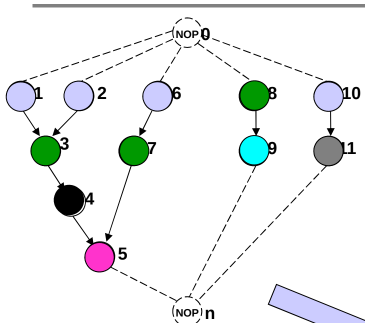
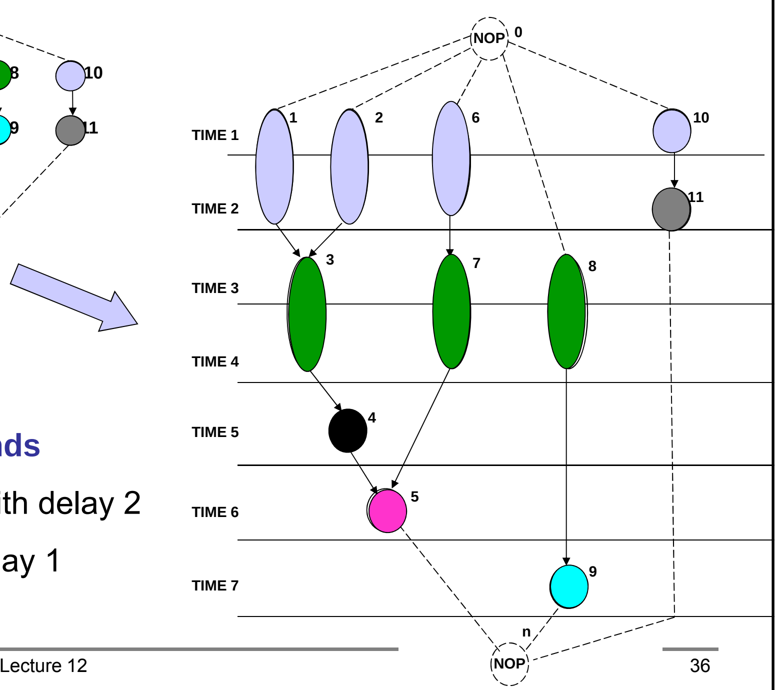
List Scheduling Example





Resource bounds

3 multipliers with delay 2

1 ALU with delay 1

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Lecture 12: Outline

- ✓ The scheduling problem
- ✓ Unconstrained scheduling
- ✓ Resource constrained (RC) scheduling
- Time constrained (TC) scheduling
  - ✓ Exact methods
    - ✓ ILP formulations
    - ✓ Hu's algorithm
  - Heuristics
    - – List scheduling
    - – Force-directed scheduling
- Advanced scheduling problems

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List Scheduling for Minimum Resources

```
LIST_R( G(V, E), λ̄) {
   a = 1;
   Compute the latest possible start times t^L by ALAP ( G(V, E), λ̄);
   if (t_0 < 0)
      return (Ø);
   l = 1;
   repeat {
         for each resource type k = 1, 2, …, n_res {
            Determine ready operations U_l,k;
            Compute the slacks { s_i = t_i – l  for all v_i ϵ U_lk};
            Schedule candidate operations with zero slack and update a;
            Schedule candidate operations not needing addt'l resources;
            }
         l = l + 1;
   }
   until (v_n is scheduled) ;
   return (t, a);
}
```

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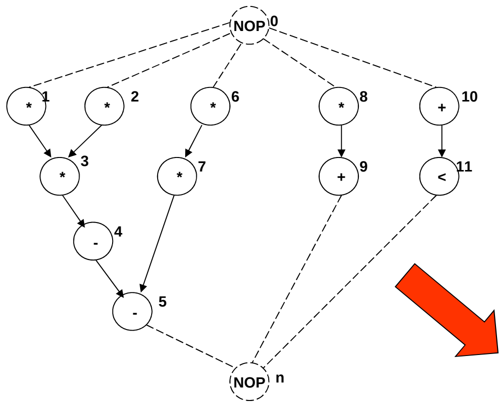
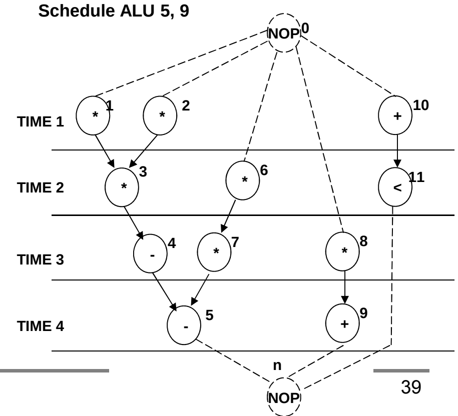
List Scheduling Example



Step 1
Two multiplications on CP
Set $a_1 = 2$
Schedule Mult 1,2
Schedule ALU 10
Step 2
Schedule Mult 3, 6
Schedule ALU 11
Step 3
Schedule Mult 7,8
Schedule ALU 4
Step 4
Set a_2=2
Schedule ALU 5, 9



- **Assumptions**
  - Unit-delay resources
  - Maximum latency = 4
- **Start with**
  - $a_1 = 1$ multiplier
  - $a_2 = 1$ ALUs

Force-Directed Scheduling (FDS)

- **Heuristic, similar to list scheduling**
  - Can handle ML-RCS and MR-LCS
  - For ML-RCS, schedules step-by-step
  - BUT, selection of the operations tries to find the *globally* best set of operations
- **Idea [Paulin]**
  - Find the mobility $\mu_i = t_i^L - t_i^S$ of operations (ALAP-ASAP)
  - Look at the operation type probability distributions
  - Try to flatten the operation type distributions
- **Definition: operation probability density**
  - $p_i(l) = \Pr\{ v_i$ executes in step $l \}$
  - Assume uniform distribution: $p_i(l) = \dfrac{1}{\mu_i + 1} \quad for\ l \in [t_i^S, t_i^L]$

Force-Directed Scheduling: Definitions

- **Operation-type distribution (sum of operation probabilities for each type)**
  - $$q_k(l) = \sum_{i:T(v_i)=k} p_i(l)$$
- **Operation probabilities over control steps**
  - $$p_i = \{p_i(0), p_i(1), \ldots, p_i(n)\}$$
- **Distribution graph of type *k* over all steps**
  - $$\{q_k(0), q_k(1), \ldots, q_k(n)\}$$
  - $q_k(l)$ can be thought of as *expected* operator cost for implementing operations of type k at step l

Force-Directed Scheduling Example

$$q_{add}(1) = \frac{1}{3} = 0.33$$

$$q_{add}(2) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$q_{add}(3) = 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 2$$

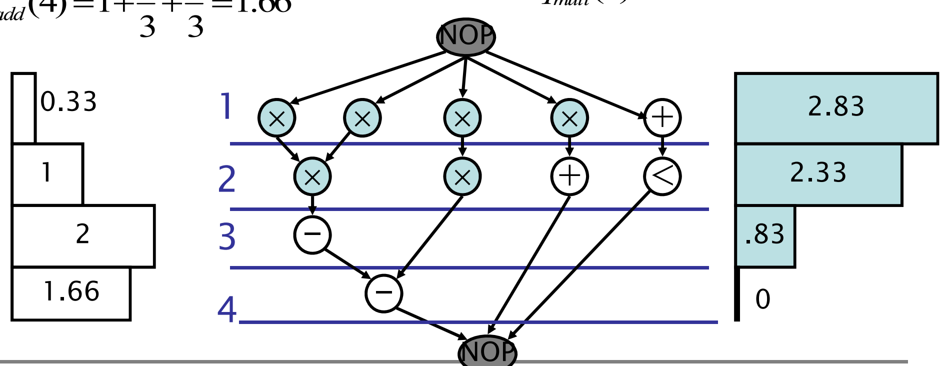
$$q_{add}(4) = 1 + \frac{1}{3} + \frac{1}{3} = 1.66$$

$$q_{mult}(1) = 1 + 1 + \frac{1}{2} + \frac{1}{3} = 2.83$$

$$q_{mult}(2) = 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = 2.33$$

$$q_{mult}(3) = \frac{1}{2} + \frac{1}{3} = 0.83$$

$$q_{mult}(4) = 0$$



Force-Directed Scheduling Algorithm

- **Very similar to LIST_L($G(V,E)$, a)**
  - Compute mobility of operations using ASAP and ALAP
  - Computer operation probabilities and type distributions
  - Select and schedule operation
  - Update operation probabilities and type distributions
  - Go to next step/operation
- **Difference with list scheduling in selecting operations**
  - Select operations with least force
  - $O(n^2)$ time complexity due to pair-wise force computations

Force

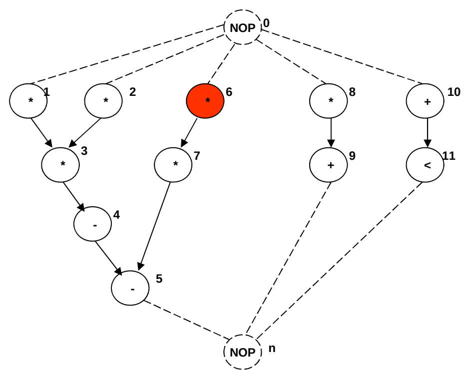
- **Used as *priority* function**
- **Force is related to concurrency**
  - Sort operations for least force
- **Mechanical analogy (spring)**
  - Force = constant × displacement
    - Constant = operation-type distribution
    - Displacement = change in probability

Two Types of Forces

- **Self-force**
  - Sum of forces to feasible schedule steps
  - Self-force for operation v_i in step l
    - Sum over type distribution × delta probability

      $\Sigma_{m \text{ in interval}}\ q_k(m)\ (\delta_{lm} - p_i(m))$
    - Higher self-force indicates higher mobility
- **Predecessor/successor-force**
  - Related to the predecessors/successors
    - Fixing an operation timeframe restricts timeframe of predecessors/successors
    - Ex: Delaying an operation implies delaying its successors
    - Computed by changes in self-forces of neighbors

Example: Schedule Operation v_6



- **Distribution graphs for multiplier and ALU**

- **Operation v_6 can be scheduled in step 1 or step 2**

Example: Operation v_6

- **Op v_6 can be scheduled in the first two steps**
  - $p(1) = 0.5; p(2) = 0.5; p(3) = 0; p(4) = 0$
- **Distribution**
  - $q(1) = 2.8; q(2) = 2.3$
- **Assign v_6 to step 1**
  - Variation in probability 1 – 0.5 = 0.5 for step 1
  - Variation in probability 0 – 0.5 = -0.5 for step 2
- **Self-force**
  - 2.8 * 0.5 – 2.3 * 0.5 = + 0.25
- **No successor force**
- **Total force = 0.25**

Example: Operation v_6

- **Assign v_6 to step 2**
  - variation in probability 0 – 0.5 = -0.5 for step 1
  - variation in probability 1 – 0.5 = 0.5 for step 2
- **Self-force**
  - - 2.8 * 0.5 + 2.3 * 0.5 = - 0.25
- **Successor-force**
  - Operation v_7 assigned to step 3
  - Succ. force is 2.3 (0- 0.5) + 0.8 (1 – 0.5) = - .75
- **Total force = -1**

Example: Operation v_6

- **Total force in step 1 = + 0.25**
- **Total force in step 2 = -1**

➢ **Conclusion:**
  - Least force is for step 2
  - Assigning v_6 to step 2 reduces concurrency

FDS for Minimum Resources

```
FDS ( G ( V, E ), λ̄ )
{
    repeat {
        Compute/update the time-frames;
        Compute the operation and type probabilities;
        Compute the self-forces, p/s-forces and total forces;
        Schedule the op. with least force;
    }
    until (all operations are scheduled)
    return (t);
}
```

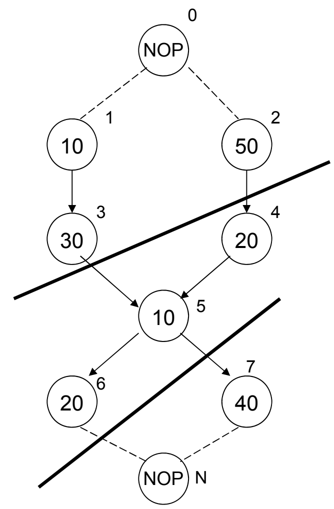
Scheduling Generalizations

- **Detailed timing constraints**
  - Protocol and interface synthesis
    - Bounds on start time differences
    - Forward & backward edges for min/max constraints
- **Operation generalizations**
  - Unbounded delay operations (e.g. synchronization)
    - Relative scheduling w.r. to anchors and combine
  - Conditional operations
- **Resource generalizations**
  - Multi-cycling and chaining
  - Pipelined resources
- **Model generalizations**
  - Hierarchy
  - Pipelining
  - Loops

Multi-Cycling and Chaining

- **Consider delays of resources not in terms of cycles**
  - Use scheduling to *chain* multiple operations in the same control step
  - Use scheduling to *multi-cycle* an operation across more than one control step
- **Useful techniques to explore effect of *cycle-time* on area/latency trade-off**
- **Algorithms**
  - ILP
  - ALAP/ASAP
  - List scheduling

Chaining Example



(a) (b)

- **Cycle-time: 50**

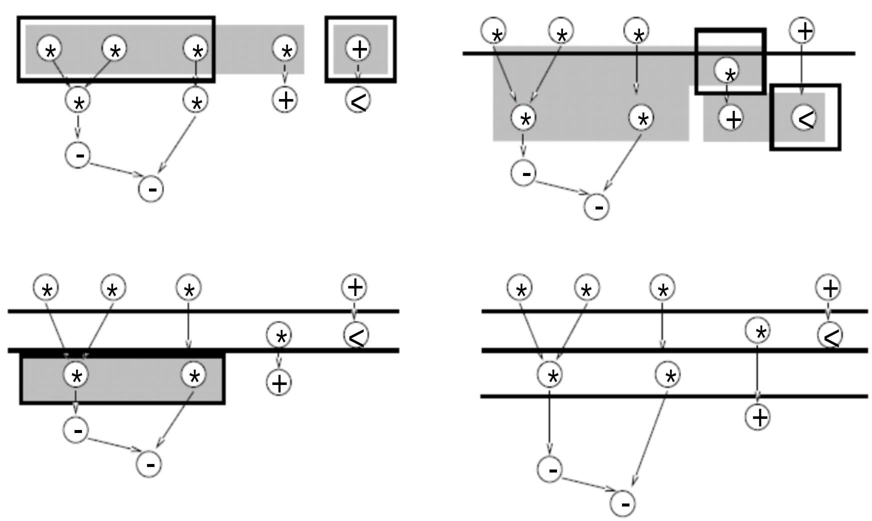
Pipelining

- **Two levels of data pipelining**
  - Structural pipelining
    - Pipelined resources
    - Non-pipelined model
  - Functional pipelining
    - Non-pipelined resources
    - Pipelined model
- **Control pipelining**
  - Pipelined control logic

Structural Pipelining

- **Non-pipelined model using pipelined resources**
- **Resources characterized by**
  - Execution delay
  - Data introduction interval: *DII*
- **Implications**
  - Operations sharing a pipelined resource are serialized (always)
  - Operations do not have data dependency
- ➢ **Solution using list scheduling**
  - Relax criteria for selection of vertices

Structural Pipelining Example



- **3 multipliers w/ 2 cycle delay and** $\boldsymbol{DII = 1}$

Functional (Loop) Pipelining

- **Pipelined model, non-pipelined resources**
- **Assume non-hierarchical graphs**
- **Model characterized by**
  - Latency
  - Initiation interval, II
- **Restart source before completing sink**
  - Implicit loop
  - Limited by loop-carried dependencies

➢ **Solutions using ILP or heuristics**
  - ILP resource constraints modified to include increased concurrency
  - List or force-directed methods

Pipelining and Concurrency

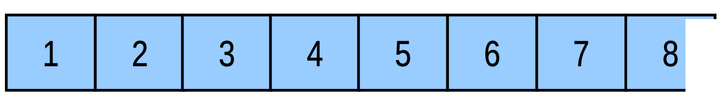
- ***II* determines resource usage**
  - Smaller II leads to larger overlaps, higher resource requirements
    $\min\{a_k\} = n_k$, for II=1 (all n_k operations are concurrent)
  - In general, $\bar{a}_k = \left\lceil \frac{n_k}{II} \right\rceil$
- **Concurrent operations**
  - Operations v_i and v_j are executing concurrently at control step l, if
    $\text{rem}\{\, t_i / II \,\} = \text{rem}\{\, t_j / II \,\} = l$
  - Affects the design of the controller circuitry

Loop Scheduling

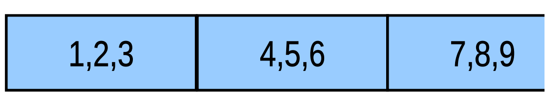
- **Potential parallelism *across* loop invocations**
- **Single loop executions**
  - Sequential execution
  - Loop unrolling (known iteration count)
    - – Merge multiple iterations into one to provide scheduling opportunities
  - Loop pipelining (iteration count might be unknown)
    - – Start next iteration while current one is still running
    - – Depends on dependencies across iterations
    - ➢ Functional pipelining
- **Merging of multiple loops**
  - Run different loops in parallel (no dependencies)

Loop Scheduling Example

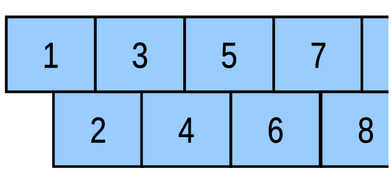
- **Sequential**



- **Unrolled**



- **Pipelined**



- Iteration count = N
- Loop latency = $N \cdot \lambda$
- Pipeline loop iterations with $II < \lambda$
- Latency of the pipelined loop
  - – $N \cdot II$ + overhead
  - – Overhead = $\lceil \lambda / II \rceil - 1$

Lecture 12: Summary

- **Scheduling determines *area/latency* trade-off**
- **Intractable problem in general**
  - Heuristic algorithms
  - ILP formulation (small-case problems)
- **Several heuristic formulations**
  - List scheduling is the fastest and most used
  - Force-directed scheduling tends to yield good results
- **Several extensions**
  - Chaining and multi-cycling
  - Pipelining

