Optimizing Timely Coverage in Communication Constrained Collaborative Sensing Systems

Jean Abou Rahal, Gustavo de Veciana, Takayuki Shimizu, and Hongsheng Lu

***Abstract*— We consider a collection of distributed sensor nodes periodically exchanging information to achieve real-time situational awareness in a communication constrained setting, e.g., collaborative sensing amongst vehicles to improve safety-critical decisions. Nodes may be both consumers and producers of sensed information. Consumers express interest in information about particular locations, e.g., obstructed regions and/or road intersections, whilst producers broadcast updates on what they are currently able to see. Accordingly, we introduce and explore optimizing trade-offs between the coverage and the space-time interest weighted average "age" of the information available to consumers. We consider two settings that capture the fundamental character of the problem. The first addresses selecting a subset of producers that maximizes the coverage of the consumers preferred regions and minimizes the average age of these regions given that producers provide updates at a fixed rate. The second addresses the minimization of the interest weighted average age achieved by a fixed subset of producers with possibly overlapping coverage by optimizing their update rates. The first problem is shown to be submodular and thus amenable to greedy optimization while the second has a non-convex/non-concave cost function which is amenable to effective optimization using the Frank-Wolfe algorithm. Numerical results exhibit the benefits of context dependent optimization information sharing among obstructed sensing nodes.**

***Index Terms*— Age of Information (AoI), non-convex optimization, producers/consumers of information, resource allocation, sensing, submodular optimization.**

I. Introduction

In the near future it is envisaged that there will be several disruptions and challenges to the automotive and wireless industries. Amongst these, an intriguing and challenging one will be the emergence of automated cars (also UAVs, robots, etc.) with the ability to collaboratively navigate through complex environments. In order to enable such functionality, it is expected that nodes will collaborate by sharing sensed information, e.g., cars share their views of obstructed locations in their environment. The aim is to achieve a high degree of "real-time situational-awareness," i.e., to detect/recognize and then effectively track dynamic objects in their vicinity so as to improve safety-critical decisions. To that end, it is expected that vehicles will not only rely on on-board multimodal sensing, but also share (raw or fused) sensing information with each other, with the goal of facilitating coordination. This may require the transport of substantial volumes of data among cars, as well as to/from the network edge and/or cloud. The optimization of information sharing in a communication constrained system will thus be a fundamental problem underlying such systems. The focus of this paper is on the modeling and analysis of this problem and its implications for collaborative sensing systems.

A key step in this direction is to identify appropriate/usable metrics to assess how well an information sharing policy is performing. This involves at least two concerns. On the one hand, one is interested in *coverage*, i.e., the fraction of the relevant region that a set of collaborating nodes (producers) is able to view. On the other hand, for dynamic environments, one is interested in the timeliness of the available information across space, e.g., the Age of Information (AoI), when sensors periodically share what they see. Intuitively high coverage is achieved by ensuring that all sensors disseminate their (possibly redundant) information to all the other relevant nodes while the minimization of age may involve giving some well positioned sensors/nodes a higher update rate or leveraging overlaps among sensors' coverage sets. Additionally it is of interest to incorporate contextual information in that nodes may want to have a higher awareness of the on-goings in close proximity or uncertain regions on their path, e.g., it is more critical for a car to have fresh information of on-goings in its neighborhood or of obstructed regions at intersections they are about to enter rather than receiving frequent updates about distant locations. Roughly speaking optimizing the "timely coverage" for a collaborative sensing system requires modeling the relative value/usefulness of each sensor's updates, e.g., in terms of the overall coverage, importance, and timeliness.

We focus on four major intertwined questions:

1) *Can one provide a model and metrics to evaluate the coverage/timeliness trade-offs achieved by a given information sharing policy for a set of collaborative sensors?*
2) *Assuming that a fixed number of sensing nodes broadcast information at a fixed rate regarding their overlapping coverage, can one determine the best subset to participate in information exchanges so as to meet the overall nodes' demand for timely updates about the regions of most interest?*
3) *Assuming a subset of sensors is chosen, can one jointly*

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optimize their update rates to minimize the interest weighted (space-time) average age over their coverage set?

4) *How do optimized information sharing policies compare to simple policies as a function of the sensor/node density, i.e., inherent overlap, and system communication capacity?*

Related Work. A key motivating application for this work is collaborative sensing in support of automated vehicles. The basic idea is to facilitate real-time exchanges of sensor information among vehicles and/or road side units to enhance their 'situational awareness' in obstructed and dynamic environments, see e.g., [1]–[5] and recent work in [6], [7] which uses stochastic geometry to model and analyze collaborative sensing coverage in obstructed environments as a function of the penetration of vehicles with sensing capabilities.

When addressing real-time situational awareness, it is key that the decision-making nodes have access to timely information. The modeling and delivery of timely information has recently received substantial attention, see e.g., [8], [9]. The newly proposed metric, Age of Information (AoI) became popular since it better captures information freshness as compared to the traditional delay metric. AoI has been extensively studied in the literature, see e.g., [10]–[13]. A close work to ours is that of [14] where the goal is to design an update policy that allows a single cache to update stored files one at a time by accessing the server. A policy geared at minimizing the average AoI of all the files stored in the cache is derived, given that the update duration time depends on the file size. The work in [15] is perhaps the closest to that in this paper in that it addresses the issue of optimizing the overall AoI by carefully choosing sensors' update rates and allocating network resources. However, by contrast with these works, in this paper, we model and explore the impact that updates from multiple sensors with possibly overlapping fields of view will have on the AoI, as well as trade-offs between coverage and timeliness.

Many instances of coverage and sensor selection problems, see e.g., [16] are known to be submodular which in turn are amenable to greedy approximations, see e.g., [17]. To our knowledge, this paper is distinct from previous work in that it introduces and explores a new fundamental trade-off between coverage and AoI in a collaborative sensing system.

Contributions. Given a set of sensors periodically broadcasting updates (at possibly different rates) regarding their coverage sets we define and characterize the interest weighted (space-time) average age for the information exchanged. To the best of our knowledge this is the first work addressing the "timely coverage" for a set of collaborative sensors. We model our network of sensing nodes as a set of information producers and consumers and explore the resource allocation and performance trade-offs in such systems. In particular we formulate and study two possible settings.

The first captures a trade-off between maximizing the interest weighted coverage and minimizing the interest weighted average age of the spatial information requested by consumers about regions where they lack timely updates, e.g., obstructed regions or regions with high uncertainty, when all the sensors have the same update rate.

We show that this weighted coverage-age trade-off optimization problem has a submodular structure which leads to efficient greedy optimization algorithms. In the second setting we fix the subset of sensors, e.g., all that are available or those selected in the first setting, which now act as producers of information, and explore the benefits of jointly optimizing their periodic update rates towards minimizing the interest weighted average age. When producers have non-overlapping coverage sets, we show that their optimal periodic update rates are proportional to the square root of their coverage's weights. However, more generally, the interest weighted average age minimization problem has a non-convex/non-concave structure. We explore the use of the Frank-Wolfe gradient method to show the potential benefits of update rate optimization for collaborative sensing.

A numerical evaluation of the benefits of these approaches from the point of view of coverage and interest weighted average age achieved by consumers is conducted. It exhibits the possible advantages that constraining both the number of active producers as well as the resource allocation amongst these active nodes in a collaborative sensing setting should play, particularly in congested environments with limited communication resources.

The present work is an extension of our previously published work [18]. We summarize major additions as follows. We introduce a new framework that involves sensing nodes that act as both consumers and producers of information, where consumers exhibit interest in information about particular locations, while producers are scheduled to satisfy the consumers' need for timely updates at the relevant locations. We devise an algorithm that operates in two stages, in the first one it selects a subset of the available producers to maximize the overall weighted coverage of the consumers' regions of interest, and in the second, it further selects producers to minimize the weighted age of the covered regions. We further analyze and provide performance guarantees for this algorithm. We finally develop a simulation platform to explore the optimization of coverage versus weighted age trade-offs in collaborative sensing applications and explore the advantage of having consumers express context-dependent interest for timely updates about specific locations and how this affects the producers' selection as well as the overall achieved weighted coverage and weighted age of the consumers' regions of interest. We consider in addition to the highway scenario, an intersection scenario, where a higher spatial correlation in the consumers' interests exists, especially in the shared obstructed regions.

Organization. The paper is organized as follows. In Section II we introduce our system model. In Section III, we develop metrics for collaborative sensing based on the AoI. In Section IV we introduce and study the problem of selecting a subset of sensors that achieves maximal coverage of the consumers' preferred regions, as well as minimal weighted age of the covered regions, while in Section V we consider the minimization of the weighted age function by jointly optimizing the update rates for a fixed set of sensors. Section VI presents our numerical results and analysis of the underlying characteristics

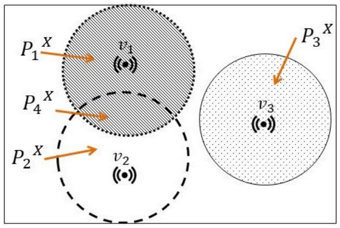

Fig. 1: Three sensors observing their environments.

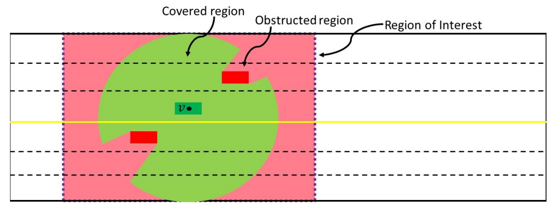

Fig. 2: Region of interest and coverage of a consumer/producer node.

of collaborative sensing, and Section VII concludes the paper.

II. System Model

We shall begin by formally describing our model for a collaborative sensing system and the associated notation.

A. Sensor coverage sets, consumers, producers and interest weighted measures

Without loss of generality we consider a set of sensors V in a given overall region $R \subset \mathbb{R}^2$. Sensors are indexed by their locations $v \in V$ and the coverage (field-of-view) of sensor v in a given environment is denoted by a subset $C_v \subseteq R$. Given a subset of sensors $X \subseteq V$, we denote X's overall coverage by $C(X) := \bigcup_{v \in X} C_v$.

The coverage sets for a subset of sensors X, i.e., $(C_v,\ v \in X)$ induce a partition of the overall coverage set $C(X)$ which we denote by $\mathcal{P}^X = \{P_i^X,\ i = 1, ..., |\mathcal{P}^X|\}$. Each subset of the partition P_i^X is such that each location $x \in P_i^X$ can be seen by the *same* subset of sensors $V_i^X \subseteq V$, i.e., such that $x \in P_i^X$ if and only if $x \in C_v$ for all $v \in V_i^X$. It should be clear that if $i \neq j$ then $P_i^X \cap P_j^X = \emptyset$. Further it should be clear that $\cup_{i=1}^{|\mathcal{P}^X|} P_i^X = C(X)$, thus we have a partition of $C(X)$. In fact, assuming it is nonempty, if we further include an additional set $R \setminus C(X)$ corresponding to the locations which are not covered by X, we get a partition of the overall region R. It is also possible that the coverage sets of two or more sensors intersect on a set of measure zero. For simplicity, and to avoid unnecessary burdens, we assume that all sets of the partition have non-zero area, or remove sets of measure zero.

Fig. 1 illustrates three sensors $X = \{v_1, v_2, v_3\}$ which for simplicity have each an unobstructed disc coverage set. Sensors v_1 and v_2 have overlapping coverage regions. The figure also exhibits the four subsets in the induced partition, $\mathcal{P}^X = \{P_1^X, P_2^X, P_3^X, P_4^X\}$.

Without loss of generality we suppose each sensor node is simultaneously a consumer and a producer of information which may broadcast periodic updates regarding regions it is able to see. A consumer indicates its interest in information regarding various locations through a *spatial interest measure*. In turn the sum of the consumers' spatial interest measures captures the aggregate interest of consumers. These are formally defined below.

Definition *1: **(Consumer's interest measure)** A consumer $v \in V$ indicates interest in timely information about the environment via a spatial interest measure $w_v(.)$ on R.*

Definition *2: **(Aggregated consumers' interest measure)** The aggregated consumers' interest measure $w(.)$ is given by the sum of the consumers' interest measures, i.e., $w(.) = \sum_{v \in V} w_v(.)$ on R.*

Definition *3: **(Weighted coverage)** The overall weighted coverage of the region covered by a subset of sensors X, i.e., $C(X)$, is given by $w(C(X)) = \sum_{v \in V} w_v(C(X))$.*

For example, if $w(.)$ corresponds to the area measure, then $w(C(X))$ denotes the area covered by the sensors in X, and if normalized, $w(C(X))/w(R)$ represents the fraction of the region R which is covered. The weight measure provides a flexible means to model the importance of various locations, and/or to model the relative importance of a region from the perspective of information sharing. Note that in general the weight measure could be continuous or discrete. In the latter case we envisage a measure placed at discrete locations corresponding to *anchor points* which based on the known geometry of the environment may have higher importance, e.g., intersections for incoming vehicles or locations obstructed by other vehicles. In some practical settings, a consumer v's interest may be limited to a smaller region, say R_v. For example, a vehicle with a response time of t_{interest} moving at a speed s would primarily care about what is happening in a region $s \cdot t_{\text{interest}}$ around it. Thus R_v might be modelled as a rectangle (centered at v's location) of length $2 \cdot s \cdot t_{\text{interest}}$ and width typically covering the road and surrounding areas. Fig. 2 illustrates the coverage of a sensor v (green region), obstructed by neighbouring vehicles (red region behind the vehicles), as well as its rectangular region of interest R_v. In this case, v's spatial interest measure would be supported by the red region. Assuming that sensor v's location on the road is $v = (x_v, y_v)$, where x_v and y_v stand for the $x - y$ coordinates of v in 2-D, and the origin 0 is at the center of R, then R_v is defined as, $R_v := R \cap \left(\left[-\frac{w_{road}}{2}, \frac{w_{road}}{2}\right] \times [x_v - s \cdot t_{\text{interest}}, x_v + s \cdot t_{\text{interest}}]\right)$, where w_{road} denotes the width of the road.

In some settings, consumers may only express an interest in obstructed regions corresponding to regions where they have significant uncertainty or blind spots, as depicted in Fig. 3. Thus for example, consumer v might place a point mass/anchor point in front of the truck. In the shown example, this anchor point falls in the coverage regions of the two other sensors (blue and green), which can thus in principle help out.

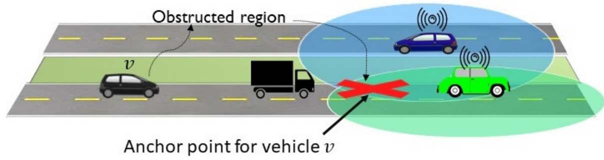

Fig. 3: Consumer requesting timely updates about the obstructed anchor point.

B. Network capacity, sensor updates, and AoI

We shall assume that each producer v generates periodic updates regarding its coverage set C_v at a rate of r_v updates per second, i.e., the update interval is thus $\frac{1}{r_v}$ seconds. The updates are either broadcast to the other sensors or shared with a central controller. For any subset $X \subseteq V$ we let $\mathbf{r}(X) = (r_v : v \in X)$ denote the vector of update rates for the sensors in X. The delay for sensor v to access a shared communication medium and transmit its update is assumed to be exactly, or at most, d_v. Thus the fraction of time sensor v holds the medium is d_v over the update interval $1/r_v$, i.e., $d_v r_v$. The selected update rates for a set of sensors X must then satisfy a "capacity" constraint,

$$\sum_{v \in X} d_v r_v \leq 1, \tag{1}$$

ensuring the medium is not overbooked. Note in practice, depending on the character of the medium, one would require a back off $\sum_{v \in X} d_v r_v \leq 1 - \epsilon$ for some $\epsilon > 0$ to ensure minimal queuing and contention delays. The required back off will depend on the details of channel access and/or scheduler. For simplicity we will suppress ϵ in the sequel, and use (1) as the capacity constraint while assuming no overlapped/collided transmissions.

A natural metric that captures the freshness of the received updates is the Age of the Information (AoI) available at the consumers. Fig. 4 exhibits the time-varying AoI at a consumer for such a periodic update process from a single producer at rate r_v and with transmission delay d_v. In the sequel, we will consider both *average* age and the probability that the age exceeds a pre-specified threshold at a random time at a consumer v. For example the average age for the process shown in Fig. 4 is given by,

$$\textit{average} \text{ age of sensor } v = d_v + \frac{1}{2r_v}. \tag{2}$$

To keep things simple we will assume $d_v = d$ for all $v \in V$, i.e., they are either identical or bounded by d. However since producers' coverage sets may overlap, consumers may receive updates from multiple producers for the same location, which may result in a reduction in the age of the available information. We define a generic age function for such overlapping regions as follows.

Definition 4: ***(Age of regions with overlapping sensor updates)*** *Suppose a set of producers X transmit periodic updates at rates $\mathbf{r}(X) = (r_v, v \in X)$. Recall that X induces a partition where the locations in P_i^X are covered by a set of sensors V_i^X. The age of P_i^X thus depends on the update rates of these sensors, i.e., $\mathbf{r}(V_i^X)$. With a slight abuse of notation we will define an **age function** which captures a proxy for the age (e.g., average or probability of exceeding a threshold) of the set P_i^X as*

$$\text{age}(P_i^X) = \text{age}(\mathbf{r}(V_i^X)), \tag{3}$$

with the intention of emphasizing its dependence on the associated sensors' update rates.

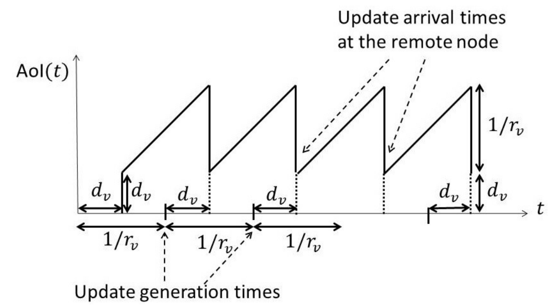

Fig. 4: Time-varying age of information.

In the next section we shall explore the characteristics of the age as a function of the number of sensors and their associated update rates.

The age of a region as introduced in Definition 4 can be viewed as a dis-utility (cost) function that is tied to the resources allocated to the set of sensors observing this region. Consumers express interest in specific locations while sensor nodes having direct access to these regions become potential producers. We define the *weighted age function* as being an aggregated dis-utility function based on a linear combination of the aggregated consumers' spatial interest in particular regions and the age of these regions.

Definition 5: ***(Aggregated interest weighted age for a set of sensors)*** *Given a weighted coverage measure vector $\mathbf{w} = (w(P_i^X),\ P_i^X \subseteq \mathcal{P}^X,\ i = 1, \ldots, |\mathcal{P}^X|)$ on partition $\mathcal{P}^X$ induced by a set of sensors X, where $\cup_{i=1}^{|\mathcal{P}^X|} P_i^X = C(X)$, and sensor update rates $\mathbf{r}(X)$, the **aggregated interest weighted age** of the coverage set $C(X)$ associated with X is given by,*

$$\begin{aligned} a(X, \mathbf{r}(X)) &:= \sum_{i=1}^{|\mathcal{P}^X|} w(P_i^X)\text{age}(P_i^X) \\ &= \sum_{i=1}^{|\mathcal{P}^X|} w(P_i^X)\text{age}(\mathbf{r}(V_i^X)). \end{aligned}$$

*If $w(.)$ is a measure corresponding to the area, we say $a(X, \mathbf{r}(X))$ is the **area weighted age** of $C(X)$. If it is further divided by $w(C(X))$ it will be referred to as the **normalized weighted age**.*
For simplicity we shall write $a(X, \mathbf{r}(X))$ as $a(X)$.

III. Characterizing the Age Function

In this section we define and characterize the properties of two possible age functions, as introduced in Definition 4.

A. Definition and computation of the age function

We first consider two simple motivational examples. Recall that the age function depends on a vector of update rates of sensors which see a given location. As discussed in the previous section, if periodic updates from only a single sensor at rate r_1 are available, then the *average* age, denoted $\overline{\text{age}}$ depends on the scalar r_1 and is given by,

$$\overline{\text{age}}(r_1) = d + \frac{1}{2r_1}.$$

Let A_1 be a random variable denoting the age of the saw-tooth function when viewed at a random time (see Fig. 4). Given the saw-tooth function's linear age growth, it should be clear that $A_1 \sim d + \frac{1}{r_1}U_1$, where $U_1 \sim \text{Unif}[0,1]$.
We now define two age functions, the average age, $\overline{\text{age}}$, and the γ-age violation, age_γ, given by,

$$\overline{\text{age}}(r_1) = \mathbb{E}[A_1] \quad \text{and} \quad \text{age}_\gamma(r_1) = \mathbb{P}(A_1 > \gamma),$$

where $\gamma \geq 0$ is a target age one would not wish to exceed.

Definition 6: ***(Age functions)*** *Consider a region observed by n sensors generating periodic updates at rates $\mathbf{r} = (r_1, \ldots, r_n)$ and with associated transmission delays d such that $d \leq \frac{1}{r_v}$ for all $v = 1, \ldots, n$. Assuming the phases of the sensors' periodic updates are independent and uniformly distributed then the* **average age** *and* **γ-age violation** *functions of locations in a region seen by sensors with update rates* $\mathbf{r}$ *are*

$$\overline{\text{age}}(\mathbf{r}) = \mathbb{E}[A] \quad \text{and} \quad \text{age}_\gamma(\mathbf{r}) = \mathbb{P}(A \geq \gamma),$$

where $A = \min_{v=1,\ldots,n}[A_v]$ and $A_v \sim d + \frac{1}{r_v}U_v$ and $U_v \sim \text{Unif}[0,1]$ are independent of each other.

Suppose there are in fact updates from two sensors covering a given set in the partition, e.g., as shown in Fig. 1, sensors v_1 and v_2 are providing updates of region P_4^X with update rates $\mathbf{r} = (r_1, r_2)$. Without loss of generality, assume $r_1 \geq r_2$. As shown in Fig. 5, the dashed and dotted saw-tooth functions correspond to the updates of the two sensors. Assuming that the phases of the saw-tooth curves are randomly distributed, and no transmission fail, it is easy to see that the average age at a typical time is given by the minimum of the two functions, i.e.,

$$\overline{\text{age}}(\mathbf{r}) = \mathbb{E}\left[\min[A_1, A_2]\right] = d + \frac{1}{r_1}\left(\frac{1}{2} - \frac{1}{6}\frac{r_2}{r_1}\right),$$

where $A_1 \sim d + \frac{1}{r_1}U_1$ and $A_2 \sim d + \frac{1}{r_2}U_2$, and where U_1, U_2 are uniformly distributed and assumed to be i.i.d., and A_1, A_2 correspond to the ages of the updates from Sensors 1 and 2 observed at a random time. The reduction in age due to redundancy in the sensors' updates is clear. The probability of γ-age violation shares similar properties as the average age.

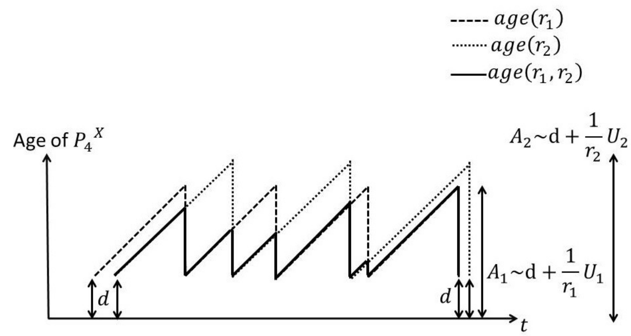

Fig. 5: Age of partition P_4^X is the minimum of both age functions $\text{age}(r_1)$ and $\text{age}(r_2)$.

We now characterize the age functions.

***Theorem** 1: **(Characterization of the age functions)** Consider a region observed by n sensors which generate periodic updates at rates $\mathbf{r} = (r_v : v = 1, \ldots, n)$, then the average age function is given by*

$$\begin{aligned} \overline{\text{age}}(\mathbf{r}) = \mathbb{E}[A] &= d + \int_d^{d+\frac{1}{r_1}} \prod_{v=1}^{n} z_v(y)\, dy \qquad (4) \\ &= d + \frac{1}{r_1}\left[\sum_{k=0}^{n}(-1)^k \frac{c(k,\mathbf{r})}{k+1}\frac{1}{r_1^k}\right], \qquad (5) \end{aligned}$$

where $z_v(y) = 1 - r_v(y-d)$, for $v = 1, \ldots, n$, and where $c(k,\mathbf{r}) = \sum_{i_1,i_2,\ldots,i_k} r_{i_1} r_{i_2} \cdots r_{i_k}$, for $k = 1, \ldots, n$, and $c(0,\mathbf{r}) = 1$.
The γ-age violation function is given by

$$\text{age}_\gamma(\mathbf{r}) = P(A \geq \gamma) = \begin{cases} 1, & \text{if } 0 \leq \gamma \leq d, \\ \prod_{v=1}^{n} z_v(\gamma), & \text{if } d < \gamma \leq d + \frac{1}{r_1}, \\ 0, & \text{if } d + \frac{1}{r_1} < \gamma. \end{cases} \qquad (6)$$

The proof of Theorem 1 can be found in [18].

As we will see, these age functions and coupling across sensors with overlapping coverage regions are somewhat complex, thus we will first characterize a few of their properties.

B. Properties of the age functions

The following corollary further characterizes the age functions.

***Corollary** 1: **(Properties of the age functions).** Suppose that $\mathbf{r} = (r_v : v = 1, \ldots, n)$, where $r_v = r$, then the average age function is given by,*

$$\overline{\text{age}}(\mathbf{r}) = d + \frac{1}{n+1}\frac{1}{r}, \qquad (7)$$

and the γ-age violation function is given by,

$$\text{age}_\gamma(\mathbf{r}) = \begin{cases} 1, & \text{if } 0 \leq \gamma \leq d, \\ (1 - r(\gamma - d))^n, & \text{if } d < \gamma \leq d + \frac{1}{r}, \\ 0, & \text{if } d + \frac{1}{r} < \gamma. \end{cases} \qquad (8)$$

The proof of this corollary is left out of the paper due to space constraints.

IV. Sensor Selection: Weighted Coverage-Age Trade-offs

In this section, we consider a setting where sensors send/broadcast updates at the *same fixed* rate r to a centralized observer/each other. We assume that only a maximum number k out of N available sensors can be active. Based on Eq. (1) and for equal transmission delays d and fixed sensor update rate r, it should be clear that $k \leq N \leq \lfloor \frac{1}{rd} \rfloor$. The goal is to select a subset of sensors which realizes a good compromise between ensuring good coverage of the consumers' regions of interest and minimizing the weighted age of these covered regions. There exist multiple approaches to achieving such trade-offs. We propose and explore the one that attempts to achieve both maximal aggregated weighted coverage and minimal weighted average age of the covered regions. Before devising the algorithm that achieves this goal, we first introduce a modified average age function given by,

$$b(\mathbf{r}) = -\overline{\text{age}}(\mathbf{r}) + d + \frac{1}{r} = \frac{n}{n+1}\frac{1}{r}, \tag{9}$$

where $\overline{\text{age}}(\mathbf{r})$ corresponds to the age function with equal rates $\mathbf{r} = (r_v : v = 1, \ldots, n)$, where $r_v = r$, as determined in Eq. (7). Note that $b(\mathbf{r})$ is strictly positive, upper-bounded, concave and increasing with respect to n. We further introduce the following *"utility" function*:

$$u(X) := \sum_{i=1}^{|\mathcal{P}^X|} w(P_i^X) b(\mathbf{r}(P_i^X)). \tag{10}$$

It should be clear that minimizing the weighted average age in Definition 5 is equivalent to maximizing the utility function in Eq. (10), i.e.,

$$\min_{X \subseteq V} a(X) \equiv \max_{X \subseteq V} u(X).$$

This transformation is useful because $u(.)$ can be shown to be a submodular set function as supported by the following result.

Theorem 2: ***(Characterization of the utility function)*** *The utility function $u(.)$ satisfies the following properties:*

(*Monotonicity*) *It is monotonically increasing, i.e., if $X \subset Y \subset V$ then, $u(X) \leq u(Y)$.*

(*Submodularity*) *It is submodular, i.e., if $X \subset Y \subset V$ and $v \notin Y$ then,*

$$u(X \cup \{v\}) - u(X) \geq u(Y \cup \{v\}) - u(Y). \tag{11}$$

The proof of this theorem can be found in [18].

We now propose Algorithm 1 that prioritizes the selection of the subset of producers that *first* maximizes the overall weighted coverage motivated by safety concerns, and then *second* maximizes the utility to further enhance the timeliness of the generated updates. The order in which the objectives are maximized is important because the target is to first meet the overall consumers' coverage demand and then to provide them with frequent updates about their interest regions.

Algorithm 1: Selecting sensors in V that maximize the weighted coverage of the consumers' regions of interest and minimize the weighted age of the covered regions.

1 $S_0 = \emptyset$
2 t=0
3 **while** $t < k$ **do**
4 $\quad V' = \{v' \in \arg\max_{v \in V \setminus S_t} (w(C(S_t \cup v)) - w(C(S_t)))\}$
5 $\quad v'' \in \arg\max_{v' \in V'} (u(S_t \cup v') - u(S_t))$
6 $\quad S_{t+1} = S_t \bigcup \{v''\}$
7 $\quad$ **if** $S_{t+1} = S_t$ **then**
8 $\quad\quad$ **break**
9 $\quad$ **end**
10 **end**
11 return S_{t+1}

As can be seen the algorithm begins with an empty set of selected sensors and proceeds by iteratively selecting producers and stopping when either k sensors have been selected (Line 3) or no new sensor has been selected in the current iteration (Lines 7-9). In each iteration, it first greedily selects the subset of sensors V' that achieve a maximal weighted coverage (Line 4) then out of the sensors in V', it selects that which further maximizes the utility $u(.)$. Overall, the algorithm consists of two phases: (1) It greedily selects $k' \leq k$ sensors so as to maximize the weighted coverage and then maximizes the utility, and (2) once the maximal weighted coverage is achieved and $k' < k$, the focus switches to solely selecting $k - k'$ sensors which further maximize the utility function (Line 5). We point out that if the consumers' aggregated interest measure is uniform, the weighted coverage maximization phase simplifies to a coverage maximization problem. We further define the following notation.

- $S_{k'}$: Denotes the set of sensors of size k' selected by Algorithm 1 in phase 1.
- $S_{k'}^*$: Denotes the optimal set of sensors of size k' achieving a maximal weighted coverage and maximal utility.
- $S_k(S_{k'})$: Denotes a set of sensors of size k returned by Algorithm 1 by the end of phase 2 given that k' sensors were selected in phase 1. For simplicity we will write $S_k(S_{k'})$ as S_k.
- $S_k^*(S_{k'})$: Denotes the optimal set of sensors that achieves maximal utility given that k' sensors were already selected by Algorithm 1 in phase 1. For simplicity we will write $S_k^*(S_{k'})$ as S_k^*.
- $\hat{a}(X)$: Denotes the normalized weighted average age of the region covered by sensors X, i.e., $\hat{a}(X) := \frac{1}{w(C(X))} a(X)$.

This proposed algorithm is guaranteed to achieve both good weighted coverage and normalized overall consumer interest weighted age. The following theorem formalizes these performance guarantees.

Theorem 3: ***(Performance guarantees for Algorithm 1)*** *Given a set of sensors V of size $N \geq k$, and equal update rate r per sensor, Algorithm 1 satisfies the following lower and upper bounds on both the weighted coverage and the* *normalized weighted average age respectively,*

$$w(C(S_k)) \geq \left(1 - \frac{1}{e}\right) w(C(S_k^*)),$$

and

$$\begin{aligned}\hat{a}(S_k) \leq &\left(1 - \frac{1}{e}\right) \hat{a}(S_k^*) \\ &+ \frac{1}{e}\left[\left(1 - \frac{1}{e}\right) \hat{a}(S_{k'}^*) + \frac{1}{e}\left(d + \frac{1}{r}\right)\right] \\ &- \frac{1}{e}\lambda\left[w(C(S_k)) - \left(1 - \frac{1}{e}\right) w(C(S_k^*))\right].\end{aligned}$$

It should be clear that $w(C(S_k)) = w(C(S_{k'}))$ and $w(C(S_k^*)) = w(C(S_{k'}^*))$, given that maximal coverage has been reached with k' sensors. We provide a detailed proof of this theorem in the appendix.
The derived upper bound on $\hat{a}(S_k)$ is closely tied to the coverage performance (through the third term). Achieving a good coverage will guarantee a good upper bound on the age of the covered regions.

V. Optimization of Sensor Update Rates

In this section, we consider a setting where the set of active sensors, say without loss of generality V is fixed, but their update rates $\mathbf{r} = (r_v : v \in V)$ can be jointly optimized subject to the communication constraint Eq. (1). Given a fixed set of sensors V, we let the weighted average age of the covered set $C(V)$ be,

$$a(V, \mathbf{r}(V)) \quad := \quad \sum_{i=1}^{|\mathcal{P}^V|} w(P_i^V)\overline{\text{age}}(\mathbf{r}(V_i^V)). \tag{12}$$

as introduced in Definition 5. Note that this section will focus on the case where age $=$ $\overline{\text{age}}$, i.e., on selecting the sensor update rates so as to minimize the weighted age which can be formally stated as follows:

Problem 1: ***(Age minimization)***

$$\min_{\mathbf{r}} \{a(V, \mathbf{r}(V)) \mid \mathbf{r} \geq 0, \sum_{v \in V} dr_v \leq 1\}. \tag{13}$$

In the producer-consumer setting, solving Problem 1 is equivalent to having a third party optimally redistribute the resources amongst the selected producers subject to communication constraints, in an attempt to minimize the overall consumer interest weighted average age of these regions.

Proposition 1: ***(Age minimization for sensors with disjoint coverage)*** *Suppose the sensor coverage sets $(C_v, v \in V)$ are disjoint then the age minimization Problem 1 is convex and reduces to,*

$$\min_{\boldsymbol{r}} \{\sum_{v \in V} w(C_v)(d + \frac{1}{2}\frac{1}{r_v}) \mid \mathbf{r} \geq 0, \sum_{v \in V} dr_v \leq 1\},$$

whose optimal joint update rates $\mathbf{r}^*$ are given by

$$r_v^* = \frac{\sqrt{w(C_v)}}{\sum_{u \in V} \sqrt{w(C_u)}} \times \frac{1}{d}.$$

The proof of this proposition follows from standard convex optimization tools and so is left out. The solution reveals the first basic insight that for sensors with disjoint coverage sets, the age minimizing rate allocation is proportional to the *square-root* of the weight (e.g., area) of the coverage set each sensor is tracking. Thus sensors covering disjoint regions with equal weights would lead to equal update rate allocations. The general case where sensors have overlapping coverage sets is more complex.

Proposition 2: ***(Characterization of the age minimization problem)*** *For the general age minimization Problem 1 where coverage sets may overlap, the objective function given in Eq. (12) is a weighted sum of a convex function and a non-convex/non-concave function, and hence belongs to the family of non-convex/non-concave functions.*

It is easy to see this by noting that the average age of a partition as given in Eq. (4) can be re-written as,

$$\begin{aligned}\overline{\text{age}}(\mathbf{r}) &= d + \int_d^{d+\frac{1}{r_1}} \prod_{v=1}^{n} z_v(y)\ dy \\ &= f(\mathbf{r}) + g(\mathbf{r}),\end{aligned}$$

where $f(\mathbf{r}) = d + \frac{1}{2r_1}$ and where

$$g(\mathbf{r}) = -\int_d^{d+\frac{1}{r_1}} z_1(x)\left[1 - \prod_{i=2}^{n} z_i(x)\right] dx.$$

It is clear that $f(\mathbf{r})$ is convex in r_1, while $g(\mathbf{r})$ is non-convex/non-concave in $\mathbf{r}$ which can be proved by finding the Hessian of the function $g(.)$ with respect to $\mathbf{r}$, $H \in \mathbb{R}^{n \times n}$, and either showing that H has a mix of positive and negative eigenvalues, or that $\mathbf{y}^T H \mathbf{y}$, for all $\mathbf{y} \in \mathbb{R}^{n \times 1}$ can either be positive or negative. We can then show that $\overline{\text{age}}(\mathbf{r})$ is a non-convex function, and since it is part of the objective function in Eq. (12), then the latter belongs as well to the same family of functions. It should be clear by now that in the case of a single sensor v observing a partition and updating at a rate r_v, the age of this partition is convex in r_v and given by $d + \frac{1}{2}\frac{1}{r_v}$. But whenever more than one sensor are observing the same partition, the function capturing the age of this partition belongs to the family of non-convex/non-concave functions. There exists a family of algorithms that addresses this type of optimization problems, from which we pick the Frank-Wolfe algorithm [19]–[21], described in Algorithm 2.

Algorithm 2: Frank-Wolfe Algorithm (with adaptive step sizes) [20]

```
Let r = (r_v, v ∈ 𝒱) ∈ 𝒟
Let 𝒟 = {∑_{v∈𝒱} dr_v ≤ 1}
for t=0,...,T do
    Compute s^(t) := arg min_{s∈𝒟} ⟨ s, ∇a(r^(t)) ⟩
    Let d_t := s^(t) − r^(t)
    Compute g_t := ⟨d_t, −∇a(r^(t))⟩
    if g_t ≤ ε then return r^(t)
    Line-search: γ_t ∈ arg min_{γ∈[0,1]} a(r^(t) + γ d_t)
    Update r^(t+1) := r^(t) + γ_t d_t
return r^(T)
```

We summarize the FW algorithm for both the cases of convex and non-convex objective functions. In [19], under the assumption of a convex and continuously differentiable function, and a compact convex domain $\mathcal{D}$, the algorithm computes at each iteration t the maximal step it can take in the direction of the gradient of the function while satisfying the constraint $\mathbf{s} \in \mathcal{D}$, and then moves in the direction of this maximizer. This process, as explained in [19], intuitively makes sense since the algorithm finds the direction in which it can maximize the improvement in the function value while remaining feasible. Additionally, one key advantage of this algorithm is that it does not need to project back into the constraint set, given that it never leaves it. On the other hand, Theorem 1 in [20] gives a simple proof that the Frank-Wolfe algorithm obtains a stationary point at a rate of $\mathcal{O}\left(1/\sqrt{t}\right)$ on non-convex objectives with a Lipschitz-continuous gradient. We refer the reader to [20] for more details on the convergence of FW on non-convex objectives.

VI. Numerical Results

We develop a simulation framework to explore the optimization of coverage vs. normalized interest weighted average age trade-offs in a collaborative sensing application and present some results in this section.

A. Model

We shall present results for a representative road intersection scenario. Vehicles are modeled as $4.8 \times 1.8\text{m}^2$ rectangles with omnidirectional sensors placed at the vehicle's center (rooftop). The unobstructed coverage set for each sensor is a disc of radius $r = 50$m with area πr^2. The coverage area of a sensor does not include the regions off the road, and the only obstructions present are those associated with vehicles blocking each others' field-of-view. We assume that vehicles are randomly placed in lanes, with spacings of at least 10m between any two vehicles in the same lane. We assume vehicles (consumers) have an interest in specific locations which might be obstructed. We refer to these as anchor points. Specifically, we will define the consumers' interests as associated with particular anchor points on the highway.

B. Communication model

We assume that each vehicle/sensor is equipped with a 360^o camera which samples at 30 frames per second with a corresponding data rate of 1 Mbyte/sec. Additionally, V2X technology is used to share sensor updates. We assume that a single producer accesses the medium at a time and broadcasts its update to all the consumers in the system. For simplicity we assume an operational bandwidth achieving a data rate of 6 Mbps, which results in a transmission delay of 44.44 msec per image frame. We combine both the channel access time ($\sim$ 20 msec) and update transmission time into a single deterministic value, d, and find $d = 64.44$ msec. We further assume no transmission failures so broadcasts are reliable.

C. Coverage and normalized interest weighted average age of the anchor points

We define the coverage of a typical consumer v as the percentage of v's anchor points that are covered. We say an anchor point is covered if either v directly sees it or if it receives updates about this anchor point from active producers that directly see it. The *normalized interest weighted average age of a typical consumer* v is the weighted average age of v's covered anchor points, normalized by their weighted coverage. We note that we will be interested in the coverage of a typical consumer and not in the weighted coverage. By contrast we use the weights when considering the weighted age.

D. Comparison of algorithms

We assume that there are N available sensors in R, all of them acting as both consumers and producers of information. We will evaluate the performance of three main algorithms.

- The *baseline* selects *all* N sensors to act as both producers and consumers of information, each of which has the same (possibly low) update rate to meet the capacity constraint. One would expect that this technique achieves the best coverage but performs poorly in terms of the normalized interest weighted average age of a typical consumer.
- The *sensor selection* algorithm selects $k \leq N$ producers, allowing for a higher update rate per sensor and hence fresher and more frequent updates for consumers. We point out that all N sensors are consumers of information.
- The *age minimization* algorithm optimally allocates the update rates amongst $k \leq N$ selected producers to further minimize the interest weighted average age of the covered regions.

We define and make use of the following three notions of aggregated consumers' interest measures.

- The *Uniform Discrete (UD) aggregated consumer interest measure* places equal weight on a set of discrete anchor points to reflect the same level of interest of consumers in those locations.
- The *General Discrete (GD) aggregated consumer interest measure* places unequal weights on a set of discrete anchor points to reflect different levels of consumer interest in those locations.
- The *Uniform Continuous (UC) aggregated consumer interest measure* corresponds to an interest weight corresponding to the area measure on the region R.

The UD sensor selection algorithm selects a set of producers that cover the consumers' anchor points which further keeps their interest weighted average age low. The GD sensor selection algorithm selects producers that cover the consumers' anchor points, given different weights on the anchor points. This algorithm exhibits the advantage of having consumers indicate their degrees of interest in locations by assigning weights proportional to their interest. It should be clear that when all the anchor points have an equal weight, the GD sensor selection algorithm reduces to the UD sensor selection algorithm. Finally, the UC sensor selection algorithm chooses a set of producers so as to cover as much as possible of the overall region R, while trying to keep the average age of the covered regions as low as possible, given a uniform consumer interest across the whole region R. This can only be achieved by judiciously optimizing producers' coverage overlaps. This

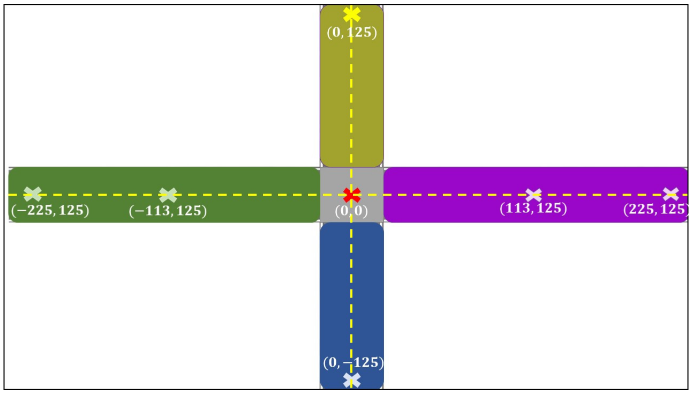

Fig. 6: Road intersection with 7 pre-set anchor points.

existing tension between spreading producers across space and overlapping their coverage sets is what makes this optimization challenging. With such a weight measure, the algorithm selects the producers independently of the consumers' preferences, i.e., it is not consumer oriented, but it provides a choice of producers which is robust across possible consumer interests.

E. Road Intersection Scenario

We consider a road intersection scenario as depicted in Fig. 6 where high spatial correlation exists between the interest locations of different consumers reflected through different weights assigned to different locations. This results in a clear improvement in the age of the consumers' covered anchor points when the algorithm that selects the producers is oriented towards meeting the consumers' interest in timely updates.

The horizontal two-way road has a width of 40m (corresponding to 10 lanes) and length of 450m, the vertical road has a width of 16m (4 lanes) and length equal to 250m. There are 7 pre-set anchor points in total. A consumer is interested in the anchors that fall within its own colored section and in the anchor at the intersection, as can be seen in Fig. 6. For example, vehicles falling in the green region express interest in the two green anchor points with coordinates $(-225, 125)$ and $(-113, 125)$ and in the red one falling at the origin. A consumer assigns a weight equal to 5 to the red anchor point at the intersection versus a weight of 1 to any other anchor point in its own section, which expresses the consumers' higher urge for updates about the intersection.

We assume there are N randomly placed vehicles equipped with 360^o cameras on any of the highways' lanes with a minimum distance of at least 10m between them.

We mainly compare the GD and UD sensor selection algorithms. Once producers are selected, we evaluate the coverage and normalized interest weighted average age of a typical consumer, assuming the aggregated interest measures are assigned according to the consumers' interests.

*1) **Effect of increasing the number of selected producers by GD, UD and UC sensor selection algorithms on both age and coverage:*** In Fig. 7, we assume there are $N = 70$ available sensors. We increase the number of selected producers k from 1 to 70 for all of GD, UD and UC sensor selection algorithms. We see that the GD sensor selection algorithm

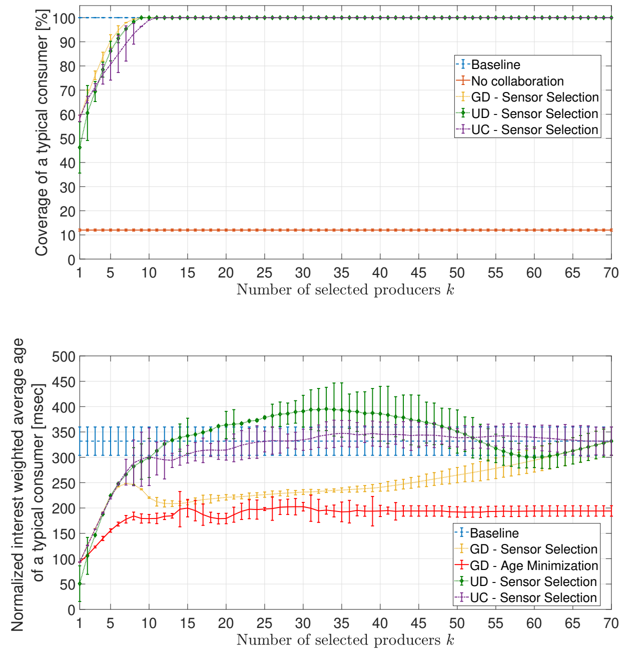

Fig. 7: Coverage and normalized interest weighted average age of a typical consumer with anchor points having unequal weights, when there is a fixed number of available sensors $N = 70$ but an increasing number of selected producers k by GD, UD and UC algorithms from 1 to 70.

achieves both a better coverage than all the others algorithms for $k \leq 6$ and a major age improvement for $6 \leq k \leq 60$. The clear drop in age achieved by the GD algorithm for $6 \leq k \leq 15$ is the result of selecting the sensors that solely minimize the age once maximal coverage is achieved. We also clearly see that both the UD and UC algorithms achieve a worse age than the one achieved by the baseline, and this occurs after the number of selected producers k achieves maximal coverage, which confirms that these two schemes are not consumer oriented. Finally, an interesting observation is that for $N = 70$, optimizing the rates among the 70 selected producers reduces the age by ~ 140 msec.

*2) **Advantage of using a consumer-oriented scheme when a fixed small number of producers is selected:*** Finally, in Fig. 8, we increase the number of available sensors from 10 to 70. GD, UD and UC sensor selection algorithms only select 10 producers at all time. As expected, the coverage achieved by all algorithms (except for the no-collaboration algorithm) are somewhat similar and maximal, with a clear age improvement achieved by the GD algorithm. With further optimization of the producers' update rates, the age achieved by this algorithm is itself further improved by around 30 msec for $N = 70$.

VII. Conclusion and Future Work

In this paper we have exhibited an approach to achieving trade-offs between coverage and timeliness in communication constrained collaborative sensing settings wherein spatially

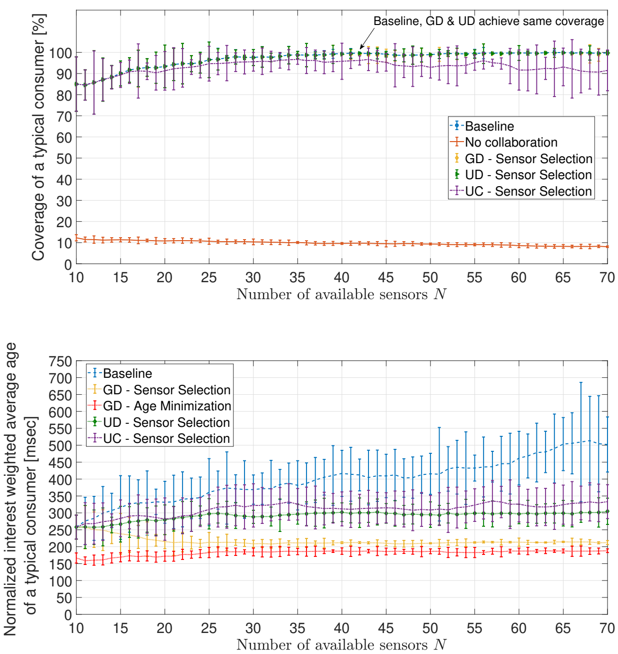

Fig. 8: Coverage and normalized interest weighted average age of a typical consumer with anchor points having unequal weights, when there is an increasing number of available sensors N from 10 to 70 but a fixed number of selected producers $k = 10$ by GD, UD and UC algorithms.

distributed sensor nodes can serve dual roles as producers and consumers of sensed information. The proposed framework allows quite a bit of flexibility in terms of capturing the underlying characteristics of information sharing, and suggests the development of a possible market place for sharing real-time sensor data in a context dependent manner, i.e., matching nodes' current interest, in order to minimize situational uncertainty so as to enhance vehicular flow and safety. A key aspect is the design of strategies to match consumers spatial interest in timely information to producers' overlapping coverage, subject to communication network capacity constraints. A key part of our future work is to make the sensor selection and update rate optimization more scalable to start addressing heterogeneous and dynamic environments.

VIII. Appendix

A. Proof of Theorem 3

Proof: We first linearly combine both the weighted coverage and normalized utility function through a positive parameter λ as shown below,

$$g^{\lambda}(X) = \frac{1}{w(C(X))} \sum_{i=1}^{|\mathcal{P}^X|} w(P_i^X) b(\mathbf{r}(P_i^X)) + \lambda w(C(X)) \quad (14)$$

$$= \hat{u}(X) + \lambda w(C(X)). \quad (15)$$

where $\hat{u}(X) = \frac{1}{w(C(X))} u(X)$.

Theorem 4: ***(Characterization of the weighted coverage-normalized utility function)*** *If $\lambda \geq 0$ then the weighted coverage-normalized utility linear combination function $g^{\lambda}(.)$ satisfies the following properties,*

(Monotonicity) *It is monotonically increasing, i.e., if $X \subset Y \subset V$ then $g^{\lambda}(X) \leq g^{\lambda}(Y)$.*

(Submodularity) *It is submodular, i.e., if $X \subset Y \subset V$ and $v \notin Y$ then,*

$$g^{\lambda}(X \cup \{v\}) - g^{\lambda}(X) \geq g^{\lambda}(Y \cup \{v\}) - g^{\lambda}(Y). \quad (16)$$

The proof is similar to the proof of Theorem 2.

We can now clearly state our weighted coverage-age optimization problem as follows

Problem 2: ***(Weighted coverage-normalized utility optimization problem)*** *The weighted coverage-normalized utility optimization problem is a submodular optimization problem with a cardinality constraint,*

$$S^* \in \arg\max_{X \subseteq V} \{ \, g^{\lambda}(X) \mid |X| \leq k \, \}.$$

Such combinatorial problems are NP hard, but may satisfy submodularity properties that make greedy approaches quite effective. Although this is a complex combinatorial problem, the classical greedy algorithm shown in Algorithm 3 panel requires $O(|V|k)$ function evaluations to determine a subset S_k which is $(1 - 1/e)$ constant factor of the optimal [17], i.e.,

$$g^{\lambda}(S_k) \geq \left(1 - \frac{1}{e}\right) g^{\lambda}(S^{\star}) + \frac{1}{e} g(S_0), \quad (17)$$

where S_0 is the initial set of selected elements with $g(S_0) = 0$ if $S_0 = \emptyset$.

Algorithm 3: Greedy submodular optimization [17]

```
Let S_0 = ∅
for i=0,...,k-1 do
    j ← arg max_j g^λ(S_i ∪ {j}) − g^λ(S_i)
    S_{i+1} ← S_i ∪ {j}
end
```

There are computationally less costly possibly distributed versions of the algorithm leveraging random sampling. There is a growing line of work to design possibly distributed algorithms with sub-linear cost which have shown to be be similarly effective [22]–[28].

The choice of λ in $g^{\lambda}(.)$ is tied to the weighted coverage-normalized utility approach one desires to apply. We provide below a λ value that meets our desired target in maximizing the weighted coverage first then breaking ties by minimizing the weighted age.

Proposition 3: ***(Characterization of λ for the maximal weighted coverage-minimal weighted average age problem)*** *Given a subset of sensors $X \subseteq V$ updating at the same rate r, and given that the smallest aggregated consumers' interest measure on a region of interest covered by sensors in X is $w_{min} = \min_{x \in X} w(C(x))$, then for $\lambda = \frac{1}{w_{min}} \frac{1}{6r} + \epsilon$, $\epsilon > 0$ very small, maximizing $g^{\lambda}(.)$ corresponds to first maximizing the weighted coverage of the consumers' regions of interest while minimizing their weighted average age, then to solely minimizing the weighted average age of the covered regions once maximal coverage is achieved.*

We provide a sketch of the proof of this proposition. It is easy to show that for λ satisfying the condition in the theorem, greedily maximizing $g^{\lambda}(.)$ prioritizes selecting a sensor that covers the uncovered region weighted by the smallest consumers' aggregated interest measure over a sensor that covers a previously covered region weighted by the maximal consumers' aggregated interest measure. Once maximal coverage is achieved and given the submodularity properties of the normalized utility function $\hat{u}(X)$ * then the focus switches to greedily maximizing $\hat{u}(X)$ or equivalently minimizing the weighted average age.

Our proposed Algorithm 1 is greedy in nature. It proceeds with greedily selecting the sensor that provides the largest marginal gains for both weighted coverage and the utility function. We make use of this property in the following lemma.

Lemma 1: For $\lambda = \frac{1}{w_{min}}\frac{1}{6r}+\epsilon$, $\epsilon > 0$ *very small, Algorithm 1 greedily approximates the solution of Problem 2.*

Lemma 1 directly follows from Theorem 4 and Proposition 3. Using the submodular greedy maximization nature of Algorithm 1 and Lemma 1, and given the submodularity property of weighted coverage [29], we lower bound $w(C(S_{k'}))$ as shown below

$$w(C(S_{k'})) \geq \left(1 - \frac{1}{e}\right) w(C(S^*_{k'})). \tag{18}$$

As already discussed, Algorithm 1 operates in two phases. Following Lemma 1, the tightest lower bound on $g^{\lambda}(S_{k'})$ after greedily selecting k' sensors in phase 1 is

$$g^{\lambda}(S_{k'}) \geq \left(1 - \frac{1}{e}\right) g^{\lambda}(S^*_{k'}),$$

which by definition of $g^{\lambda}(X)$ in Eq.(14) gives

$$\begin{aligned} \hat{u}(S_{k'}) \geq & \left(1 - \frac{1}{e}\right) \hat{u}(S^*_{k'}) \\ & - \lambda \left[w(C(S_{k'})) - \left(1 - \frac{1}{e}\right) w(C(S^*_{k'}))\right]. \end{aligned} \tag{19}$$

Phase 2 of the algorithm consists of greedily maximizing $\hat{u}(.)$ over the remaining unselected sensors in V, given k' selected sensors achieving maximal weighted coverage. A lower bound on the normalized utility function by the end of the algorithm is

$$\begin{aligned} \hat{u}(S_k) &\geq \left(1 - \frac{1}{e}\right) \hat{u}(S^*_k) + \frac{1}{e}\hat{u}(S_{k'}) \\ &\geq \left(1 - \frac{1}{e}\right) \hat{u}(S^*_k) + \frac{1}{e}\left(1 - \frac{1}{e}\right) \hat{u}(S^*_{k'}) \\ &\quad - \frac{1}{e}\lambda \left[w(C(S_{k'})) - \left(1 - \frac{1}{e}\right) w(C(S^*_{k'}))\right], \end{aligned}$$

where the first inequality follows from Eq. (17) and the second inequality follows from Eq. (19). Using the fact that $\hat{a}(X) = -\hat{u}(X) + d + \frac{1}{r}$, we finally derive an upper bound on the normalized weighted average age,

$$\begin{aligned} \hat{a}(S_k) \leq & \left(1 - \frac{1}{e}\right) \hat{a}(S^*_k) \\ & + \frac{1}{e}\left[\left(1 - \frac{1}{e}\right) \hat{a}(S^*_{k'}) + \frac{1}{e}\left(d + \frac{1}{r}\right)\right] \\ & - \frac{1}{e}\lambda \left[w(C(S_{k'})) - \left(1 - \frac{1}{e}\right) w(C(S^*_{k'}))\right], \end{aligned}$$

which concludes the proof of Theorem 3. ■

*$\hat{u}(X)$ is submodular when maximal coverage is achieved, i.e., $w(C(X))$ is maximal and a constant.

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