Analysis of Cancellation Error for Successive Interference Cancellation with Imperfect Channel Estimation

Project Report

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Abstract—Successive interference cancellation (SIC) is a technique, which can increase the capacity of a DS-CDMA system. Increase in capacity has been shown to approach theoretical limits for an AWGN channel [1]. For a practical channel from the viewpoint of cellular systems, significant increase in capacity using SIC has been shown in [2] even in the presence of estimation errors up to 50%. This capacity increase in [2] is maximized if the statistics of cancellation error is known. In this paper, cancellation error for SIC for the system model proposed in [2] has been analyzed mathematically and verified through simulation for an Additive White Gaussian Channel (AWGN) an also for a fading channel. The results have shown that statistics of cancellation error in SIC has a zero mean with standard deviation and probability density function dependent on the channel statistics. The knowledge and insight gained about cancellation error statistics provide better power allocation schemes required for a typical system using SIC. The results can be used to model the estimation of fractional cancellation error for each user. It is shown in [2] that the knowledge of fractional cancellation error is important for optimum power allocation scheme in a system employing SIC. The results also suggest that better power control schemes, used in 3G systems in the uplink, can reduce the estimation error and thus provide greater capacity enhancements relative to systems not employing SIC.

I. Introduction

At the physical layer of second and third generation cellular systems Code-Division Multiple Access (CDMA) is the primary multiple access scheme. CDMA is a flexible multiple access method suitable for supporting many services (speech, data, video, etc) which are becoming increasingly important for mobile communications. In order to achieve higher capacity and robustness different flavors of CDMA is being implemented in the 3G cellular systems to cater for different needs. Limitation in capacity in second generation cellular systems, which employ a conventional receiver technique, is due the fact that these systems are limited by multiple access interference (MAI) and the near-far effect. In the past 10-15 years an enormous amount of work has been done in order to increase the capacity of a CDMA wireless system. Multiuser Detection (MUD) is a broad field, which studies the demodulation of one or more digital signals in the presence of multiuser interference. Although MUD is applicable to situations where co-channel interference arises from channel non-ideal effects and out-of-cell transmissions but primarily it addresses interference cancellation where MAI is present by design (non-orthogonal CDMA). Therefore, most of the literature on MUD has been with a viewpoint to increase the capacity of the uplink in the CDMA system.

Successive Interference Cancellation is a nonlinear type of MUD scheme in which users are decoded successively. The approach successively cancels strongest users by re-encoding the decoded bits and after making an estimate of the channel, the interfering signal is recreated at the receiver and subtracted from the received waveform. In this manner successive user does not have to encounter MAI caused be initial users. SIC improves performance for all users, initial users improve because the later users are given less power which means less MAI for the initial users, and later users improve because early users interference have been cancelled out. An optimum power control scheme as shown in [2] can be employed so that all users are decoded with the same Signal to Interference Ratio (SIR).

In the past two to three years a growing interest has been shown in SIC and presently its implementation in the industry is being pursued. The main reason for its popularity is its low complexity and in its simplest form, SIC uses decisions produced by single-user matched filters. SIC as proposed in [1] is different from much of the MUD research in that it doesn't rely on dimensional separation or short-period spreading sequences in order to distinguish users from one another. Further, as proposed in [2] it is highly suited to an uncoordinated, noisy, asynchronous environment such as the uplink in a cellular system.

However, there is a concern in industry about SIC providing capacity enhancements in a fading channel and in the presence of estimation errors. The requirement for unequal power amongst users in order to provide same performance to all users is also considered an implementation issue. Increase in capacity as proposed in [2] is shown to degrade if the statistics of cancellation error is not known or incorrectly guessed for the optimum power control allocation as shown in fig 8 of [2] which is reproduced below.

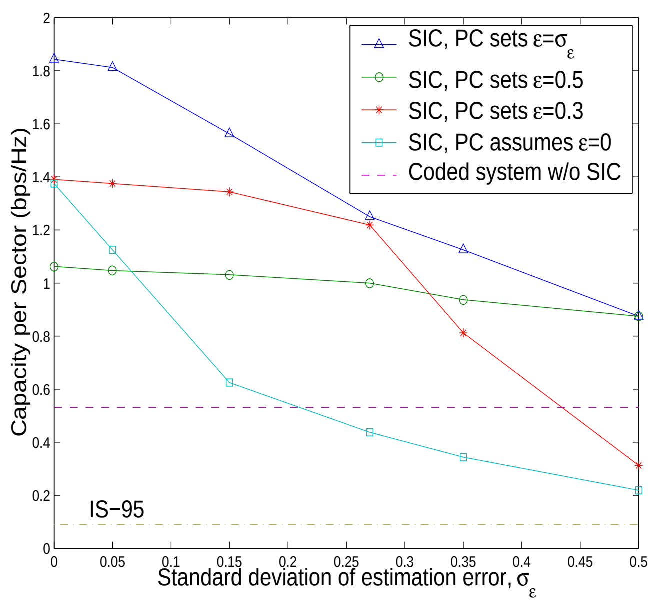

Fig. 1 Spectral Efficiencies for SIC

A simple iterative power control scheme for SIC has been proposed in [3] and is shown that it can be incorporated in a conventional CDMA system with no additional complexity. Capacity enhancements in SIC for a fading channel even in the presence of estimation error has been shown in [2] and can be seen from Fig 1. In this paper we focus on the analysis of cancellation error with a viewpoint to gain knowledge on its statistics so that future work on SIC related to 3G and 4G cellular systems could exploit the results obtained in this paper. Similar system model as in [2] has been used for the simulation and analytical treatment of cancellation error. Optimum power control proposed in [2] is also implemented. However, Power Control Error (PCE) model has been modified. In order to realistically model the received power over a fading channel, it is assumed that users amplitudes are Rayleigh distributed with unit mean value. In order to keep the model simple, it is assumed that there is only one path from transmitter to receiver, but results could be extended if multi-path were to be taken into account using SIC with multi-path combining as in [4]. Most of the parameters and conventions used in the paper are based on the work done in [2]. It is assumed that reader is familiar with the work presented in [2].

II. System Model

A. Transmitter and Receiver

The transmitter and receiver models are shown in Fig. 2, details of which can be seen in [2].

B. Channel

The channel is modeled as an asynchronous fading channel with additive white gaussian (AWGN). It is assumed that the users can be aligned at the receiver. Each users signal under goes an independent fading channel where the amplitudes are assumed to be Rayleigh distributed with unit mean, its probability density function is given by

$$f(x) = x/_{\alpha} . e^{x^2/2\alpha^2} \quad \text{where}\ \alpha^2 = 2/\pi$$

Fading incorporated for each user is for a Doppler equivalent to 25 mph object.

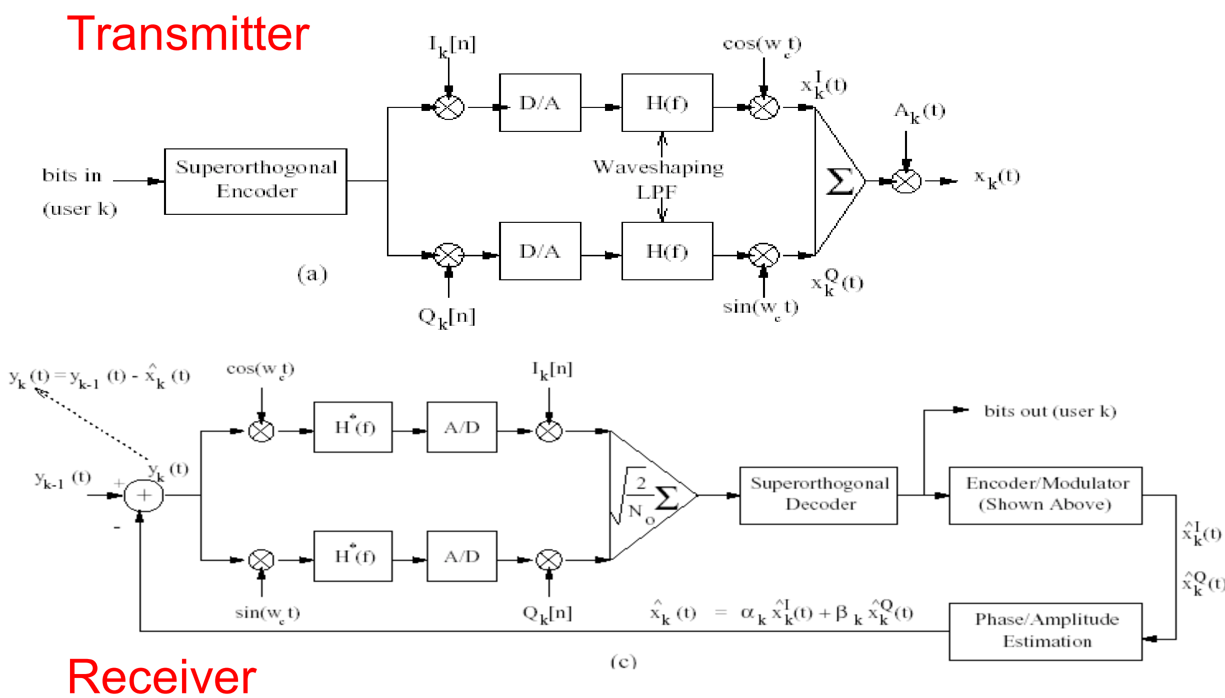

Fig. 2

III. Cancellation Error

SIC attempts to remove the interference of the k^{th} user (the most recently decoded user) from the current composite received signal $y_{k-1}[n]$, by re-encoding the decoded bit sequence for user k, modulating it with the appropriate amplitude and phase adjustment, and subtracting it out from $y_{k-1}[n]$ this can be seen in Fig. 2. Cancellation error for user k is defined as the residual signal of user k in the remaining composite signal after the subtraction of the recreated signal. Cancellation error is because of the limitation that the amplitude and phase estimation are never perfect, therefore accurate channel estimates cannot be made in any realistic system. The other source for cancellation error is incorrect bit decisions. In the presence of bit decisions errors the contribution of SIC is to enhance the MAI instead of removing it. Because the BER is assumed to be low, virtually all of the cancellation error comes from amplitude and phase estimation error [2] .

Estimate of the recd signal; $\hat{x}_k[n] = \alpha_k \hat{x}_k^I[n] + j\beta_k \hat{x}_k^Q[n]$ (1)

where α_k and β_k are the amplitude estimates of user k's in-phase and quadrature branches respectively and are formed:

$$\alpha_k = \frac{1}{M}\sum_{n=1}^{M} y_k[n].\hat{x}_k^I[n] \quad \text{where } M = No \text{ of symbols in a frame} \qquad (2)$$

$$\beta_k = \frac{1}{M}\sum_{n=1}^{M} y_k[n].\hat{x}_k^Q[n] \qquad (3)$$

The stored composite signal is updated as

$$y_k[n] = y_{k-1} - \hat{x}_{k-1}[n] = y_o - \sum_{i=1}^{k-1} \hat{x}_i[n] \qquad (4)$$

Error for user k is defined as : $e_k[n] = x_k[n] - \hat{x}_k[n]$ (5)

with fading $x_k[n] = A_k[n]\left(I_k[n]h_k^I[n] + jQ_k[n]h_k^I[n] \right) c_k[n]$ (6)

where $h_k[n]$ is modeled as a Rayleigh envelope distribution for the amplitude for user *k* and is for a Doppler speed of 25mph, based on the Clarke's model.

Looking at error for only I Branch & assuming low bit errors i.e $c_k[n] = \hat{c}_k[n]$

$$e_k[n] = A_k[n] I_k[n] c_k[n].h_k[n] - \alpha_k I_k[n] c_k[n] \qquad (7)$$

Table 1 gives the parameters used for the simulation. If the error $e_k[n]$ for each user is normalized by $A_k[n]$ which is assumed to remain constant for the duration of the frame, an Error Matrix **E** was generated whose each row corresponds to the normalized error vector for a user. The error matrix **E** for a ten-user SIC system was simulated and its histogram was generated. Shown in Fig. 3 is the simulated result for histogram of normalized cancellation error.

TABLE I

Simulation Parameters

Symbol	Description	Value
v	Super orthogonal code constraint length	7
J	Spreading factor = 2^{v-2}	32
M	Number of symbols/frame (100 bits)	3200
K	Number of full rate users	10
	Path loss exponent	Not considered
ε_k	Fractional cancellation error for user *k*	0.3
$\hat{\varepsilon}_k$	Receiver estimate of fractional cancellation error for user *k*	
z_k	A gaussian random variable	Zero mean, std=1
$h_k[n]$	Raleigh sequence of 3200 symbols for user k generated using Clarke's Model for a fading channel	
h_k	A Rayleigh random variable	Unit mean
A_k	Gain factor due to power control, for user k, it is assumed it remains constant for the duration of the frame	

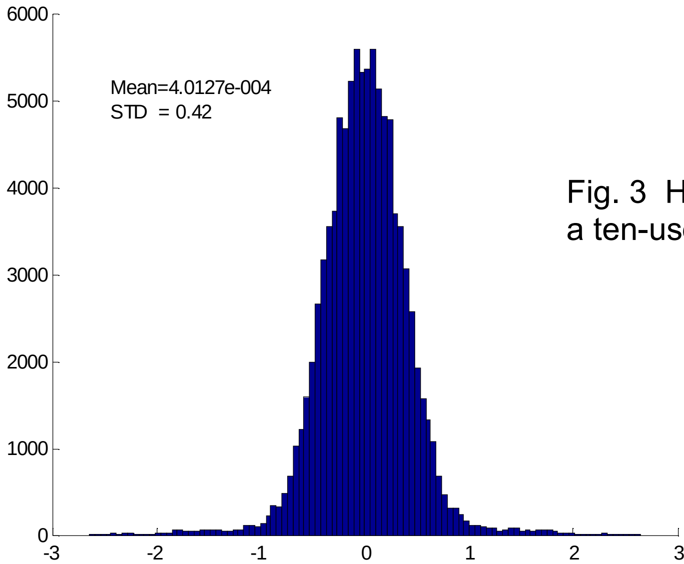

Fig. 3 Histogram of normalized cancellation error for a ten-user SIC system in a fading channel

It can be seen from the simulation that the mean of the cancellation error is zero, and normalized standard deviation =0.42. BER for the same simulation were found to be less then 10^-3. Therefore, it can be said that the above cancellation error is mainly due to the estimation error. The same simulation was also repeated for a channel with only AWGN. The histogram keeping all other parameters same is shown in fig 4

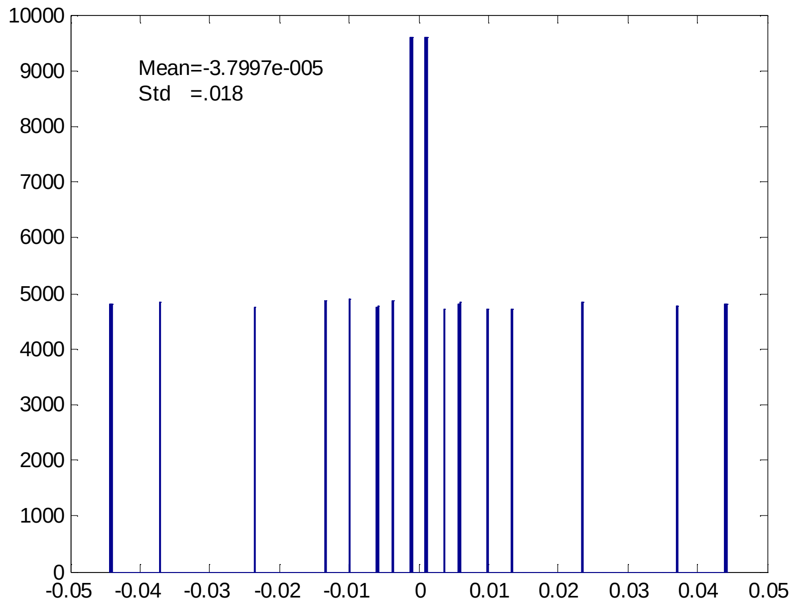

Fig. 3 Histogram of normalized cancellation error for a ten-user SIC system in an AWGN channel

It can be seen from Fig. 4 that the mean is again zero but the normalized standard deviation is much less as compared to the fading channel.

IV. Analysis of Cancellation Error

Since perfect separation between the in-phase and quadrature phase channels is assumed, so all digital-domain analysis can be considered for uncorrelated I and Q branch.

Considering error $e_k[n]$ only for the I branch, it can be shown that α_k which is the estimate of the amplitude for user k is a random variable. It is shown in appendix A that α_k can be modeled as a gaussian random variable and can be expressed as:

$$\alpha_k = A_k\left[z_k.\sigma/\sqrt{M} + \mu\right] \text{ where } \sigma \ \& \ \mu \text{ are Std dev and Mean of } h_k \quad (8)$$

where z_k is a Gaussian random variable with zero mean and unit Variance

This shows that α_k is also a Gaussian random Variable with mean A_k and whose variance decreases with higher M. This was also verified through simulation where 10,000 samples of α_k were taken and Histogram of normalized α_k by $A_k[n]$ was plotted and is shown in Fig. 5

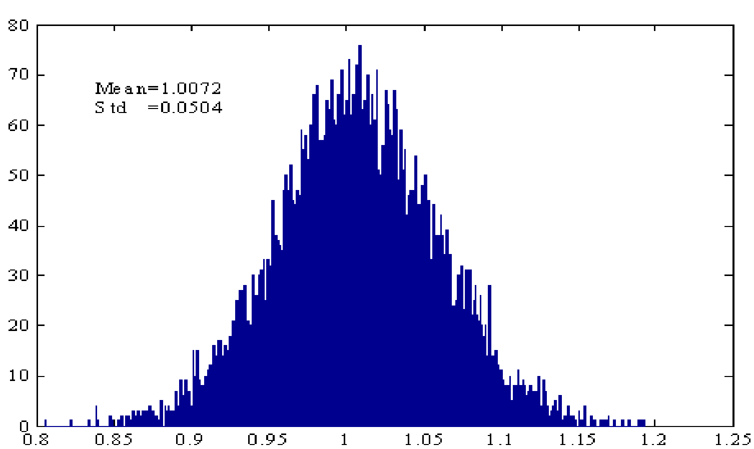

Fig. 5 histogram of normalized α_k

It was also verified through simulation that the standard deviation of α_k reduces with the increase in the frame size M. This can be seen from appendix A that the standard deviation of α_k is inversely proportional to $1/\sqrt{M}$, therefore the estimate of the amplitude α_k for user k is approximately equal to A_k with standard deviation approaching zero.

If we substitute α_k in (7) and then normalize $e_k[n]$ by $A_k[n]$, we can see from Appendix A that normalized $e_k[n]$ is given by

$$\hat{e}_k[n] = I_k[n]c_k[n].[h_k[n] - \hat{\alpha}_k] \quad (9)$$

where $\hat{\alpha}_k$ is the normalized α_k

Looking at (9) gives an insight to the random sequence $\hat{e}_k[n]$. If we assume $h_k[n]$ to a be an uncorrelated Rayleigh sequence with the same unit mean as modeled in the simulation , and if we subtract $\hat{\alpha}_k$ which is approximately equal to unit mean gaussian RV with standard deviation approaching zero, the result would be a shifted Rayleigh envelope with zero mean, each symbol is then multiplied by a random Bernoulli sequence, with probability ½ for $I_k[n]c_k[n]= +1$ and with probability ½ for $I_k[n]c_k[n]= -1$. Probability density function for h_k, $h_k - \hat{\alpha}_k$, $I_k[n]c_k[n].[h_k[n] - \hat{\alpha}_k]$ conditioned with $I_k[n]c_k[n]= +1$ and $I_k[n]c_k[n].[h_k[n] - \hat{\alpha}_k]$ conditioned with $I_k[n]c_k[n]= -1$ is shown in Fig.6 (a), (b), (c), (d) respectively. Finally if we add the two pdf's of Fig. 6 (c) & (d), we get the pdf for $\hat{e}_k[n]$. This is shown in Fig. 7 along with the histogram generated for the normalized cancellation error through simulation. A chi-square test for the data of Fig. 7 (a) and (b) were done for a frame size of 200 bits and the fitness was found under 1% for all 10 users. However, for a frame size of 100 bits, which were used for the simulation, the fitness test was between 1-10% for all 10 users. The chi-square test was carried out with 20 intervals.

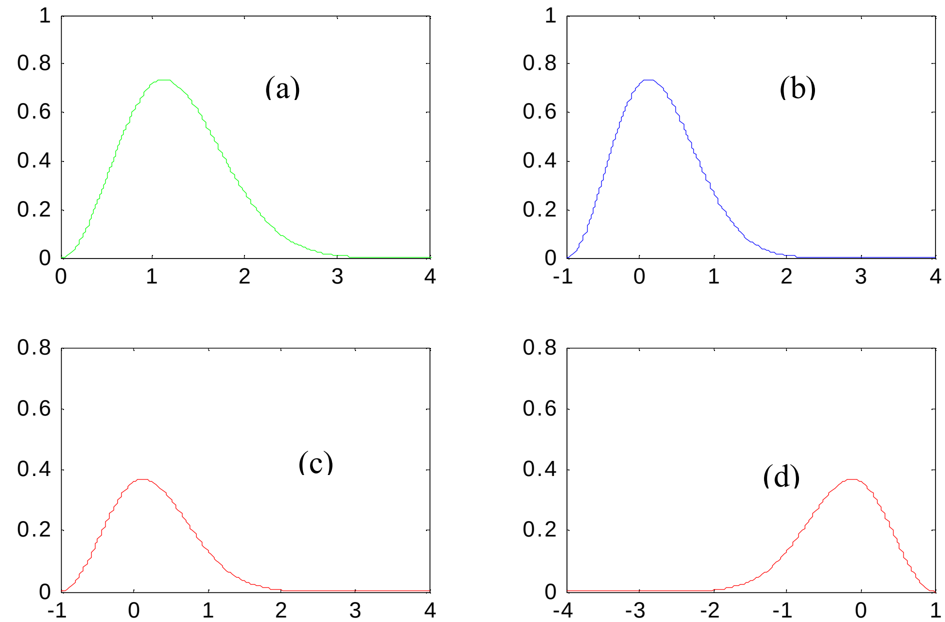

Fig. 6

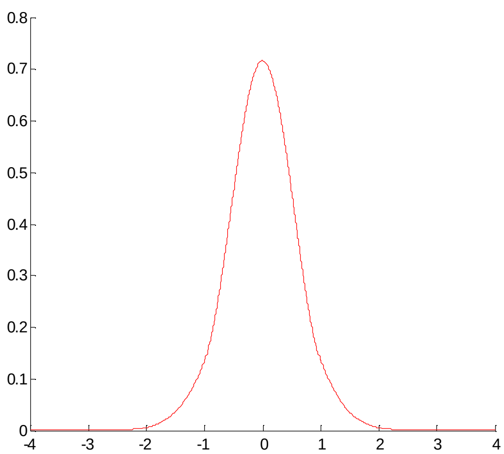

Fig. 7 (a) pdf of $\hat{e}_k[n]$
Analytical

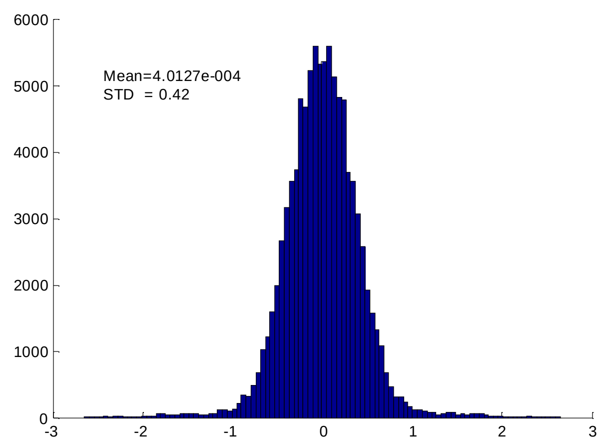

Fig. 7 (a) pdf of $\hat{e}_k[n]$
Simulation

If α_k is equal to $A_k[n]$, for large frame size M, this can be seen from Appendix A, and also verified through simulation shown in Fig. 5, $h_k[n] - \alpha_k$ can then be computed. If the resulting sequence is multiplied by the respective users $I_k[n]c_k[n]$, error sequence for each user can be formed. This error sequence was compared with error sequence generated with simulation. The standard deviation for each user, computed mathematically is compared with the simulated results and is shown in Fig. 8.

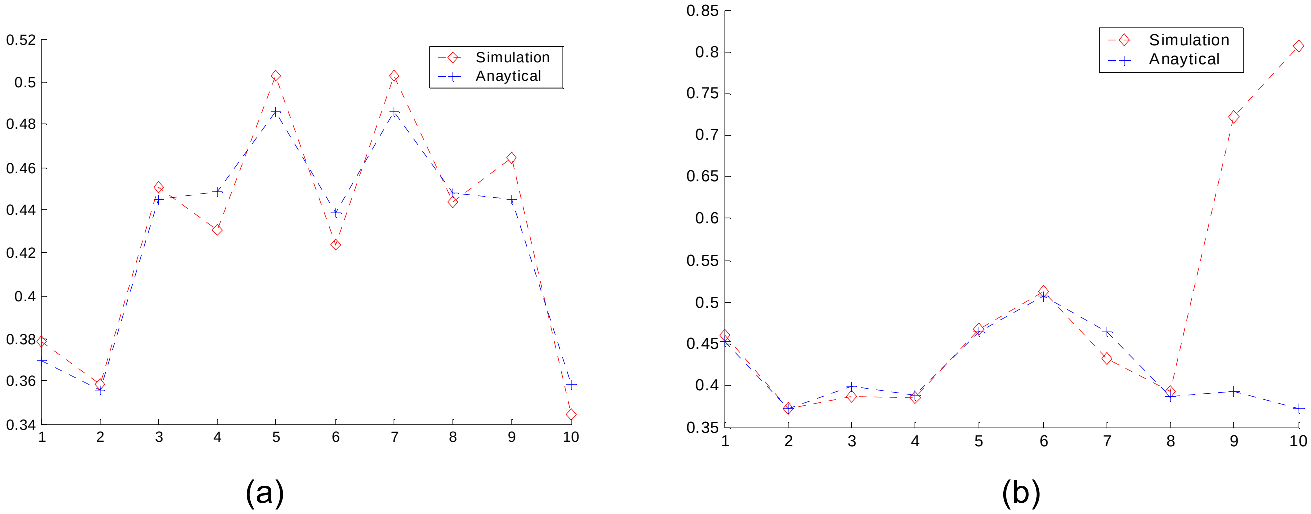

Fig. 8 Std dev for each user (a) $\hat{\varepsilon}_k$=0.3 (b) $\hat{\varepsilon}_k$=0

It can be seen from Fig 8(b) that when receiver estimate of residual fractional error is set to zero, the optimum power allocation is done assuming that perfect cancellation is done, later users are allocated very less power, this can be seen from the optimal power distribution for SIC in [2]. Therefore, later users suffer from bit errors and the assumption $c_k[n] = \hat{c}_k[n]$ is no more valid. This is why standard deviation from simulation for user No 9 and 10 was not able to follow the calculated standard deviation.

V. Conclusion

It has been shown that the statistics of the cancellation error for low bit error rates is related to the statistics of the channel. Mean of the cancellation error is zero and its standard deviation is equal to the standard deviation of the channel. In this paper the channel was modeled as a fading channel with a unit mean Rayleigh envelope, with standard deviation of 0.4 with a Doppler speed of 25 mph. The model used for fading was based on the Clarke's model [5]. It has been shown mathematically and verified through simulation that standard deviation of the cancellation error is approximately equal to the standard deviation of the channel. The standard deviation obtained through simulation is 0.42 which is very close to the STD dev of the channel =0.4. An insight to the pdf of cancellation error based on the receiver used in [2] is also been analytically obtained and verified through simulation. This pdf, which is somewhat like cauchy distribution, can be seen in Fig 7(a) and can be compared with 7 (b).

It can also be seen that the spread in the cancellation error can be reduced if the spread of the amplitude envelope is reduced, this amounts to better power control techniques. The optimum power allocation scheme required for SIC does not add additional complexity relative to a conventional CDMA system as shown in [3].

Therefore in a realistic CDMA system where power control is being implemented to overcome a fading channel, an accurate model for the power control error will have similar second order statistics as that of the cancellation error. With perfect power control it will be an AWGN channel where we have seen that there are negligible estimation/cancellation error.

Appendix A

$$x_k(t) = A_k(t)\sum_{n=-\infty}^{n=\infty} C_k[n].[I_k[n]h(t-nT)Cos(w_c t) + Q_k[n]h(t-nT)Sin(w_c t)] \quad (10)$$

$$x_k[n] = A_k[n](I_k[n] + jQ_k[n])c_k[n] \longrightarrow x_k^I[n] + jx_k^Q[n] \quad (11)$$

$$y_o[n] = \sum_{k=1}^{K} h_k[n].x_k[n-\tau_k] + \eta[n] \quad (12)$$

$I_k[n], Q_k[n]$ are pseudorandom Bernoulli { +1,-1} sequences

Estimated bits for user k are re-encoded, and estimates for the amplitude of the I and Q branches are formed,

$$\alpha_k = 1/M \sum_{n=1}^{M} y_k[n].\hat{x}_k^I[n] \quad \text{where } M = No \text{ of symbols in a frame} \quad (13)$$

$$\beta_k = 1/M \sum_{n=1}^{M} y_k[n].\hat{x}_k^Q[n] \quad (14)$$

Estimate of the recd signal; $\hat{x}_k[n] = \alpha_k \hat{x}_k^I[n] + j\beta_k \hat{x}_k^Q[n]$ (15)

The stored composite signal is updated as $y_k[n] = y_{k-1} - \hat{x}_{k-1}[n] = y_o - \sum_{i=1}^{k-1} \hat{x}_i[n]$ (16)

Error for user k is defined as: $e_k[n] = x_k[n] - \hat{x}_k[n]$ (17)

with fading $x_k[n] = A_k[n]\left(I_k[n]h_k^I[n] + jQ_k[n]h_k^I[n]\right)c_k[n]$ (18)

Looking at error for only I Branch & $c_k[n] = \hat{c}_k[n]$ i.e assuming BER to be low

$$e_k[n] = A_k[n]I_k[n]c_k[n].h_k[n] - \alpha_k I_k[n]c_k[n] \quad (19)$$

Lets look at $\alpha_k = 1/M \sum_{n=1}^{M} y_k[n].\hat{x}_k^I[n]$; since $y_k[n] = y_o - \sum_{i=1}^{k-1} \hat{x}_i[n] = \sum_{k=1}^{K} x_k[n] - \sum_{i=1}^{k-1} \hat{x}_i[n] + \eta[n]$ (20)

$$y_k[n] = \sum_{j=1}^{k-1} e_j[n] + \sum_{i=k+1}^{K} x_i[n] + x_k[n] + \eta[n] \quad (21)$$

$$\alpha_k = \frac{1}{M}\sum_{n=1}^{M}\left[\sum_{j=1}^{k-1} e_j[n] + \sum_{i=k+1}^{K} x_i[n] + x_k[n] + \eta[n]\right].\hat{x}_k^{I} \text{ where } \hat{x}_k^{I} = I_k[n].c_k[n] \quad (22)$$

$$\alpha_k = \frac{1}{M}\sum_{n=1}^{M}\left[\begin{array}{l}\left[\sum_{j=1}^{k-1}\left[A_j[n]h_j[n] - \alpha_j\right].I_j[n]c_j[n].I_k[n].c_k[n]\right] + \left[\sum_{i=k+1}^{K}\left[A_i[n]h_i[n]\right].I_i[n]c_i[n].I_k[n].c_k[n]\right] \\ + \left[\eta[n].I_k[n].c_k[n]\right] + \left[A_k[n]I_k[n]c_k[n]h_k[n].I_k[n].c_k[n]\right]\end{array}\right] \quad (23)$$

$$\frac{1}{M}\sum_{n=1}^{M}\left[\sum_{j=1}^{k-1}\left[A_j[n]h_j[n] - \alpha_j\right].I_j[n]c_j[n].I_k[n].c_k[n]\right] + \left[\sum_{i=k+1}^{K}\left[A_i[n]h_i[n]\right].I_i[n]c_i[n].I_k[n].c_k[n]\right]$$

$+\left[\eta[n].I_k[n].c_k[n]\right]$ This can be approx as a Gaussian Random Variable ξ_k with mean (24)

is zero and std $\cong 0$ as M increases.

Therefore, $\alpha_k = \frac{1}{M}\sum_{n=1}^{M}\left[A_k[n]h_k[n]\right] + \xi_k$ (25)

Ignoring ξ_k & Applying Central Limit Theorem, α_k is

$\alpha_k = A_k\left[z_k.\sigma/\sqrt{M} + \mu\right]$ where σ & μ are Std dev and Mean of h_k (26)

z_k is a Gaussian random variable with zero mean and unit Variance

This shows that α_k is also a Gaussian random Variable with mean A_k and whose variance decreases with higher M,

Using the above result in $e_k[n] = A_k[n]I_k[n]c_k[n].h_k[n] - \alpha_k I_k[n]c_k[n]$ (27)

$e_k = A_k[n]I_k[n]c_k[n].\left[h_k[n] - \hat{\alpha}_k\right]$ where $\hat{\alpha}_k$ is normalized α_k

if we normalize $e_k[n]$ by A_k

$$\hat{e}_k[n] = I_k[n]c_k[n].\left[h_k[n] - \hat{\alpha}_k\right] \quad (28)$$

Expected value of $\widehat{e}_k = 0$; This is consistent with the simulated results

Variance of $\widehat{e}_k$ = variance of Channel h_k

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