EE382M.20: System-on-Chip (SoC) Design

Lecture 13 – Resource Allocation and Binding

Source: G. De Micheli, Integrated Systems Center, EPFL
"Synthesis and Optimization of Digital Circuits", McGraw Hill, 2001.

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Lecture 13: Outline

- **Allocation, binding and sharing**
    - Problem formulation
- **Functional unit sharing**
    - Flat graphs
    - Hierarchical graphs
- **Register sharing**
    - Multi-port register files/memories
- **Bus sharing**
- **Extensions**
    - Module selection
    - Datapath and control synthesis

Allocation and Binding

- **Allocation**
  - Number of resources available
- **Binding**
  - Mapping of operations onto resources
- **Sharing**
  - Many-to-one relation
- **Selection**
  - Type to implement each operation

Binding

- **Limiting cases**
  - Dedicated resources
    - One resource per operation
    - No sharing
  - One multi-task resource
    - ALU
  - One resource per type
- **Closely related to scheduling**
- **Optimum binding/sharing**
  - Minimize the resource usage
  - ➢ Scheduled sequencing graphs
    - Operation concurrency well defined
  - ➢ Consider *operation types* independently
    - Problem decomposition
      - » Perform analysis for each resource type

ILP Formulation of Binding

- **Boolean variable b_{ir}**
  - Operation i bound to resource r

$\sum_r b_{ir} = 1$ for all operations i

- **Boolean variables x_{il}**
  - Operation i scheduled to start at step l

$\sum_i b_{ir} \sum_{m=l-d_i+1..l} x_{im} \leq 1$ for all steps l and resources r

Compatibly and Conflicts

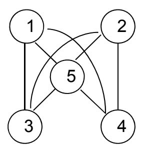
- **Operation compatibility:**
  - Same type
  - Non concurrent

t1	x=a+b	y=c+d	1	2
t2	s=x+y	t=x-y	3	4
t3	z=a+t		5	

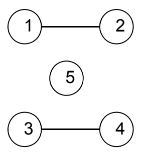
- ***Compatibility* graph:**
  - Vertices: operations
  - Edges: compatibility relation

- ***Conflict* graph:**
  - Complement of compatibility graph

Compatibility graph



Conflict graph



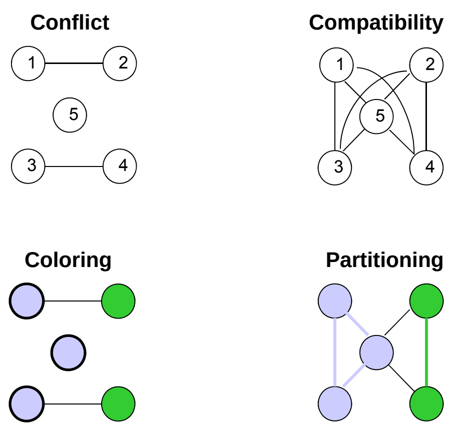
Compatibility and Conflicts

- **Compatibility graph clique partitioning**
  - Partition the graph into a minimum number of cliques
  - Find *clique cover number* κ (G_+)
- **Conflict graph coloring**
  - Color the vertices by a minimum number of colors.
  - Find *chromatic number* χ (G_-)

➢ **NP-complete problems**
  - Heuristic algorithms

Compatibility and Conflict Example

t1	x=a+b	y=c+d	1	2
t2	s=x+y	t=x-y	3	4
t3	z=a+t		5	



ALU1: 1,3,5
ALU2: 2,4

Perfect Graphs

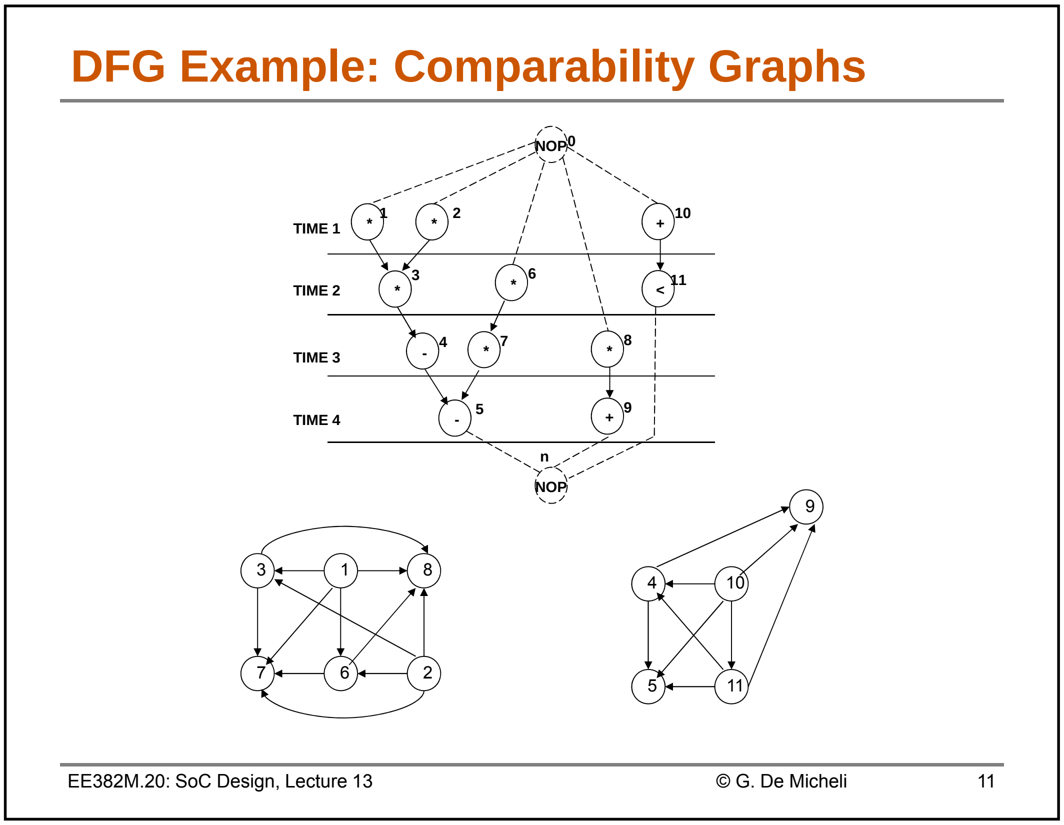
- ***Comparability*** **graph**
  - Graph $G\,(V, E\,)$ has an orientation $G\,(\,V, F\,)$ with the transitive property

    $(v_i, v_j) \in F$ and $(v_j, v_k) \in F \rightarrow (v_i, v_k) \in F$
- ***Interval*** **graph**
  - Vertices correspond to *intervals*
  - Edges correspond to interval intersection
  - Subset of *chordal* graphs
    - Every loop with more than three edges has a chord (edge that is not part of the cycle but connects two nodes in the loop)

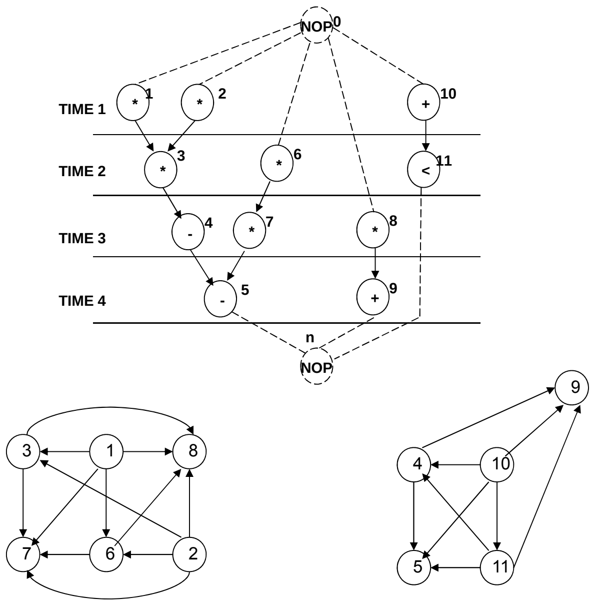
Data Flow Graphs (DFGs)

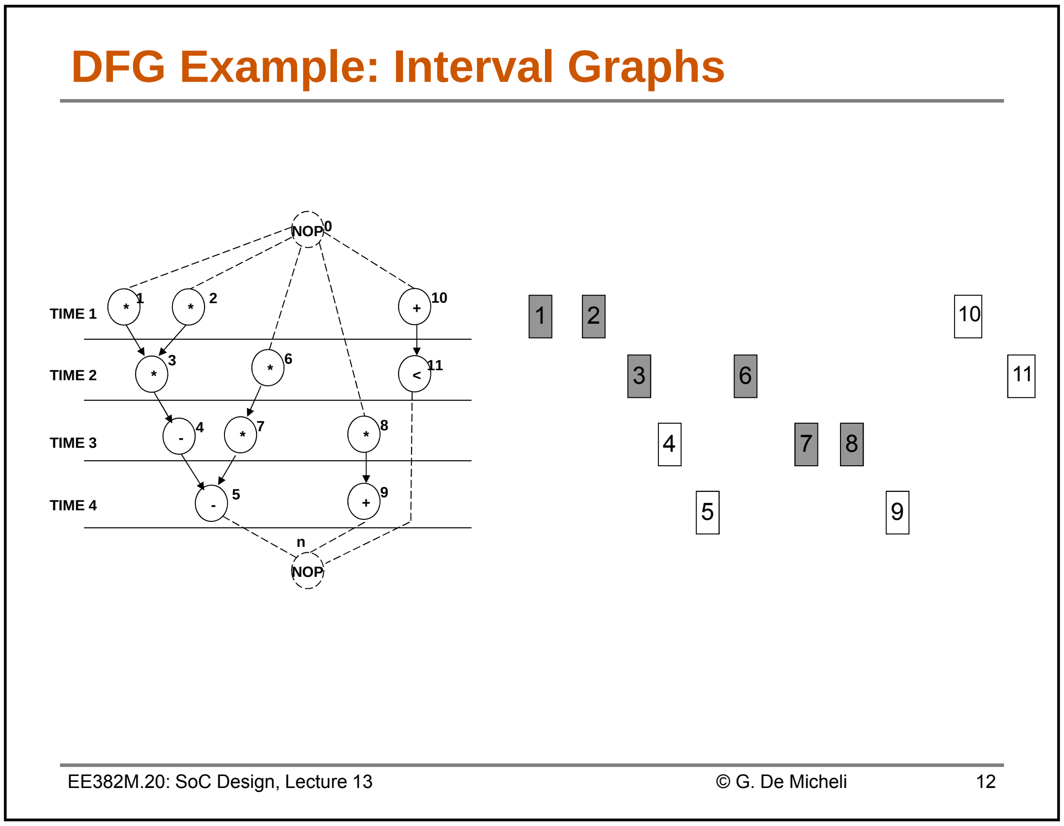
- **The compatibility/conflict graphs have special properties**
  - Compatibility
    - Comparability graph
  - Conflict
    - Interval graph

➢ **Polynomial time solutions**
  - Golumbic's algorithm for partitioning of compatibility graphs
  - Left-edge algorithm for coloring of interval graphs

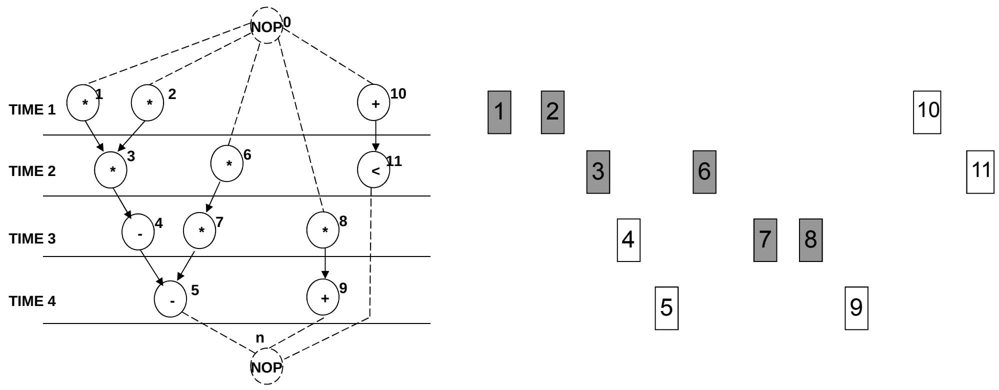


DFG Example: Comparability Graphs





DFG Example: Interval Graphs



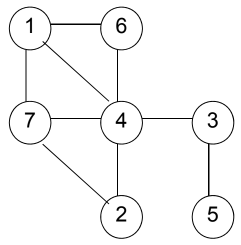
Left-Edge Algorithm

- **Input:**
  - Set of intervals with *left* and *right edge*
  - A set of *colors* (initially one color)
- **Rationale**
  - Sort intervals in a *list* by *left* edge
  - Assign non overlapping intervals to first color using the list
  - When possible intervals are exhausted,
    increase color counter and repeat

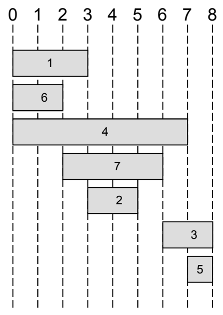
Left-Edge Algorithm

```
LEFT_EDGE($I$)
{
    Sort elements of $I$ in a list $L$ in ascending order of $l_i$;
    $c = 0$;
    while (some interval has not been colored) do {
        $S = Ø$;
        $r = 0$;
        while ( exists $s \in L$ such that $l_s > r$) do {
            $s$ = First element in the list $L$ with $l_s > r$;
            $S = S \cup \{s\}$;
            $r = r_s$;
            Delete $s$ from $L$;
        }
        $c = c + 1$;
        Label elements of $S$ with color $c$;
    }
}
```

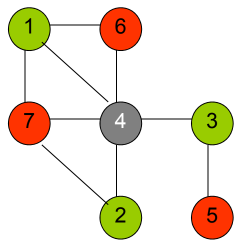
Left-Edge Example



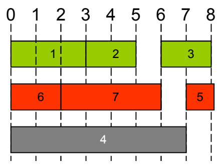
Conflict graph



Intervals



Colored conflict graph



Coloring

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Lecture 17: Outline

- ✓ **Allocation, binding and sharing**
  - ✓ Problem formulation
- **Functional unit sharing**
  - ✓ Flat graphs
  - Hierarchical graphs
- **Register sharing**
  - Multi-port register files/memories
- **Bus sharing**
- **Extensions**
  - Module selection
  - Datapath and control synthesis

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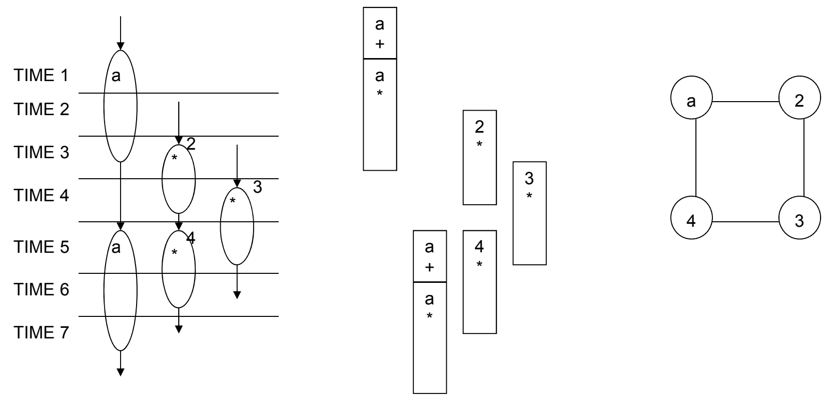
Hierarchical Sequencing Graphs (CDFGs)

- **Hierarchical conflict/compatibility graphs**
  - Easy to compute
  - Prevent sharing across hierarchy
- **Flatten hierarchy**
  - Bigger graphs
  - Destroy nice properties

Hierarchical Graph Example

- **Calls of sub-models**

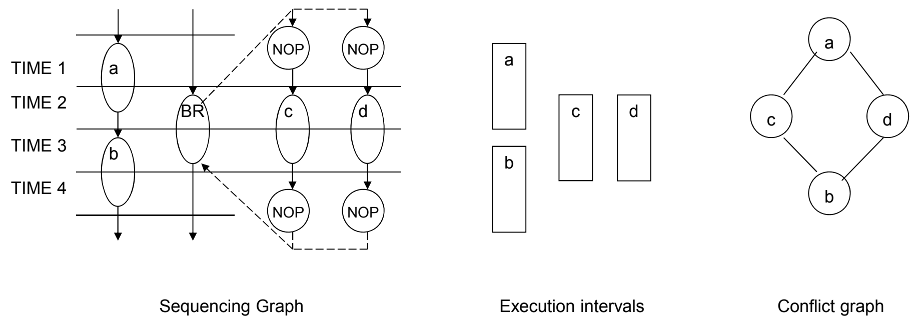


Sequencing Graph Execution intervals Conflict graph

➢ **Not perfect graphs any more**
  - Intractable, use of heuristics

Hierarchical Graph Example

- **Branching constructs**



➢ **Not perfect graphs any more**
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Storage Elements

- **Registers**
  - Hold data across cycles
  - Data: value of a variable
  - Variable lifetime in scheduled graph
  - Can be re-used (shared) across variables
- **Memory blocks, register files**
  - Limited number of read/write ports

Register Binding Problem

- **Given a schedule**
  - *Lifetime intervals* for variables
  - Lifetime overlaps
- **Conflict graph (interval graph)**
  - Vertices $\leftrightarrow$ variables
  - Edges $\leftrightarrow$ overlaps
  - Interval graph
- **Compatibility graph (comparability graph)**
  - Complement of conflict graph

Register Sharing

- **Given**
  - Variable lifetime conflict graph
- **Find**
  - Minimum number of registers storing all the variables
- **Key point**
  - Interval graph
    - Left-edge algorithm (polynomial-time complexity)

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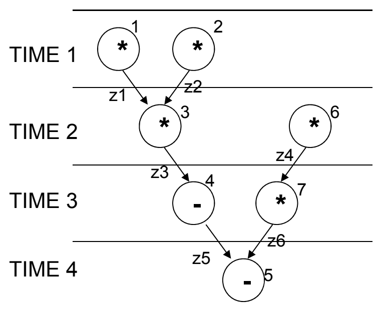
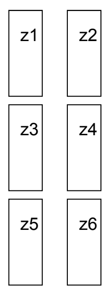
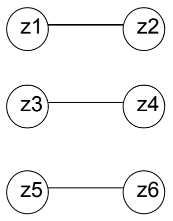
Register Sharing Example



Sequencing graph



Variable lifetimes



Conflict graph

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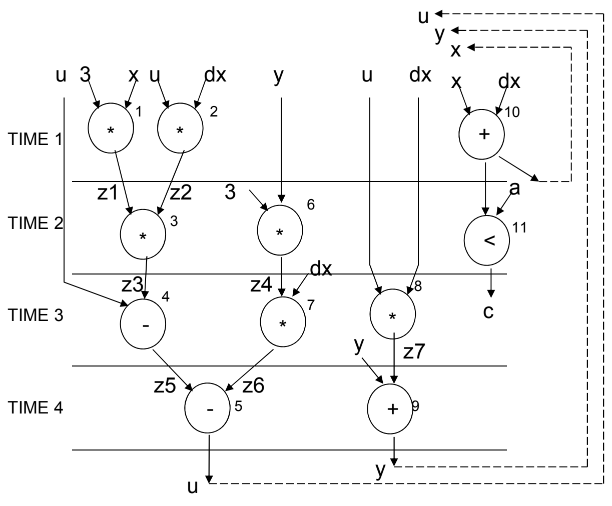
Register Sharing in Loops

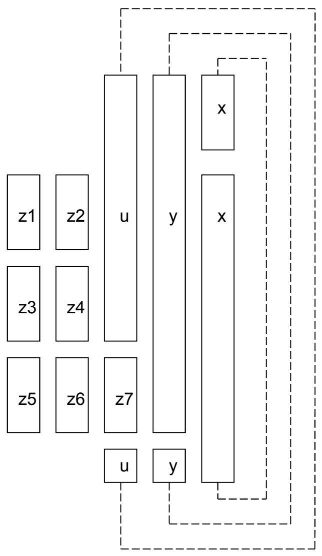
- **Iterative conflicts through loop-carried dependencies**
  - Preserve values across iterations
  - Circular-arc conflict graph
    - Coloring is intractable
- **Hierarchical graphs**
  - General conflict graphs
    - Coloring is intractable

➢ **Heuristic algorithms**

Loop Example

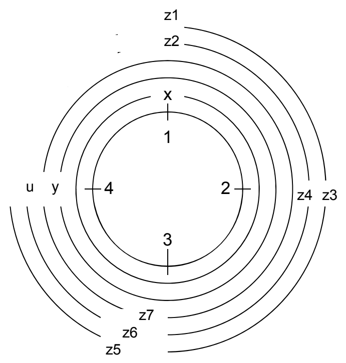


Sequencing graph

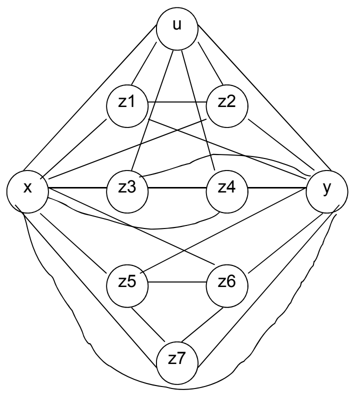


Variable lifetimes

Loop Variable Lifetimes



Variable lifetimes



Circular-arc conflict graph

Multi-Port Memory Binding

- **Find *minimum number of ports* to access the required number of variables**
- **Variables use the same port**
  - Port compatibility/conflict
  - Similar to resource binding
- **Variables can use any port**
  - Decision variable x_{il} id TRUE when variable i is accessed is step l
  - Optimum: $\max_{1 \leq l \leq \lambda + 1} (\sum_{i=1..nvar} x_{il})$

Dual Multiport-Memory Binding Problem

- **Find *max number of variables* to be stored through a fixed number of ports a**
  - Boolean variables $\{ b_i, \; i = 1, 2, \ldots, n_{var} \}$:
    - Variable i with b_i=1 will be stored in register file
  - maximize $\sum_{i=1..nvar} b_i$ such that $\sum_{i=1..nvar} b_i \, x_{il} \le a \qquad l = 1,2,\ldots,\lambda + 1$

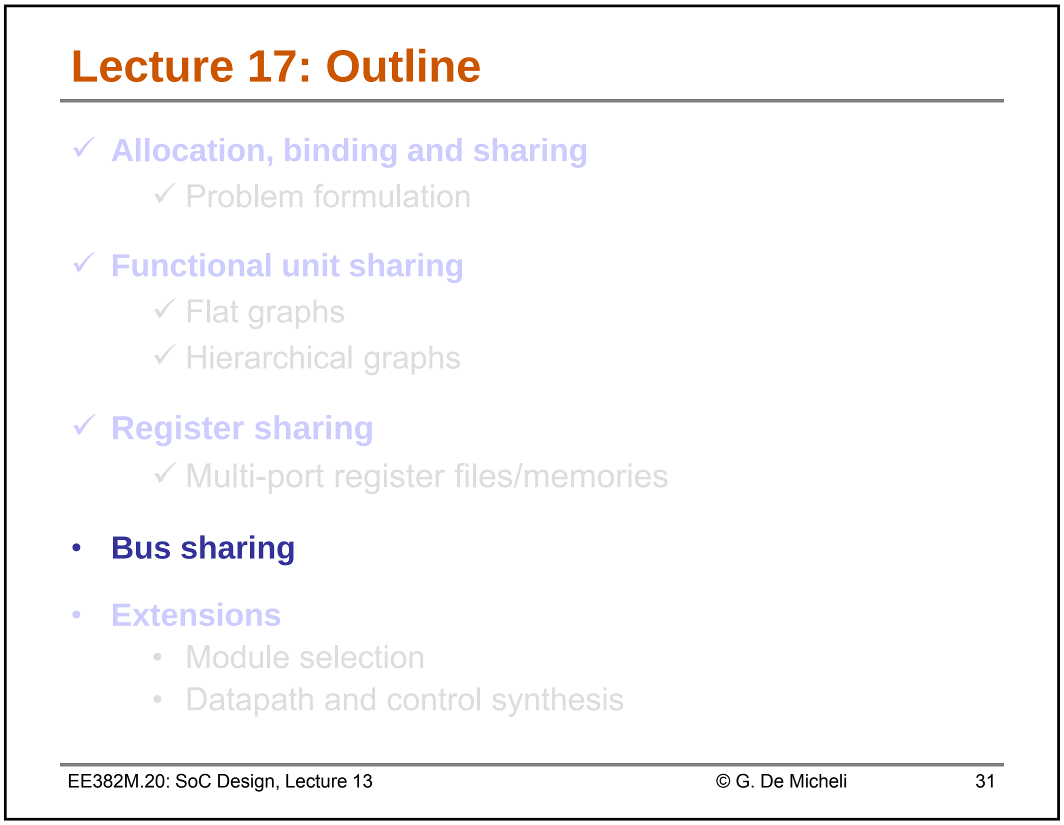
Multi-Port Memory Binding Example

***Time – step* 1 *:* $r_3 = r_1 + r_2$ *;* $r_{12} = r_1$**
***Time – step* 2 *:* $r_5 = r_3 + r_4$ *;* $r_7 = r_3 * r_6$ *;* $r_{13} = r_3$**
***Time – step* 3 *:* $r_8 = r_3 + r_5$ *;* $r_9 = r_1 + r_7$ *;* $r_{11} = r_{10} / r_5$**
***Time – step* 4 *:* $r_{14} = r_{11} \,\&\, r_8$ *;* $r_{15} = r_{12} \,|\, r_9$**
***Time – step* 5 *:* $r_1 = r_{11}$ *;* $r_2 = r_{15}$**

$\max \sum_{i=1}^{15} b_i$ **such that**

$$\begin{aligned}
b_1 + b_2 + b_3 + b_{12} &\le a \\
b_3 + b_4 + b_5 + b_6 + b_7 + b_{13} &\le a \\
b_1 + b_3 + b_5 + b_7 + b_8 + b_9 + b_{10} + b_{11} &\le a \\
b_8 + b_9 + b_{11} + b_{12} + b_{14} + b_{15} &\le a \\
b_1 + b_2 + b_{14} + b_{15} &\le a
\end{aligned}$$

- **One port $a = 1$:**
  - $\{ b_2, b_4, b_8 \}$ non-zero
  - 3 variables stored: v_2, v_4, v_8
- **Two ports $a = 2$:**
  - 6 variables stored: $v_2, v_4, v_5, v_{10}, v_{12}, v_{14}$
- **Three ports $a = 3$:**
  - 9 variables stored: $v_1, v_2, v_4, v_6, v_8, v_{10}, v_{12}, v_{13}$



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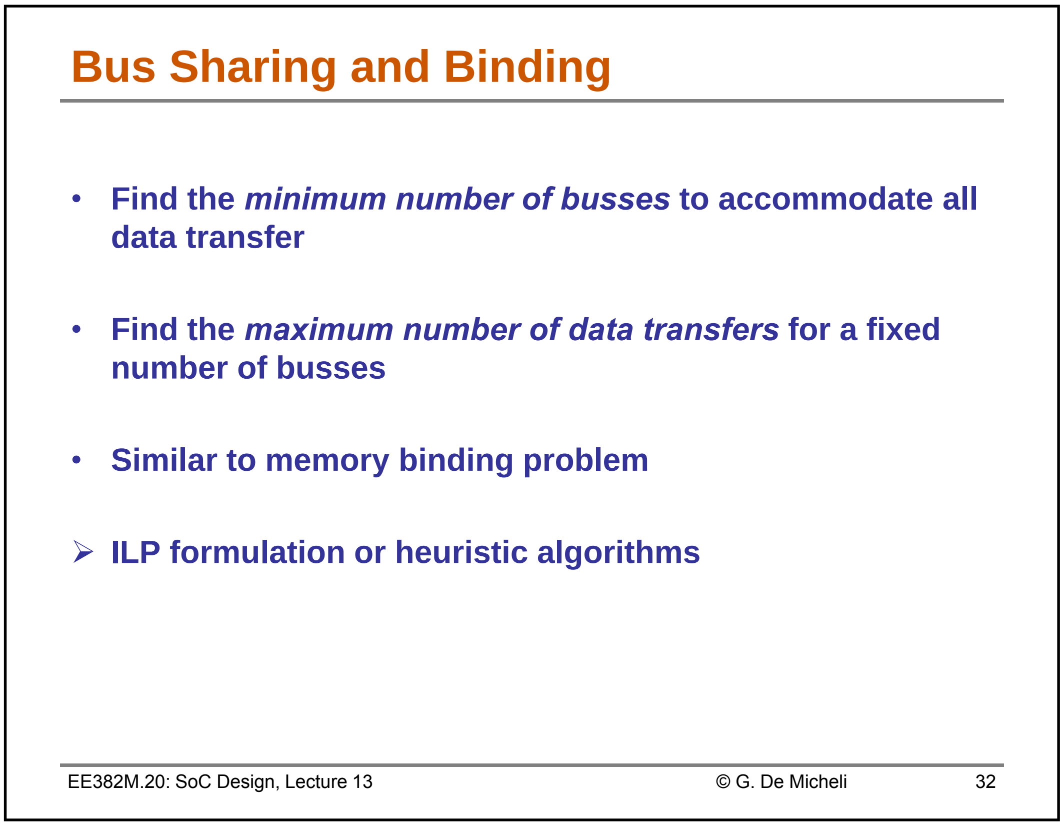
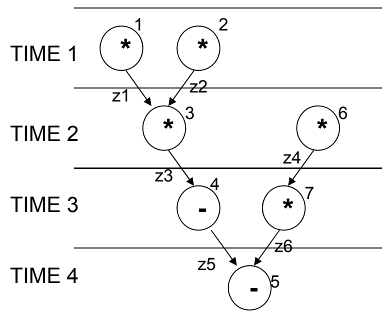


Bus Sharing and Binding

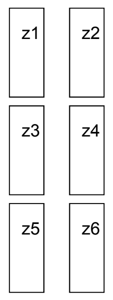
- **Find the *minimum number of busses* to accommodate all data transfer**
- **Find the *maximum number of data transfers* for a fixed number of busses**
- **Similar to memory binding problem**

➢ **ILP formulation or heuristic algorithms**

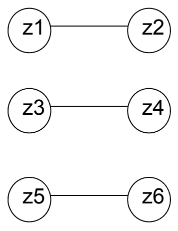
Bus Sharing Example



Sequencing graph



Connection usage



Conflict graph

- **One bus:**
  - 3 variables can be transferred
- **Two busses:**
  - All variables can be transferred

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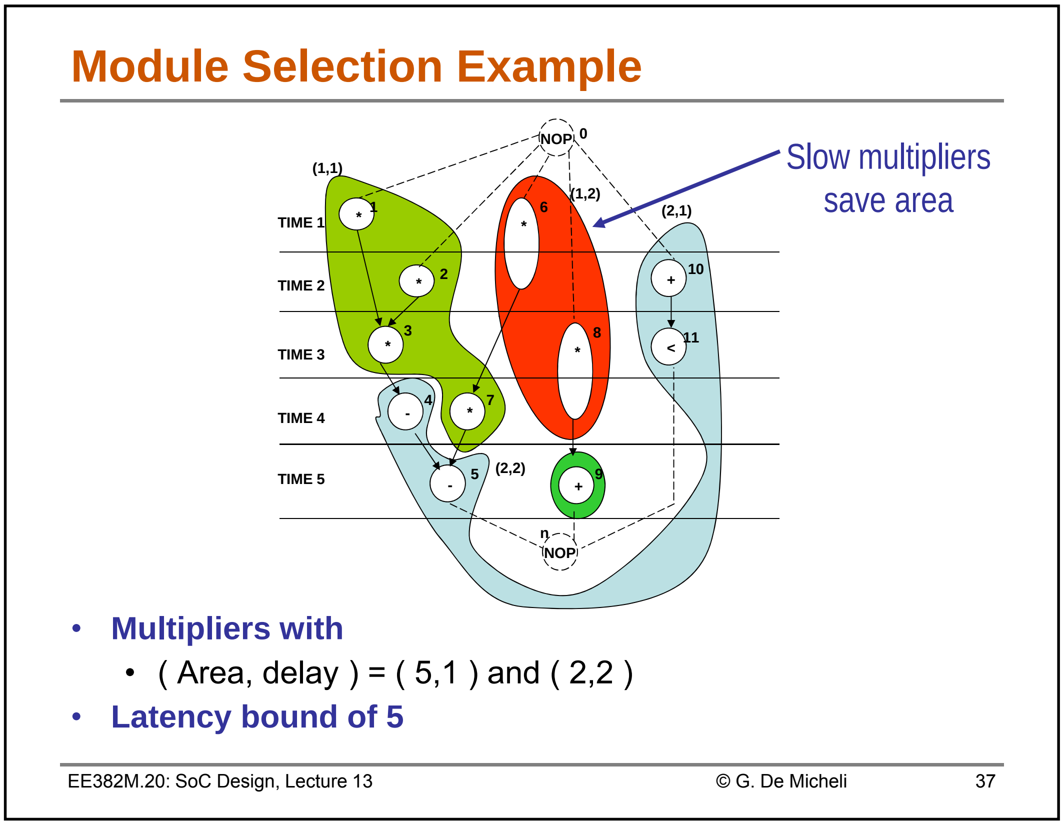
Module Selection Problem

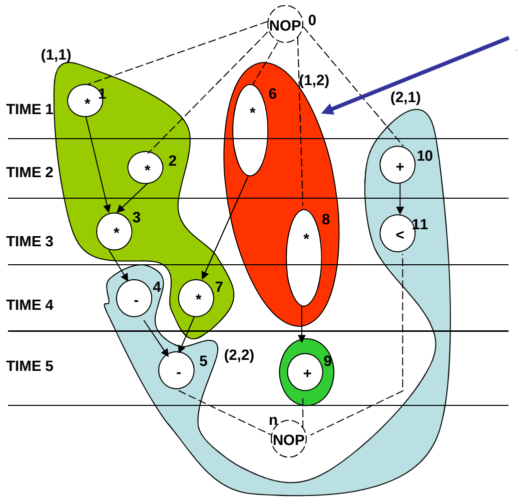
- **Extension of resource sharing**
  - Library of resources
  - More than one resource per type
- **Example**
  - Ripple-carry adder
  - Carry look-ahead adder
- **Resource modeling**
  - Resource *subtypes* with
    - (area, delay) parameters

Module Selection Solutions

- **ILP formulation**
  - Decision variables
    - Select resource sub-type
    - Determine (*area, delay*)
- **Heuristic algorithm**
  - Determine *minimum latency* with fastest resource subtypes
  - Recover area by using slower resources on non-critical paths



Module Selection Example



Slow multipliers save area

- **Multipliers with**
  - (Area, delay) = (5,1) and (2,2)
- **Latency bound of 5**

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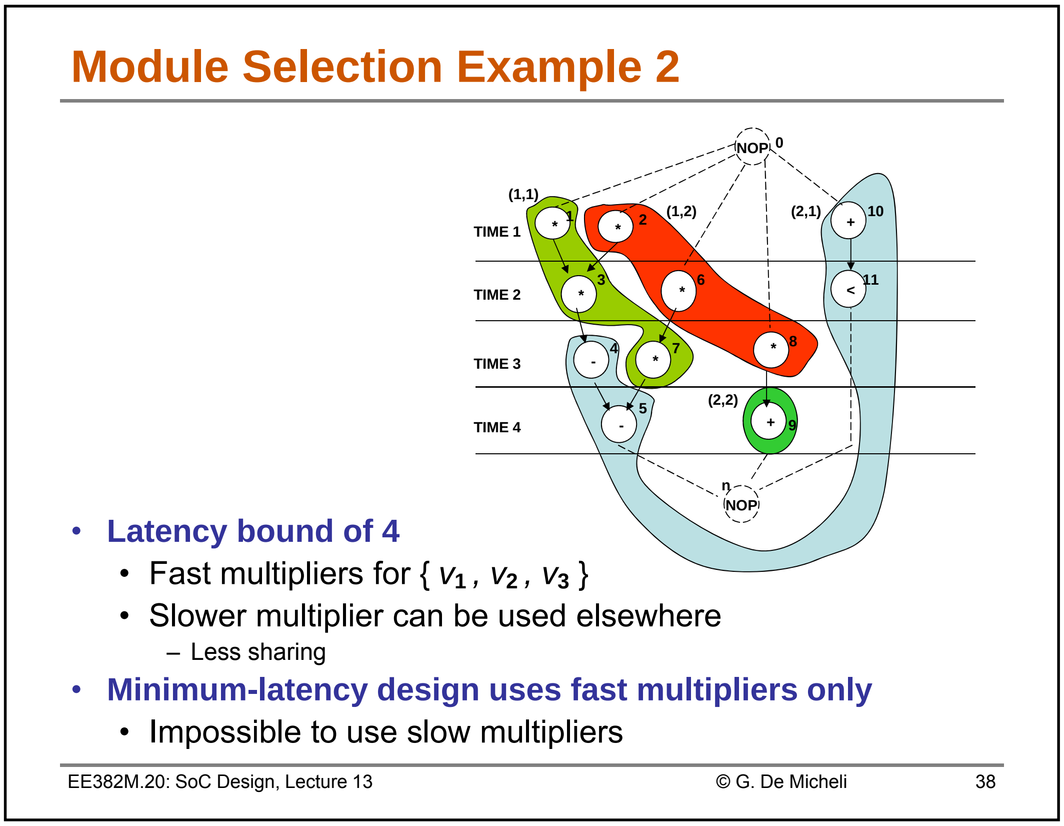
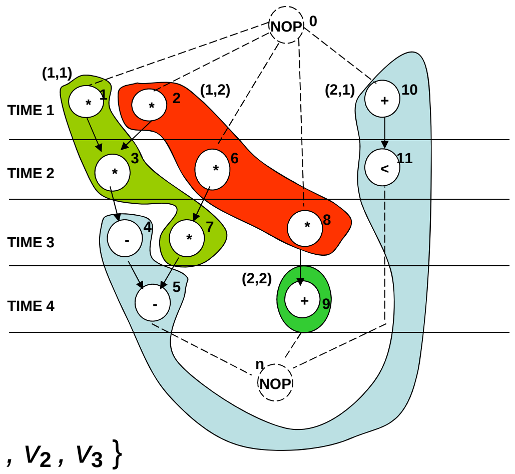


Module Selection Example 2



- **Latency bound of 4**
  - Fast multipliers for { v_1 , v_2 , v_3 }
  - Slower multiplier can be used elsewhere
    - Less sharing
- **Minimum-latency design uses fast multipliers only**
  - Impossible to use slow multipliers

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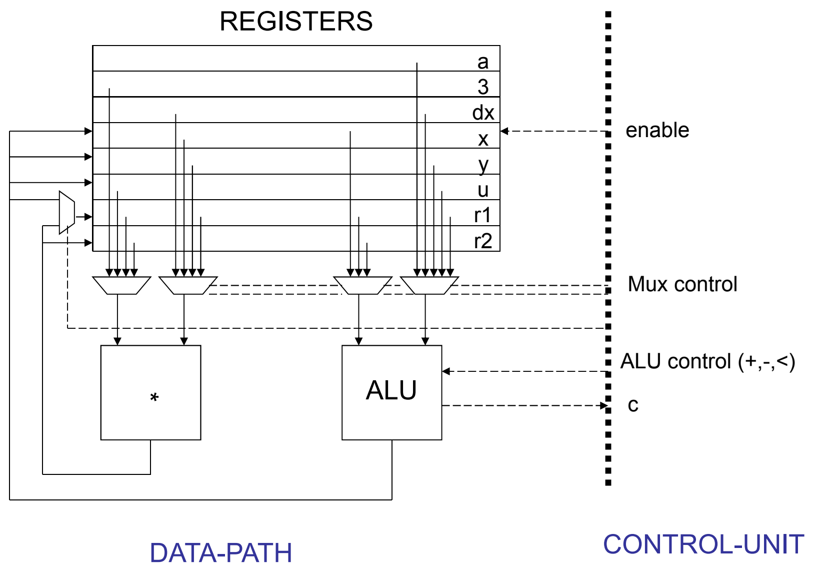
Data Path Synthesis

- **Applied after resource binding**
- **Connectivity synthesis**
  - Connection of resources to *multiplexers, busses* and *registers*
  - Control unit interface
  - I/O ports
- **Physical data path synthesis**
  - Specific techniques for regular datapath design
    - Regularity extraction

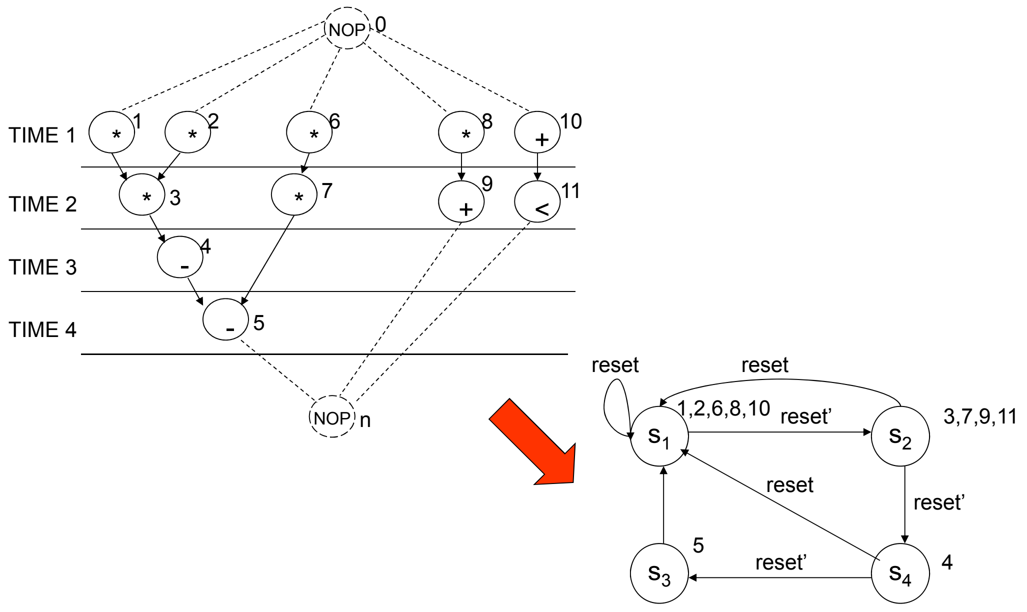
Data Path Synthesis Example



Control Synthesis

- **Synthesis of the control unit**
  - Sequencer
- **Logic model**
  - Synchronous FSM
- **Physical implementation**
  - Hard-wired or distributed FSM
  - Microcode

Control Synthesis Example



Lecture 17: Summary

- **Resource sharing is reducible to vertex coloring or to clique covering**
  - Simple for flat graphs
  - Intractable, but still easy in practice, for other graphs
  - Resource sharing has several extensions
    - Module selection
- **Data path design and control synthesis are conceptually simple but still important steps**
  - Generated data path is an interconnection of blocks
  - Control is one or more finite-state machines