Is Rate Adaptation Beneficial for Inter-Session Network Coding?

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***Abstract*—This paper considers the interplay between rate adaptation and inter-session network coding gains in wireless mesh or ad hoc networks. Inter-session network coding opportunities at relay nodes depend on packets being overheard by surrounding nodes – the more packets nodes overhear, the more opportunities relays have to combine packets, resulting in a potential increase in network throughput. Thus, by adapting its transmission rate, a node can increase the range over which its packets are overheard, enabling additional opportunities for coding and increased overall throughput. This paper considers inter-session coding, restricted to a single relay (bottleneck) node, or star network topology. Even for such simple topologies the optimal joint rate adaptation and network coding policy is known to be NP-hard. Optimal rate vector selection is a combinatorial optimization problem, which is NP-hard, and finding optimal coding scheme turns out to be a clique partitioning problem which is also NP-hard. So, we provide heuristics to find a sub-optimal rate vector and coding scheme. Additionally, we provide a linear programming formulation for network coding when only pairwise intersession coding is allowed. We evaluate the averaged throughput in two different scenarios, in which relays have different access opportunities, giving some intuition on the impact of rate adaptation in lightly and heavily loaded systems. The gains of joint rate adaptation and network coding are marginal when the relay has a higher access opportunity than other nodes, or when the MAC operates ideally it ranges from 9% to 19% as compared to a network without network coding and 4% over a network using regular network coding. While, when the relay has equal access opportunity as other source nodes, which is more typical of todays MAC protocols under heavy loads, the gain ranges from 40% to 62% as compared to the standard relaying case and is upto around 20% as compared to a network with regular pairwise network coding. This can further be increased to a gain of 40% to 120% by replacing pairwise coding with sub-optimal general network coding scheme.**

***Index Terms*—network coding, rate adaptation, relay, ad hoc network, mesh network, clique partitioning, combinatorial optimization**

I. Introduction

SINCE Ahlswede's seminal work [1], network coding has received significant attention. It has been shown that network coding achieves maximum capacity for multicast sessions in wired networks while increasing the reliability of lossy networks [2]–[6]. Most work to date has focused on, *intra-session* network coding, where only packets in the same session are encoded together, e.g., [4]–[6]. However, the work of Katti and Katabi which proposed the scheme called COPE, does allow coding across different sessions or *inter-session* network coding [7]. They observe that *overheard* packets, in the context of broadcast wireless media, can be effectively exploited to enable network coding resulting in further throughput improvements. This was followed by [8]–[10] where efforts were made to quantify the possible gains in general wireless networks with analytical frameworks. There has been also been much work toward diverse practical approaches for network coding working under weaker conditions than COPE, see [11], [12].

In this paper, we explore the potential gains of joint rate adaptation and network coding in random star network topologies. The star network topology is particularly interesting because there are abundant overhearing opportunities between neighboring nodes and a node acting as relay may take advantage of these to improve network performance. Several network coding schemes have been first explored in the context of such topologies, see [7], [11]–[13]. The topology is also be viewed as a part of a larger network, that is, one hop transmitters and receivers associated with any node form a star network. So, it is not only of intrinsic interest but also as a building block for larger wireless networks.

The key idea is to transmit at a reduced rate so as to allow additional neighboring nodes to overhear transmissions creating additional opportunities for network coding at a relay node. To this end, we propose a polynomial time heuristic to jointly determine sub-optimal Tx rates and two complementary inter-session coding schemes: pairwise coding and a more general heuristic coding. We study the performance of the proposed schemes in two regimes with different assumptions on MAC fairness, i.e., the relay node's access opportunities to the medium. In this paper, we do not endorse a specific scheduling scheme. It is assumed that coding scheme is separately designed on top of a MAC which may achieve one of two extremes in terms of access fairness. We find that joint rate adaptation and network coding is more effective when the relay has an equal access opportunity to the medium, or when the relay is congested due to limited access opportunities. Our contributions can be summarized as follows.

- We show that rate adaptation can improve network coding opportunities and increase overall network throughput.
- We characterize the network coding problem as a clique partitioning problem, which is NP-hard.
- We propose a pairwise coding scheme and show that its performance is very close to optimal when congested relays have higher access opportunities than other nodes.

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We also show that the pairwise coding problem is equivalent to well known maximum weighted matching problem which can be solved in polynomial time.

- We further propose a heuristic to allow a more general coding combinations and show that its performance is very close to optimal when a relay has the same access opportunity as other nodes.
- We characterize the joint Tx rate selection problem as combinatorial optimization problem and propose a heuristic to determine a good set of optimized transmission rates enabling better ”overhearing” scenarios.

The rest of this paper is organized as follows. In Section II, we develop the key intuition for coding aware rate adaptation and its potential gain in terms of increased coverage area and rate region. We then formally describe a system and formulate a problem in Section III. In Section IV, we propose two coding schemes, pairwise and then a more general heuristic. In Section V, we propose a polynomial time heuristic algorithm for sub-optimal transmission rate adaptation. The proposed algorithm is evaluated in Section VI and, we conclude in Section VII.

II. Coding Aware Rate Adaptation

A. Inter-session network coding

We begin by briefly summarizing the COPE approach [7]. In a network with COPE, destination nodes overhear packet transmissions from neighboring nodes and store them. Information about overheard packets is subsequently sent to neighboring relay nodes. Using this information a relay node combines packets such that the intended destination nodes can decode them. When destination nodes receive a combined packet, they can extract the desired packets from the combined packet using the packets they have previously overheard. This approach is effective at increasing throughput and alleviating congestion in relay nodes acting as traffic ‘hubs’.

B. Intuition underlying inter-session network coding in wireless environments

Note that inter-session coding opportunities at relay nodes depend on the packets overheard by surrounding nodes. That is, if nodes overhear more packets, a relay may have more opportunities for combining packets, potentially increasing network performance. Until now, researchers have assumed that only nodes in the transmission (Tx) range of the transmitter could overhear packets. So, node placement and the Tx range determined the possibility of such overhearing. However, we argue that one can increase coding opportunities by dynamically increasing the Tx range so that more nodes can overhear a transmission. This can be achieved by increasing the Tx power or reducing the Tx rate. Increasing the Tx power[1] may result in a reduction of spatial reuse, so it is not clear if one can realize increased performance. However by decreasing the Tx rate, one can increase the Tx range while keeping interference power at the same level as before. So the key insight in this paper can be summarized as follows:

[1]We consider fixed Tx power.

A node reducing its instantaneous Tx rate can increase its overhearing range, leading to possible increase in network coding opportunities and thus in network throughput.

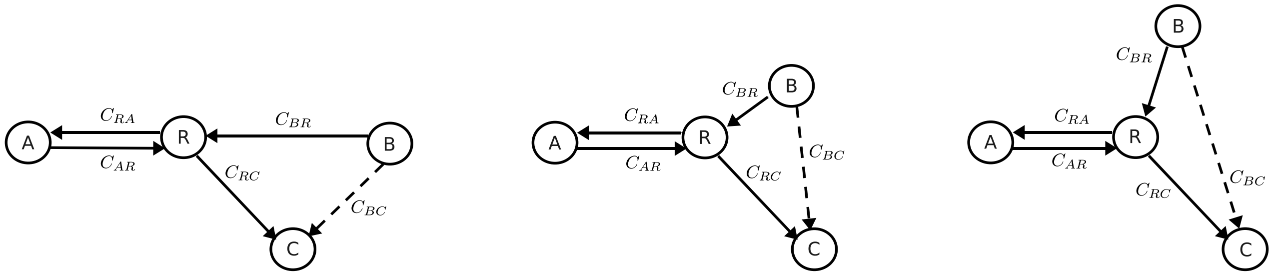
This statement appears counter intuitive since reducing Tx rate would typically decrease network throughput. Yet, interestingly, it turns out that one can, not only increase network throughput, but also increase the average individual throughputs of nodes. This is best explained through the simple example networks shown in Fig. 1. Using these examples, we will also show that the Tx rate needs to be reduced carefully, otherwise, it can lead to a deterioration of network performance. From now on, we will denote a MAC protocol which uses joint rate adaptation and network coding by RANC.

C. Rate reduction and performance

Consider the network configuration in Fig. 1 which includes three nodes and one relay. Node A transmits packets to node C and node B transmits packets to node A. Both transmissions are relayed through node R. Let C_{XY} denote the link capacity between node X and Y, and let $C_{AR} = C_{RA} = C_{RC} = 1$bps. Suppose node A and B each transmit one packet to relay R respectively. Then, node R relays the two packets to node C and A using either network coding or simple relaying. For each case, we will calculate *the network throughput* defined as the total number of bits transported divided by the total amount of time to transport the bits. For simplicity, we assume the instantaneous Tx rate on a link is given by its link capacity. We also assume that if node B transmits at rate C_{BC} to node C, then any node X with link capacity $C_{BX} > C_{BC}$ can overhear the transmission.

In Case (a) shown in Fig. 1 where $C_{BR} = 0.5$ and $C_{BC} = 1$, node B transmits with rate C_{BR} and node C can overhear the transmission since node B is closer to node C than to node R. In this case, node R can perform a bit-by-bit XOR over the two packets from node A and B and broadcast. In turn, node C receives the coded packet and can decode it since it has the overheard the packet from node B. The network throughput for this case is $T_{NC}^{(a)} = \frac{2b}{\frac{b}{1}+\frac{b}{0.5}+\frac{b}{1}} = \frac{1}{2}$, where the $2b$ in numerator is the number of bits transported in the network and $\frac{b}{1} + \frac{b}{0.5}$ in denominator is a time required to send two packets from source nodes A and B to relay R. The additional term, $\frac{b}{1}$ in the denominator is the time required for the relay to broadcast the combined packet. In a similar way, the network throughput when only relaying is used can be calculated and is $T_R^{(a)} = \frac{2b}{\frac{b}{1}+\frac{b}{0.5}+\frac{b}{1}+\frac{b}{1}} = \frac{2}{5}$. Clearly when overhearing occurs naturally, inter-session coding increases network throughput.

In Case (b) where $C_{BR} > C_{BC}$, overhearing no longer comes for free. Unless node B decreases its instantaneous Tx rate, it can not ensure node C overhears its transmissions. Once it reduces its instantaneous Tx rate (e.g, by reducing the modulation order or decreasing channel coding rate) node C will overhear the transmission and node R can perform network coding. Suppose, $C_{BR} = 1$ and $C_{BC} = 0.8$. Then, if B reduces its rate to R from 1 down to 0.8, we have $T_{NC}^{(b)} = \frac{8}{13}$, otherwise we have $T_R^{(b)} = \frac{1}{2}$. The benefit of rate reduction should be clear in this case. Hence, for both Case (a) and (b), network coding is beneficial.



(a) $C_{BR} < C_{BC}$ (b) $C_{BR} > C_{BC}$ (c) $C_{BR} \gg C_{BC}$

Fig. 1: Three simple toy networks where A sends a packet to C and B sends a packet to A through R:(a) C's overhearing B comes for free, (b) capacity can be increased with rate reduction from C_{BR} to C_{BC}, and (c) rate reduction degrades network performance.

Our last Case (c) is different. In this case, even though we can ensure node C overhears by reducing node B's instantaneous Tx rate, this will not increase the network throughput since B's transmission would be excessively slow. In this case, relaying gives higher network throughput: $T_{NC}^{(c)} = \frac{2}{5} < T_{R}^{(c)} = \frac{1}{2}$. These examples show that the TX rate should be carefully determined.

In Fig. 2, we illustrate the potential locations where instantaneous rate adaptation is beneficial for the simple network topology discussed above. To generate these figures we assumed, a static free space channel model with attenuation factor of 3.5 without fading and shadowing were considered. A transmit power of 1W and noise power spectral density of -174dBm were used. The minimum required SNR for decoding was set to as 3dB. The Tx rate of any link was set to the Shannon capacity of the link. Fig. 2a and Fig. 2b show what is the optimal relaying strategy for all possible locations for node B. Here, nodes A, R and C are fixed and we move node B to all possible locations in a two dimensional square of size $400 \times 400\text{m}^2$. The distance between nodes A and R is equal to that of node R and C - 80 meters. For any given location for node B, we first check whether node B can communicate with either node R or A. If node B can not communicate with either of them, then, we declare it "not reachable." If node B can communicate with either of them, then, three possible relaying strategies were compared: direct delivery, simple relaying, and network coding. The throughput is computed as previously. The best relaying method, i.e., giving the highest throughput, was selected. One can visualize the expanded region where joint rate adaptation and network coding are used by comparing Fig. 2b to Fig. 2a.

Fig. 2c exhibits throughput gain of rate adaptive network coding compared to direct relaying method. Clearly the gain depends on node B's location. We get the highest gain when node B is at the midpoint of the line connecting nodes R and C, since at that location node B can send at its highest rate without rate reduction. Note that for network coding, node B should make sure that both nodes R and C hear the transmission. Fig. 2d exhibits the gain of network coding with rate adaptation versus regular network coding without rate adaptation. One can see that the region where the gain is positive is the same as the expanded region in Fig. 2b. Depending on the location of node B, one can expect various gains upto 33%. Note that this is an additional gain from rate adaptation for network coding. More generally one might expect this result to hold for network topologies with more nodes around relay hubs where there are more coding opportunities. In other words, it appears that rate adaptation should reduce the area of region where relaying and direct delivery are better than network coding.

D. Rate Region

In this section, we consider the gain of joint rate adaptation and network coding in terms of the increased average rate region for users with fixed locations. We consider the rate region for the two sessions in the network shown in Fig. 1b. The resulting rate region exhibits the intuition explained above: reducing instantaneous rates can increase network throughput, i.e., the sum of the average rates.

In particular, we consider Case (b) in Fig. 1 where rate adaptation is necessary to enable network coding. A bit stream from node A flows to node C and another bit stream from node B flows to node A through R. Let R_A and R_B denote the average rate of these streams. Let λ_1, λ_2, λ_3, and λ_4 be the fraction of time transmissions from node A to R, from node B to R, from node R to A, and from node R to C occur, respectively. These must satisfy $\sum \lambda_i = 1$.

Direct Relaying: When there is no rate adaptation, we can not perform network coding. So, the associated rate region Λ can be determined as follows. The rates R_A and R_B should be smaller than the product of the link capacity's traversed by the streams and the time fractions allocated to each flow, i.e.,

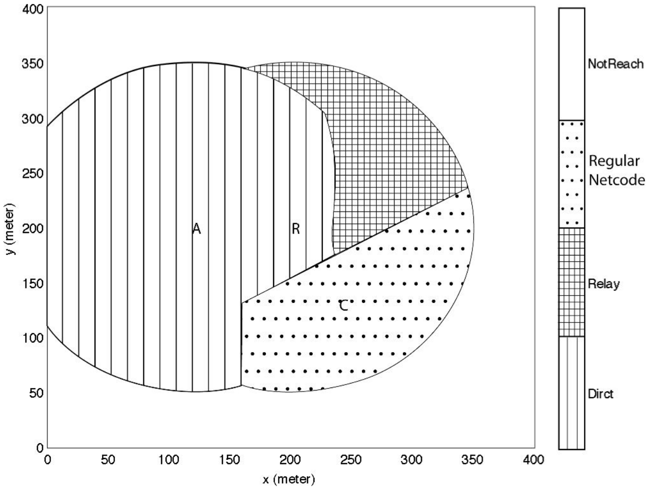
$$\Lambda = \Big\{(R_A, R_B) \mid R_A \leq \lambda_1 C_{AR}, \\ R_B \leq \lambda_2 C_{BR},\ R_B \leq \lambda_3 C_{RA}, R_A \leq \lambda_4 C_{RC}, \\ \sum \lambda_i = 1,\ R_A \geq 0,\ R_B \geq 0\Big\}.$$

Combining the first five constraints gives following inequality:

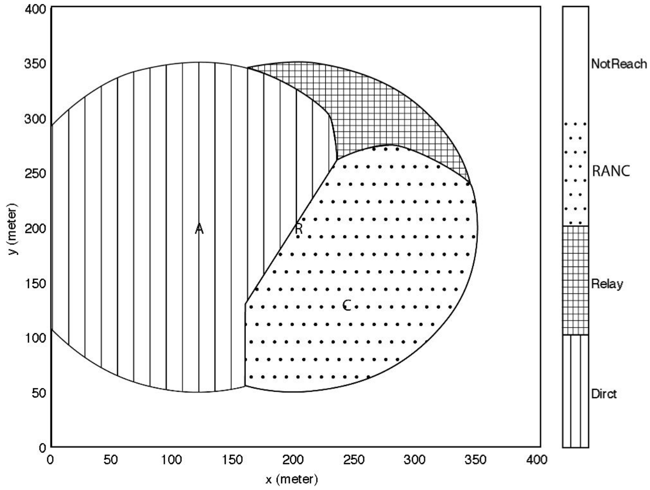
$$\frac{R_A}{(C_{AR}^{-1} + C_{RC}^{-1})^{-1}} + \frac{R_B}{(C_{BR}^{-1} + C_{RA}^{-1})^{-1}} \leq 1$$

The rate region Λ is illustrated by the triangular region connecting three points $(0, 0)$, $(0, (C_{AR}^{-1} + C_{RC}^{-1})^{-1})$, and $((C_{BR}^{-1} + C_{RA}^{-1})^{-1}, 0)$, which is the region with vertical hatching in Fig. 3.

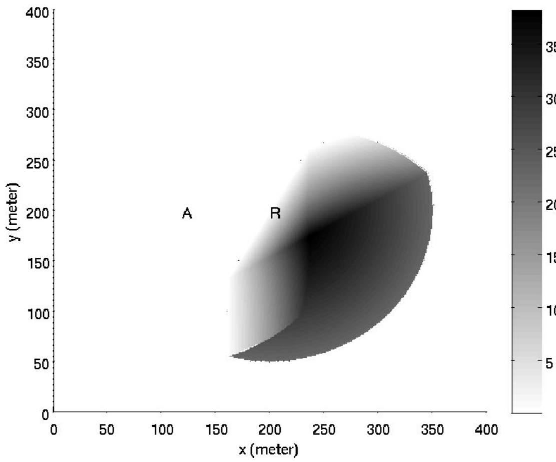
RANC: If rate adaptation is performed, node B can reduce its Tx rate so that node C can overhear its transmission.



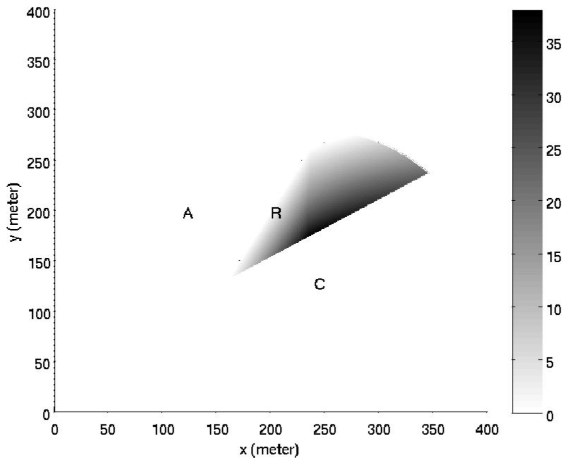
(a) Optimal relaying method in a network with regular network coding.



(b) Optimal relaying method in a network with network coding with rate adaptation.



(c) Throughput gain (%) of rate adaptive network coding compared to without network coding.



(d) Throughput gain (%) of rate adaptive network coding compared to regular network coding.

Fig. 2: Optimal relaying strategy for the four nodes network in Fig. 1 as the position of node B is varied.

This would in turn allow network coding to be performed at node R. In this case, the rate R_B should be less than λ_2 times the minimum of two link capacities, that is $R_B < \lambda_2 \min\{C_{BC}, C_{BR}\}$. The new rate region Λ^{RANC} is given as follows:

$$\Lambda^{\text{RANC}} = \Big\{(R_A, R_B) | R_A \leq \lambda_1 C_{AR}, R_B < \lambda_2 \min\{C_{BC}, C_{BR}\}, R_A \leq \lambda_3 C_{RC}, R_B \leq \lambda_3 C_{RA}, \sum \lambda_i = 1, R_A \geq 0, R_B \geq 0\Big\}.$$

Again the above constraints require the following inequalities to be satisfied:

$$\frac{R_A}{(C_{AR}^{-1} + C_{RC}^{-1})^{-1}} + \frac{R_B}{C_{BC}} \leq 1,$$

$$\frac{R_A}{C_{AR}} + \frac{R_B}{(C_{BC}^{-1} + C_{RA}^{-1})^{-1}} \leq 1,$$

$$\frac{R_A}{(C_{AR}^{-1} + C_{RC}^{-1})^{-1}} + \frac{R_B}{C_{BR}} \leq 1, \quad \text{and}$$

$$\frac{R_A}{C_{AR}} + \frac{R_B}{(C_{BR}^{-1} + C_{RA}^{-1})^{-1}} \leq 1.$$

Each of these corresponds to a half space and the intersection of them and the first quadrant, the region filled with horizontal hatching, is given as rate region Λ^{RANC}.

So, the entire rate region is given as the convex hull of the union of the two rate regions, i.e., $\text{conv}(\Lambda \cup \Lambda^{\text{RANC}})$. As shown in Fig. 3, rate adaptation allows node R to perform network coding resulting in an increase of the rate region. The additional regions shaded in gray can be achieved by time sharing between network coding and relaying. Note that if we let $C_{BC} \rightarrow \infty$, we have the two-way relay network's rate region studied in [14]. From the above discussion, it should be clear that careful selection of Tx rates can increase network throughput. So, the next question is what rates should be used in a general network. In the next section, this problem is formulated and explored.

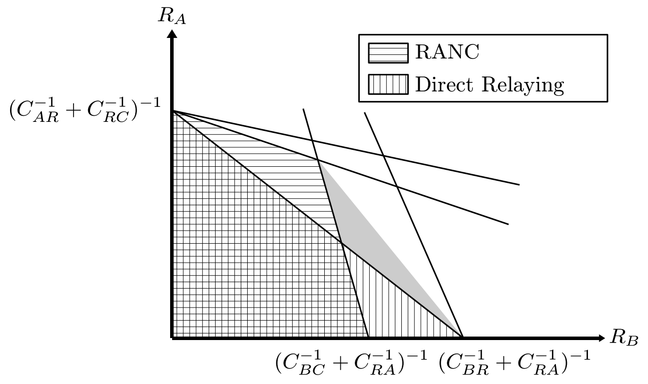


Fig. 3: The Rate region of network in Fig. 1b.

III. System Model and Problem Formulation

A. Network Model

We consider a fixed wireless network having a relay z via which a set of source nodes S communicate with a set of destination nodes D with $D \cap S = \phi$. This model precludes the possibility of direct transmission from source to destination node since performance gains from interaction between relays and neighboring nodes are our main concern. Each source node $x \in S$ has a corresponding distinct destination node $y \in D$. A pair of source and destination nodes specify a session. Let $s(y)$ denote the corresponding source node of a node $y \in D$. Each source node has only one destination node and vice versa[2]. We assume an ideal MAC without contention, i.e., each source nodes take its turns to transmit its packet. The number of bits per packet is fixed.

B. Transmission rate vector

We define $\mathcal{R} = (r^i, 1 \le i \le |\mathcal{R}|)$ to be an ordered set of discrete rates supported by nodes, where $|\mathcal{R}| < \infty$ denotes the cardinality of the set $\mathcal{R}$. So, r^1 and $r^{|\mathcal{R}|}$ are the lowest and the highest supported rates, respectively. The assumption that $\mathcal{R}$ is a finite set is intended to model today's systems, where the number of Tx rates supported is limited due to modulation, slot length, coding rate, etc [15]. Note that even though we have a rate set $\mathcal{R}$, the actual Tx rate at any link is determined by the transmit power, distance, thermal noise, bandwidth, interference, etc. The maximum Tx rate between node x and y is formally defined as

$$r_{xy}^m = \max\left\{ r \in \mathcal{R} | r \le \frac{w}{2}\log(1+\frac{\rho\|x-y\|^{-\alpha}}{\eta + I}) \right\},$$

where ρ is the Tx power of a node, η is noise power spectral density, w is system bandwidth under consideration, α is the attenuation factor, $\|x-y\|$ is distance between two nodes x and y, and I is the amount of external interference power. Losses from imperfect measurements of interference power and thus incorrect rate adaptation will be reflected as a packet error probability[3]. So, the effective Tx rate is given by r_{xy}^m times the packet delivery probability between x and y. For $x \in S$, let

$$\mathcal{R}_x = \left\{ \min(r_{xz}^m, r_{xy}^m) | y \in D \cup \{z\} \right\}$$

be the set of the highest rates from node x to relay z that allow at least one node $y \in D \cup \{z\}$ including z to hear node x's transmission. This can be understood as the set of node x's possible highest Tx rates to relay z which allow overhearing by others. The objective is to determine a vector $\mathbf{r} = (r_{xz} : x \in S)$ of Tx rates that each node will use to transmit to the relay. We have at most $\prod_{x \in S} |\mathcal{R}_x|$ such vectors since each node $x \in S$ has $|\mathcal{R}_x|$ different potential Tx rates.

C. Overhearing Graph and Clique Partition

For each Tx rate vector $\mathbf{r}$, a set of destination nodes that overhear the transmission can be determined, which then also determines which packets the relay node can combine. Based on the overhearing status of each destination node, we construct a graph $G(\mathbf{r})$ such that any two destination nodes which overhear each other's source node are connected by an undirected edge. We identify sets of packets that can be coded together with valid partitions $\mathbf{V}(\mathbf{r})$ of the graph. These are explained below.

The graph $G(\mathbf{r}) = G(D, E_{\mathbf{r}})$ is composed of a set of destination nodes D and a set of undirected edges $E_{\mathbf{r}}$. The link $(i, j) \in D \times D$ is in $E_{\mathbf{r}}$ if and only if node i overhears node $s(j)$'s transmission and node j overhears node $s(i)$'s transmission. We say node i overhears node $s(j)$ if node i is in the transmission range of node $s(j)$, or equivalently if $r_{s(j)i}^m \ge r_{s(j)z}$. So given a Tx rate vector $\mathbf{r}$ uniquely determines an overhearing graph. The set of links $E_{\mathbf{r}}$ is formally defined as

$$E_{\mathbf{r}} = \left\{ (i,j) \in D^2 \mid r_{s(i)j}^m \ge r_{s(i)z}, r_{s(j)i}^m \ge r_{s(j)z} \right\}.$$

The overhearing graph is used to find optimal sets of sessions to combine via network coding. Fig.4 exhibits an example of an overhearing graph for four sessions 1, 2, 3 and 4. Note that, by construction, sessions corresponding to nodes in any valid clique in $G(\mathbf{r})$ can be combined because each session or destination node in the valid clique can overhear other destination nodes' source nodes. In the example shown in Fig.4, $\{a,b,d\}, \{b,d\}, \{a,d\}, \{a,c\}, \{a,b\}, \{a\}, \{b\}, \{c\}$ and $\{d\}$ are valid cliques. So, finding which sessions to combine corresponds to finding a partition $\mathcal{V} = (C_1, C_2, \cdots)$ of $G(\mathbf{r})$ such that each set C_i of the partition is a clique. We shall refer to such partition as a *clique-partition*. For example, $(\{a,b,d\},\{c\})$ is a valid clique partition of the overhearing graph in Fig.4. Note that there may be one or more valid clique-partitions in the graph. We let $\mathbf{V}(\mathbf{r})$ denote the set of all valid clique-partitions of $G(\mathbf{r})$.

D. Cost Function and Formulation

For a given $\mathbf{r}$ and $\mathcal{V} \in \mathbf{V}(\mathbf{r})$, we define the *uplink cost* as a total time required for all source nodes to transmit their packets to the relay, i.e., $u(\mathbf{r}) \equiv \sum_{x \in S} \frac{1}{r_{xz}}$. The uplink cost is a function of rate vector $\mathbf{r}$. Similarly, the *downlink cost* is a total time required for relay to send the

[2]This assumption can be justified by separating a physical source(destination) node with two different destination(source) nodes into logical two source(destination) nodes.

[3]In the evaluation conducted Section VI, we ignore interference I since we consider a star topology network without contention.

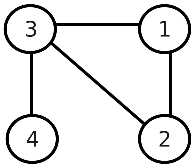


Fig. 4: An example of overhearing graph $G(\mathbf{r})$

received packets to corresponding destination nodes, i.e., $d(\mathcal{V}) \equiv \sum_{C\in\mathcal{V}} \frac{1}{\min_{y\in C} r_{zy}}$, where for each $C \in \mathcal{V}$ we take the minimum rate of r_{zy} for $y \in C$ since we want to ensure all destination nodes of the combined packets receive their associated packets. Note that downlink cost depends on the selected combination of sessions or clique partion $\mathcal{V} \in \mathbf{V}(\mathbf{r})$. Our objective is to find an optimal rate vector $\mathbf{r}^*$ and an associated clique partition of $G(\mathbf{r})$ which minimize the sum of the uplink and the downlink cost, i.e., maximize a throughput. This optimization problem is formally stated as follows:

$$\min_{\substack{\mathbf{r}\in\mathbf{R}\\ \mathcal{V}\in\mathbf{V}(\mathbf{r})}} \sum_{x\in S} \frac{1}{r_{xz}} + \sum_{C\in\mathcal{V}} \frac{1}{\min_{y\in C} r_{zy}}. \tag{1}$$

Let $\mathbf{r}^*$ denote a solution of the above problem. This combinatorial optimization problem can be chracterized as follows. For a given $\mathbf{r} \in \mathbf{R}$, we first need to find the minimum downlink cost in order to evaluate the total cost for the $\mathbf{r}$. Let $\mathcal{V}^*_{\mathbf{r}}$ be the optimal clique partition giving the minimum cost of the downlink under the rate vector $\mathbf{r}$. Then, our original problem can be rewritten as follows:

$$\min_{\mathbf{r}\in\mathbf{R}} \; u(\mathbf{r}) + d(\mathcal{V}^*_{\mathbf{r}}), \tag{2}$$

where $\mathcal{V}^*_{\mathbf{r}}$ is given as

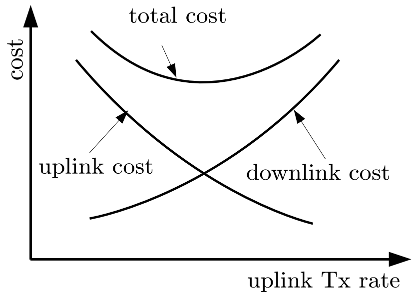
$$\mathcal{V}^*_{\mathbf{r}} \in \arg \min_{\mathcal{V}\in\mathbf{V}(\mathbf{r})} d(\mathcal{V}). \tag{3}$$

Fact 1: Determining an optimal partition $\mathcal{V}^*_{\mathbf{r}}$ is an NP-hard problem.

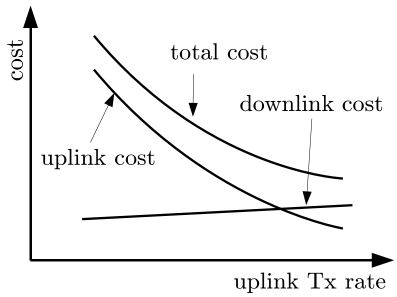
The problem is a variant of well known clique partition problem (CPP). CPP partitions a graph G into disjoint cliques with a minimum number of cliques. Determining the minimum clique partition is known to be a NP-hard problem. Indeed problem (3) is reduced to the classical CPP if cost for each clique is equal to 1, see [16], [17]

E. Tradeoff between uplink and downlink costs

Note that decreasing the Tx rate for uplink transmissions makes it easier for each other nodes to overhear these transmissions, making the overhearing graph more connected. This in principle results in an increase of the possibilities for coded transmissions, which may end up decreasing the downlink delay. So, there is a tradeoff between the uplink and the downlink costs(delay). Fig.5 shows two typical tradeoff relations between uplink and downlink costs along side the corresponding total cost. Our objective is to find optimal uplink Tx rates where the sum of those two costs are minimized. In Fig. 5a, the optimal strategy is indeed to reduce the Tx rate will be the reduced rate, while in Fig. 5b, the optimal rate is highest possible - backing off does not provide a substantial network coding opportunity and/or substantial gains.



(a)



(b)

Fig. 5: Typical tradeoff relations between uplink and downlink costs. In Case (a), rate reduction increases network coding opportunities and corresponding total cost reduction. While, in case (b), rate reduction does not give positive gain.

IV. Coding Schemes

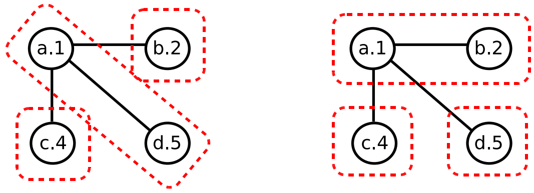
The NP-hardness of CPP lead us to consider examining other simplified clique partitioning schemes. We first consider pairwise coding, in which up to two sessions can be combined. We then consider a second heuristic approach in which the number of combined sessions is not limited.

A. Pairwise Network Coding

Suppose we restrict the space of clique partitions $\mathbf{V}(\mathbf{r})$ to partitions including cliques of size less than or equal to two, i.e., we determine only an approximate clique partition, $\mathcal{V}^{p*}_{\mathbf{r}}$. In this case, (3) can be rewritten as the following binary integer program:

$$\begin{aligned} &\min_{a_l, l\in E_{\mathbf{r}}} \sum_{y\in D} \frac{1}{r_{zy}} + \sum_{l\in E_{\mathbf{r}}} a_l c_l \\ &\quad\text{s.t.} \sum_{l\in\{(y,i)\in E_{\mathbf{r}} | i\in D\}} a_l \le 1, \; \forall y \in D \\ &\quad a_l \in \{0,1\}, \quad \forall l \in E_{\mathbf{r}} \\ &\quad c_l = \frac{1}{\min\left\{r_{zy_1(l)}, r_{zy_2(l)}\right\}} - \frac{1}{r_{zy_1(l)}} - \frac{1}{r_{zy_2(l)}} \; \forall l \in E_{\mathbf{r}}, \end{aligned} \tag{4}$$

where $y_1(l)$ and $y_2(l)$ are the two nodes at two end points of undirected link l. Also a_l is binary variable for link $l \in E_{\mathbf{r}}$,



(a) As a tie breaking rule, a node with maximum Tx rate from relay to itself is selected as p. So, d is selected as p and merged with a. Nodes b and c forms individual cliques. Downlink cost: $d(\mathcal{V}) = \frac{1}{1} + \frac{1}{4} + \frac{1}{2}$.

(b) As a tie breaking rule, a node with minimum Tx rate from relay to itself is selected as p. So, b is selected as p and merged with a. Nodes c and d forms individual cliques. Downlink cost: $d(\mathcal{V}) = \frac{1}{1} + \frac{1}{4} + \frac{1}{5}$.

Fig. 6: Partitions induced by two different tie breaking rules: the $u.w$ inside each node denote a node name u and the maximum Tx rate w from relay to node u respectively.

if packets destined to $y_1(l)$ and $y_2(l)$ then $a_l = 1$, otherwise $a_l = 0$. Our restriction makes the sub-problem (3) easier.

Fact 2: There is a polynomial time algorithm to determine an optimal set of sessions over which to perform pairwise coding.

In fact the maximization version of (4) corresponds to the well known maximum weighted matching problem (MWMP). MWMP finds a matching $M \subset E$ of graph $G = (V, E)$ such that the sum of weight of $l \in M$ is maximized. The MWMP is the first "true" binary integer programming problem with polynomial time algorithm. It is "true" binary integer problem in the sense that it can not be solved merely by linear programming relaxation, see [18]. The linear programming relaxation of (4) with additional constraints is given as follows:

$$\begin{aligned} \min_{a_l, l \in E_{\mathbf{r}}} & \sum_{y \in D} \frac{1}{r_{zy}} + \sum_{l \in E_{\mathbf{r}}} a_l c_l \\ \text{s.t.} & \sum_{l \in \{(y,i) \in E_{\mathbf{r}} | i \in D\}} a_l \le 1, \ \forall y \in D, \\ & \sum_{l \in E(H)} a_l \le \left\lfloor \frac{|H|}{2} \right\rfloor \quad \forall \text{ odd sets } H \subseteq D \\ & a_l \in \mathbb{R}_+, \quad \forall l \in E_{\mathbf{r}}, \end{aligned} \tag{5}$$

where c_l is defined in (4), $E(H)$ is the set of edges with both ends in H. Note the role of additional constraints called odd set constraints, odd set is a set with k nodes, for $k = 3, 5, 7, \cdots$. The constraints restrict the number of pairs in any odd set $H \subseteq D$ be less than or equal to $\lfloor |H|/2 \rfloor$, i.e., it prevents odd cycles. The LP-relaxation without this restriction may not result in integer solution.

B. Heuristic Coding

We provide a heuristic solution for CPP, which is a modified clique partitioning algorithm based on [19]. Our algorithm is shown in Alg.1. It starts with a clique composed of single node p in N, the set of all nodes. A node p with a minimum number of edges is selected. This is done in order to induce as many cliques as possible (Step 3). If there are several such nodes, we break ties by selecting a node which has a minimum Tx rate. Selecting such a node is likely to result in lower downlink cost. Indeed such nodes will have to operate at their associated low transmission rates, other nodes in their associated cliques, having potentially higher transmission rates, can be viewed as getting a "free ride." So pairing such node with other nodes having a higher Tx rates is likely to give a decreased downlink cost. Fig.6 shows two examples of clique partitions resulting from different tie breaking rules, in which it turns out that selecting a node with minimum Tx rate is likely to lead to a better (lower) downlink cost. This also applies to tie breaking in Step 4. Then, a neighbor of node p's, say q is selected and is merged with p. The neighbor q with a maximum number of common edges with p is selected since it is likely to maximize the current clique size (Step 4). Edges from p and q that do not connect to their common neighbors are deleted (Step 5) and the two nodes are merged. Then, the remaining common neighbors will be candidate nodes for the creation of other cliques in the partition. The merged node is renamed as p and its Tx rate is set to the minimum of Tx rates from the relay to the merged nodes (Step 6). Note that sets of nodes that are merged together form a clique. The size of the clique is increased through the proposed merging process. If there is no more candidate nodes to combine, the node p is removed from N and all the merged nodes into p form a clique (Step 7). If N is not empty, a new node p is selected and the above merging process is repeated. This algorithm results in a valid clique partition $\hat{\mathcal{V}}_{\mathbf{r}}^*$ which should have a reasonably small cost. Note that this heuristic is deterministic polynomial time algorithm with time complexity of $O\left(|N|^4\right)$.

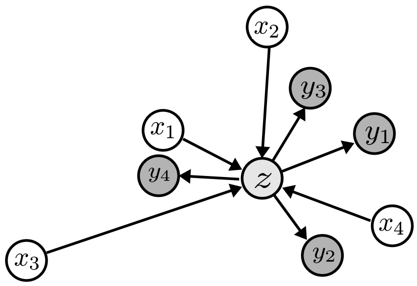
Algorithm 1 Sup-optimal Clique Partition Algorithm

1: $N \Leftarrow D$
2: **while** $N \neq \phi$ **do**
3: Select $p \in N$ with minimum degree. Tie breaking: select p with minimum tx rate from relay to p.
4: Select a node q, a neighbor of p, with the maximum common edges with p. Tie breaking: select q with minimum Tx rate from relay to q
5: Delete edges from p and q that do not connect to their common neighbor.
6: Merge p and q and rename it as p with transmission rate $\min\{r_{zp}, r_{zq}\}$.
7: If p has any remaining edge goto step 4 otherwise p become a new clique. Remove p from N.
8: **end while**

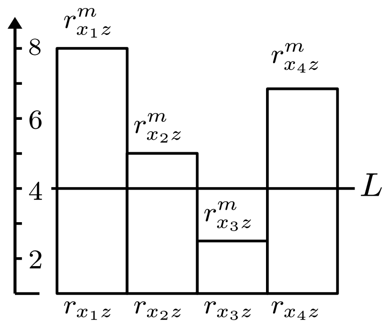
V. Tx Rate Vector Selection

A. Assumption

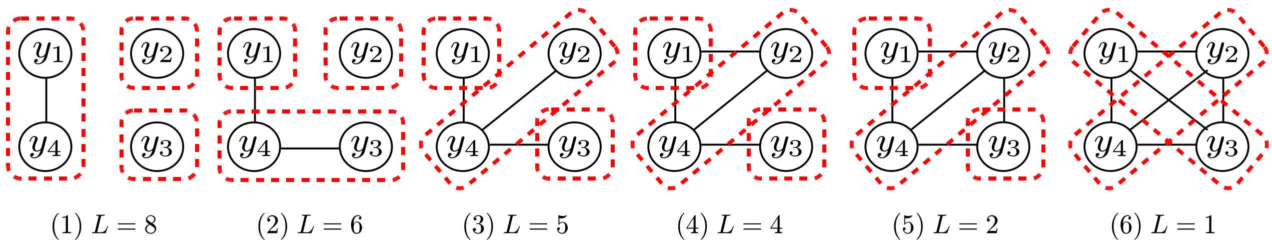
We assume that any source node in this network knows the average link rate between itself and other reachable destination nodes including the relay. These might be estimated by observing signaling messages or pilot signals. This information is shared by each source node with the relay node, which in turn runs following algorithms to perform joint rate adaptation and network coding.



(a) Network with four sessions



(b) Max Tx rates of four source nodes and Tx rate bar L. The actual Tx rate of node x_i is determined as $\min(r^m_{x_iz}, L)$.



(c) As Tx rate bar L decreases from top to bottom overhearing graph (four nodes and black edges between them) changes from (1) to (6) in order. Corresponding optimal pairwise cliques (in dotted red) also change.

Fig. 7: Tx rate bar lowering and corresponding transformation of overhearing graph in the network with four sessions

B. Suboptimal Rate Selection Algorithm

In this section, we provide a heuristic algorithm to find sub-optimal rate vector $\hat{\mathbf{r}}^*$, for problem (2). Note that there are $\prod_{x\in S}|\mathcal{R}_x|$ possible Tx rates vectors. Rather than doing an exhaustive search, we shall take advantage of the structure of $\mathbf{R}$. First, note the relation between the Tx rate vector and the overhearing graph in a given network. The maximum Tx rate vector $\mathbf{r}^m = (r^m_{xz}|x \in S)$ results in an overhearing graph $G(\mathbf{r}^m) = G(D, E_{\mathbf{r}^m})$, with a minimum number of edges. Note that this is a subgraph of all possible overhearing graphs that can be generated for all feasible Tx rate vectors. At the other extreme if $\mathbf{r}^1 = (r_{xz} = r^1|x \in S)$, then the corresponding graph $G(\mathbf{r}^1) = G(D, E_{\mathbf{r}^1})$ is the supergraph of all possible overhearing graphs in this network. So, these two graphs are at extreme ends. For $\mathbf{r} \neq \mathbf{r}^m, \mathbf{r}^1$, the set of edges $E_\mathbf{r}$ satisfies $E_{\mathbf{r}^m} \subseteq E_\mathbf{r} \subseteq E_{\mathbf{r}^1}$. Our basic strategy is to consider sequence of vectors $\mathbf{r_1}, \mathbf{r_2}, \cdots$, such that $E_{\mathbf{r}^m} \subseteq E_{\mathbf{r_1}} \subseteq E_{\mathbf{r_2}} \subseteq \cdots \subseteq E_{\mathbf{r}^1}$. We shall do this by gradually decreasing the Tx rates by limiting individual Tx rate with Tx *rate bar* L. Initially, L is set to the highest rate $r^{|\mathcal{R}|}$, and gradually decreased. For a given L, the transmission rate from source to relay may not be feasible, so, the Tx rate of a source node $x \in S$ is set to $\min\{r^m_{xz}, L\}$. With this choice of rates we can quickly estimate the value cost function. Note that this heuristic is deterministic polynomial time algorithm with time complexity $O\left(|N|^4\right)$.

*1) **Clique Partition and Cost Evaluation**:* Every time we lower Tx rate bar L, a new graph which possibly includes additional edges is produced. Then, a pairwise clique partition of the graph is found by solving (5). Based on this clique partition, we can evaluate the corresponding downlink cost. To evaluate uplink, we need to only optimize over Tx rates from the sources S to relay that result in the same overhearing graph and thus the same downlink cost. If a clique is of size two, e.g., $\{y_1, y_2\}$, then, $r_{s(y_1)z}$ is be chosen as high as possible, i.e., $\min\{r^m_{s(y_1)z}, r_{s(y_1)y_2}\}$. Note that if $r_{s(y_1)z}$ is increased more than that, then, one can not ensure y_2 will overhear its transmission, and the clique is broken. The same applies to y_2. If a clique size is one, e.g., $\{y\}$, $r_{s(y)z}$, is reverted to its original max rate $r^m_{s(y)z}$, which may be higher than L. This procedure determines the minimum uplink cost.

Algorithm 2 Sup-optimal Rate Selection Algorithm

$i \Leftarrow |\mathcal{R}|$
while $i > 0$ **do**
  $L \Leftarrow r^i$
  $r_{xz} \Leftarrow \min\{r^m_{xz}, L\}, \forall x \in S$
  $\hat{\mathcal{V}}^*_\mathbf{r} \Leftarrow$ FindCliquePartition $(\mathbf{r})$
  $r_{s(j)z} \Leftarrow \max_{y\in C\cup\{z\}\setminus\{j\}} r^m_{s(j)y}, \forall j \in C, \forall C \in \hat{\mathcal{V}}^*_\mathbf{r}$
  $\mathbf{r}^i \Leftarrow (r_{xz}|x \in S)$
  $\mathcal{E}(\mathbf{r}^i) \Leftarrow \sum_{x\in S}\frac{K}{r_{xz}} + \sum_{W\in\hat{\mathcal{V}}^*_\mathbf{r}}\frac{1}{\min_{y\in W} r_{zy}}$
  $i \Leftarrow i - 1$
end while
$\hat{\mathbf{r}}^* \Leftarrow \arg\min_{1\le i\le|\mathcal{R}|}\mathcal{E}(\mathbf{r}^i)$

*2) **Choosing Sub-optimal Rate Vector** $\hat{\mathbf{r}}^*$:* Every time the Tx rate bar decreases a step, a total cost is evaluated. This continues until the bar hits the bottom of the supported rate set $\mathcal{R}$. At this point the rate vector value resulting in the minimum cost is selected as the approximation of the optimal Tx rate vector $\hat{\mathbf{r}}^*$. This algorithm is formally described in Alg.2, where FindCliquePartition is a function which takes the rate vector $\mathbf{r}$

as an input and produces optimal pairwise clique partition $\mathcal{V}_{\mathbf{r}}^{p*}$ as its output, i.e., it solves problem (5), and K is parameter depending on relay's access opportunity.

*3) **Example**:* The algorithm is explained with the example network shown in Fig. 7a. We have four source sessions and corresponding maximum Tx rates from source nodes to relay z: $r^m_{x_3 z} < r^m_{x_2 z} < r^m_{x_4 z} < r^m_{x_1 z}$. The Tx rates from the relay to the destination nodes are given as $r_{zy_4} < r_{zy_2} < r_{zy_3} < r_{zy_1}$. The set of supported Tx rates $\mathcal{R}$ is defined as $\{1, 2, \cdots 8\}$.

To check whether there exists any better choice for the Tx rate vector, we first set the Tx bar L to 8 and compute overhearing graph. This graph is shown as (1) in Fig. 7c. Without rate adaptation, node y_1 and y_4 are overhearing with each other's source nodes so they can be combined. While, node y_2 and y_3 can not be grouped. If L is lowered below 6, so that the Tx rate of node x_1 and x_4 are limited by L, then, we can make sure that y_3 overhears x_4's transmission. This introduces a new link between y_3 and y_4. When the clique partition is found over this graph, y_4 is paired with y_3 rather than y_1 since $r_{zy_3} < r_{zy_1}$. If we further decrease L to 4, so the Tx rate from x_2 is limited by L, then, y_4 overhears x_2's transmission, which adds a new link between y_4 and y_2 as shown in (3) of Fig. 7c. Again we can perform clique partitioning over the graph and evaluate minimum cost. In a similar way, if we continue to lower L to 1, then, we obtain the complete graph (6) in Fig. 7c and corresponding clique partition. Among all Tx rates evaluated, the one giving minimum cost is selected. Note that the L is the control parameter for the tradeoff between the two costs.

VI. PERFORMANCE EVALUATION

A. Simulation Environment

In this section, we evaluate the performance of a network using a joint rate adaption and network coding scheme. In particular, we consider a star topology network where a relay node receives packets from source nodes and transmits them to destination nodes. Source and destination nodes are randomly placed within the Tx range of the relay such that the source and destination node can not directly communicate with each other. We use randomly distributed nodes because in reality the placements of nodes around a relay usually can not be controlled. We will evaluate the average performance over such randomly placed nodes. Each node can support 12 Tx modes, with different modulation and coding rates, transmitter chooses the highest Tx rate supportable based on the received average SNR at the receiver. We say a Tx mode is "supportable" if a desired target PER is achieved under AWGN channel for given average SNR. Static AWGN channel with a path loss attenuation factor of 3.5 was considered. We consider two extreme scenarios, in which relays have different transmission opportunities. We do not explicitly model the MAC and associated overheads, instead we consider possible access fairness scenarios amongst the nodes in the network. These scenarios are idealized not specific to centralized or distributed scheduling schemes, yet we expect practical MAC to fall somewhere between these two extremes. In the first scenario, we allow the node acting as relay to have a higher access opportunity than neighboring nodes acting as data sources. This policy allows us to evaluate the pure gain of network coding, which is not affected by MAC. While, in the second scenario, we assume all nodes including the relay have equal access to the medium. This case shows how network performance is affected by joint rate adaptation and network coding and how these interact with the MAC. Note that, in both cases, we give access opportunity to each source node under max-min fair policy in long term average Tx rate. The only difference is the relay's access opportunity. As a performance metric, network throughput is calculated as the number of bits transported divided by the amount of time spent on uplink and downlink packet transmissions.

B. Scenario 1: Relay with more access opportunity than other nodes

*1) **Unequal access assumption**:* In this case, each node takes its turn to transmit a packet to the relay on the uplink and then the relay consumes as many Tx opportunities as it needs to serve the received packets to their associated destinations on downlink. The uplink and downlink transmissions form one cycle, which repeats. This maintains max-min fairness in terms of on long term average rate achieved by source nodes. Note the predetermined Tx orders removes contention, and accordingly there is no throughput loss from contention for the medium. This allows us a saturated network which achieves its maximum throughput. In this sense, the gains of joint rate adaptation and network coding over simple network coding or no coding at all, can be viewed as "pure" gains.

*2) **Cost**:* Recall that the cost $\mathcal{E}(\mathbf{r}^i)$ in step 8 of Alg.2 depends on the parameter K. In this scenario, the total cost is given as the cost during one cycle, which is the sum of uplink and downlink costs. So, K is equal to 1.

*3) **Network Throughput**:* Figs. 8 and 9 show an average throughput and an average gain for the rate adaptive network coding (RANC.P) and regular network coding (RNC.P) both constrainted to using only pairwise coding. As expected, when the number of sessions, $|N|$, increases, the coding opportunities at the relay increase, resulting in an increased throughput. The throughput gain of RANC.P compared to no network coding (noNC) ranges from 9% to 19% and the gain compared to RNC.P is around 4% as $|N|$ ranges from 2 to 8. But, note that the absolute gain of RNC.P ranges 5.5% to 15%, which is much smaller than 33%, the theoretical maximum gain. The throughput averaging over random node placement reduces the average gain to 6-15% depending on $|N|$. This implies that network coding is not helpful for substantial number of the possible node placements or that the coding opportunity highly depends on node placement. In fact, we observe that around 35% of node placements had no coding opportunity at all under RNC.P when $|N| = 2$ as shown in Fig.10. The performance curves for the heuristic coding scheme and optimal coding scheme are almost identical to that of the pairwise coding case, so they are not plotted. This interesting result suggests that coding with degree larger than two hardly happens. The distribution of coding degree, explained in next section, is helpful in understanding this. Fig. 10 shows the distribution of throughput gains across random network topologies. It is clear from this picture that rate

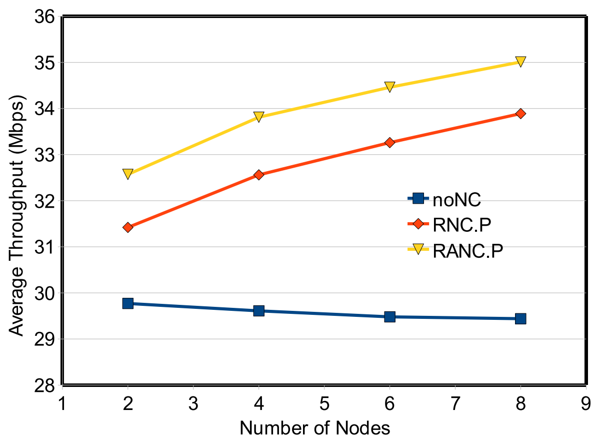


Fig. 8: The average throughput in Scenario 1

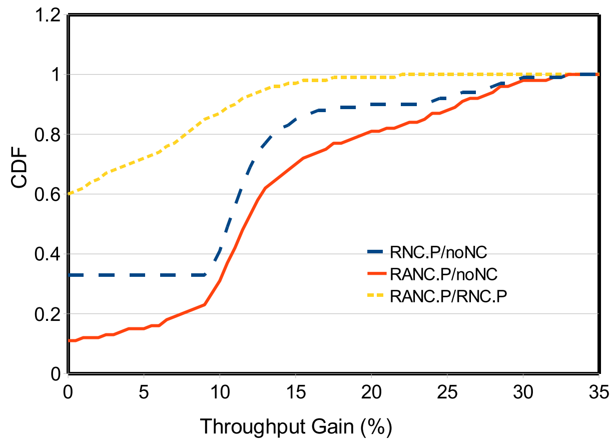


Fig. 10: The distribution of throughput gain in Scenario 1, $|N| = 2$.

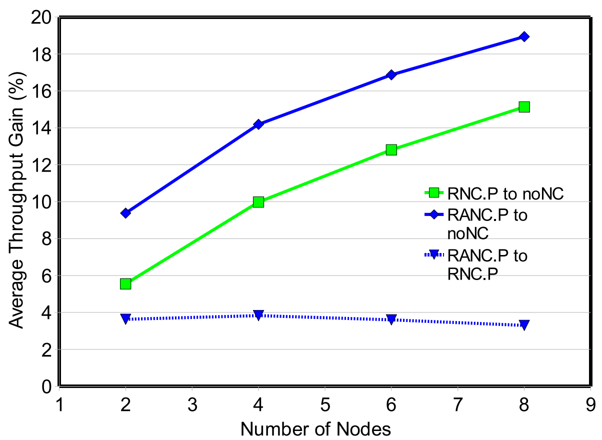


Fig. 9: The average gain in in Scenario 1

adaptation reduces the number of cases without any coding opportunity and resulting in performance increase. However most of gains for RNC.P are under 33% which implies that the two Tx rates being combined are imbalanced.

*4) **Performance of Coding Schemes**:* Table.I shows the distribution of coding degree (the number of packets combined in one encoded packet) for three clique partitioning schemes(CPS). 'P', 'H', and 'O' denote a pairwise and a heuristic and an optimal coding scheme respectively. In the optimal coding scheme, the optimal clique partition was found using an exhaustive search method. And the result for $|N| = 2$ was omitted since the pairwise coding is optimal. In Scenario 1, the distributions of three schemes in RNC case are quite similar, so as commented above, the associated network throughputs are almost the same. This is becuase coding with degree larger than two hardly happens under RNC case. This is true even in RANC case, this results in marginal gain of RANC.

C. Scenario 2: Relay with equal access opportunity as other nodes

*1) **Equal access assumption**:* In our second scenario, we assumed that all nodes including the relay have equal access to the medium. That is, the relay, as other nodes, has only one Tx opportunity per cycle. So, the relay behaves like a bottleneck node, in which unsent packets are dropped immediately. The relay is supposed to choose one coded or non-coded packet each turn so that it keeps fairness in the long run Tx rates across sessions. This can be understood as the extreme case where the relay node can not serve all received packets from neighbors, and network throughput collapse. This scenario gives an idea on the performance of joint rate adaptation and network coding in a heavily loaded networks. Note that all nodes in such a network are assumed to be backlogged and have equal access opportunities.

*2) **Cost**:* K is set to $|\hat{\mathcal{V}}^*_{\mathbf{r}}|$, the number of cliques in the selected clique partition $\hat{\mathcal{V}}^*_{\mathbf{r}}$ or equivalently the number of downlink transmissions required to serve all the nodes. That is, the total cost, $\mathcal{E}(\mathbf{r}^i)$, is the sum of uplink and downlink delay until all neighboring nodes of the relay are served at least once. We call this duration a super cycle. The super cycle is the minimum duration during which fairness for long term Tx rate can be maintained.

*3) **Network Throughput**:* Figs. 11 and 12 shows the throughput and the throughput gain of RNC.P, RANC.P, RNC.H and RANC.H for Scenario 2, where RNC.H and RANC.H denote regular and rate adaptive network coding using our heuristic coding scheme. We did not plot the throughput under the optimal coding scheme since the heuristic coding scheme achieved within 99% of the optimal throughput for $|N|$ up to 8. Observe that the throughput decreases as $|N|$ increases. If $|N|$ increases, the number of dropped packets also increases and accordingly the throughput decreases. The gain of RNC.P to noNC ranges from 20-42%. Once a rate adaptation is applied, the gain increases to 40-62%. However, the relative gain (RANC.P to RNC.P) gradually decreases. This is because as the the number of nodes increases, packets are easily paired with another packet even without rate adaptation. Even though pairwise coding looks quite effective as compared to previous scenario, it still does not fully resolve the congestion at the relay. This motivates us to use the heuristic network coding scheme. Note that the coding with high degree resolves the congestion, and increases throughput. The gain (RANC.H to noNC) ranges 40% to 120% and, the relative gain to RNC.P also increases as the number of nodes increases.

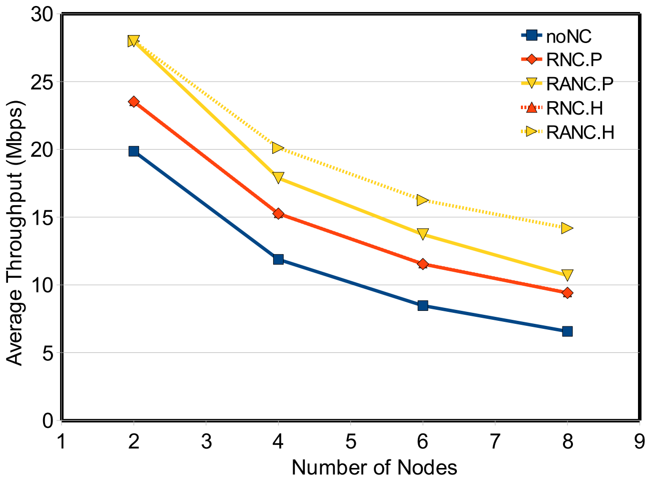


Fig. 11: The average throughput in Scenario 2

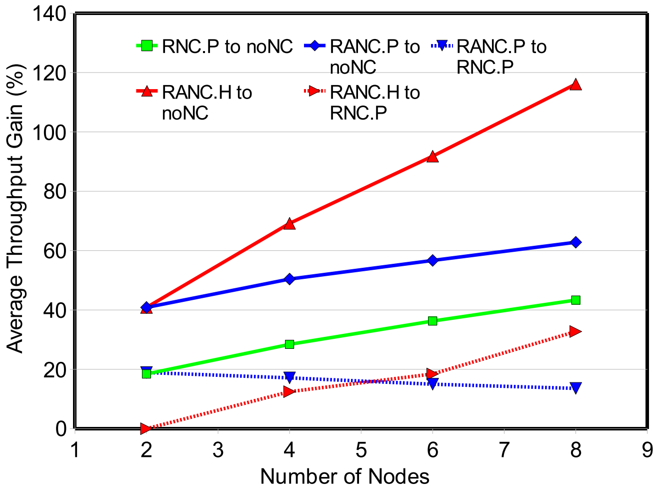


Fig. 12: The average throughput gain in Scenario 2.

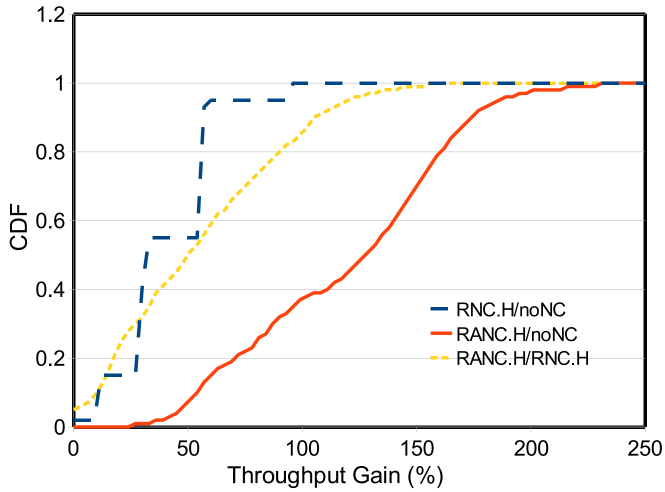


Fig. 13: The distribution of coding degree with heuristic coding scheme in Scenario 2, $|N| = 8$.

Fig.13 shows the distribution of coding degree under the optimal coding scheme with $|N| = 8$. It shows that the increased coding opportunity gives a large variance for the average throughput, and a throughput gain of up to 250%.

*4) **Performance of Coding Policy**:* The distribution of coding degree in Scenario 2 is given in Table.I. But, the degree for RNC was omitted since it is the same as that for Scenario 1. It is clear that the increasing coding degree from 2 to 4, 6, or 8 shifts the distribution to right side and it will be helpful for improving performance. One notable observation is that the probability of selecting higher degree is very high, which is different from what we saw for Scenario 1. This is because increasing coding degree one step higher reduces the set of uplink transmissions for one cycle, which will be otherwise eventually dropped from buffer. So, increasing coding degree has higher impact as compared to Scenario 1. Definitely, transmission rate on the uplink is reduced so that high coding degree is achieved.

VII. Conclusion

In this paper, we studied the joint rate adaptation and inter-session network coding in star topology wireless network. We showed that rate adaptation is effective at increasing network coding gain and in resolving congestion at relay, in particular, in networks where standard MAC protocols are used in which all nodes have equal access opportunities. Good gains were obtained in configurations where nodes are spread out e.g., to avoid interference and/or achieve good coverage and the traffic is spatially diverse. In these cases, there were no coding opportunity without rate adaptation. So, we conclude that rate adaptation, is yet another "degree of freedom" which along with routing, can be exploited to provide enhanced opportunities for network coding. Unfortunately the complexity of such joint rate adaptation is high, yet for systems where node are stable, e.g., infrastructure wireless networks, the cost can be amortized of the long term, and may be commensurate with the costs to determine appropriate inter-session network coding.

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TABLE I: Distribution of coding degree (%)

		Scenario 1						Scenario 2							
		Degree(RNC)			Degree(RANC)			Degree(RANC)							
$\|N\|$	CPS	1	2	3	1	2	3	1	2	3	4	5	6	7	8
4	P	76.8	23.2		61.5	38.5		35.0	65.0						
	H	76.8	23.2		61.7	38.0	0.3	18.3	35.4	4.8	41.5				
	O	76.8	23.2		62.4	37.0	0.6	19.0	33.8	5.5	41.7				
6	P	66.7	33.3		50.6	49.4		28.0	72.0						
	H	66.7	33.3		51.3	48.2	0.5	16.8	29.7	7.9	2.6	3.6	39.4		
	O	66.7	33.2	0.1	52.2	46.7	1.1	17.9	27.4	7.8	3.6	3.8	39.5		
8	P	58.5	41.5		42.6	57.4		23.6	76.4						
	H	58.5	41.5		43.6	55.9	0.5	14.9	34.4	8.9	5.2	2.9	1.5	2.9	29.4
	O	58.5	41.4	0.1	44.5	54.2	1.3	15.8	30.7	9.4	4.7	3.5	2.7	3.2	30

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