Multiuser OFDM Capacity Analysis with Partial Channel Information

Multiuser Wireless Communications Course Project

Zukang Shen

shen@ece.utexas.edu

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Abstract

This paper studies multiuser orthogonal frequency division multiplexing (OFDM) system using adaptive subchannel and power allocation with partial channel information. Previous research on multiuser OFDM includes margin adaptive (MA) optimization and rate adaptive (RA) optimization. The capacity gain using adaptive resource allocation over fixed resource allocation such as time division multiple access (TDMA) and frequency division multiple access (FDMA) was evaluated. However, all the previous work assumes perfect channel information available at the base station. In reality, system delays (such as processing time, channel information collection time, etc) are unavoidable. Only partial/sampled channel information is available at the base station. The delays make the current subchannel and power allocation scheme not optimal to the current channel condition. While the maximum Doppler frequency spread captures the channel variation speed, the capacity of dynamic multiuser OFDM system with partial channel information vs. Doppler frequency is simulated in this paper. In addition, since it is not possible to solve the optimization problem on-line, suboptimal algorithms are always of interest. In this paper, a new suboptimal algorithm to solve the rate adaptive optimization problem is developed.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is becoming very promising for the next generation of wireless communication systems. OFDM divides the whole wide bandwidth into N orthogonal/parallel subchannels. Multipath effect is greatly eliminated by introducing a cyclic prefix (CP) to each OFDM symbol. The existence of cyclic prefix makes the channel appear to be circular and each subchannel thus can be modeled as a gain plus additive white Gaussian noise (AWGN). Besides the advantages (such as less noise, interference enhancement and greater immunity to faster fadings [5]) brought by the multicarrier property of OFDM systems, multiple access is also made possible since the subchannels are independent to each other.

Multiuser OFDM system is a system that explores the multiple access property of OFDM. Multiuser OFDM allows K users to share an OFDM symbol. Two classes of resource (subchannel and power) allocation scheme exist: fixed resource allocation and dynamic resource allocation. Fixed resource allocation schemes, such as time division multiple access (TDMA) and frequency division multiple access (FDMA), assign an independent dimension (time slot or subchannel) to each user. It is obvious that fixed resource allocation scheme is not optimal since the scheme is fixed regardless of the current channel condition. On the other hand, dynamic resource allocation scheme allocates a dimension adaptively to the users whose channel gain is good. Due to varying nature of the wireless channel, dynamic resource allocation makes full use of the multiuser diversity (essentially space diversity) to achieve higher capacity.

Currently, two optimization problems exist in the dynamic multiuser OFDM literature: margin adaptive (MA) [1] and rate adaptive (RA) [2]. The margin adaptive optimization's objective is to achieve the minimum overall transmit power with the constraints on the user data rates [1]. The rate adaptive optimization's goal is to maximize the minimum user's capacity on the total transmit power constraint [2]. These optimizations are non-linear and computationally intensive to solve. In [3], the non-linear optimization problems were transformed into linear optimization problem with integer variables. The optimal solution can be achieved by integer programming. However, even with integer programming, the complexity increases exponentially with the number of users. Suboptimal algorithms are always of interest to balance the tradeoff between complexity and performance.

A key assumption made in all the previous work is that channel condition is perfectly known at the base station. However, various delays in real systems make only partial and delayed channel information available at the base station. Based on the partial information, the resource allocation scheme adopted might not be the optimal one for the current channel status. One of the contributions of this paper is that a more realistic assumption (partial channel information) is taken and the capacity gain of dynamic resource allocation over fixed TDMA is evaluated under this assumption.

This paper is organized as following: section II talks about multiuser OFDM system model and presents the objective function (rate adaptive optimization) concerned in this paper. A suboptimal subchannel allocation scheme is developed in section III. In section IV, optimal single user power distribution (water-filling) is formulated. Section V briefly discusses the various factors that cause the channel information to be imperfect. Various simulation results are presented and discussed in section VI. And we conclude this paper in section VII.

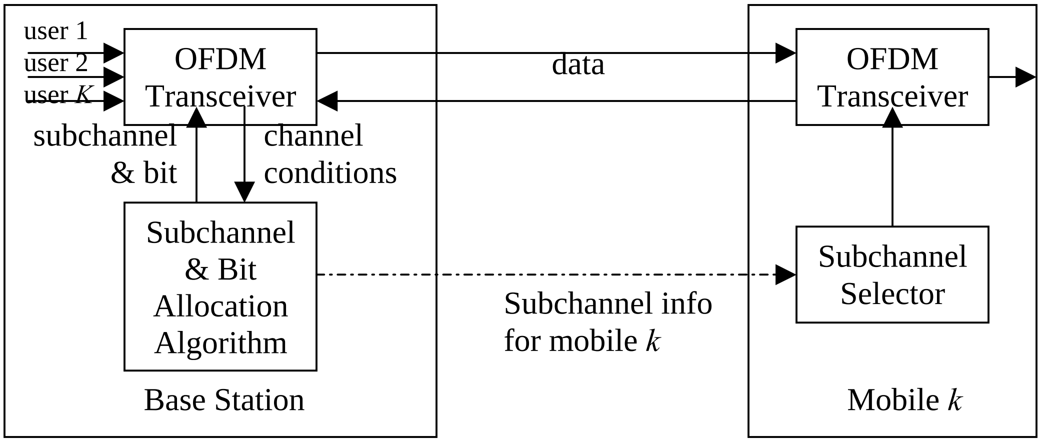

Figure 1: Multiuser OFDM System Diagram

II. System Model and Objective

The multiuser OFDM system is shown in figure1. In the base station (BS), all channel information is collected and sent to the subchannel and bit allocation algorithm block. The resource allocation scheme made by the algorithm is fed back to the OFDM transceiver. The transceiver then selects different number of bits from different users to form an OFDM symbol. Channel information is collected and sent to the subchannel and bit allocation block to update the resource allocation scheme. We also assume that the subchannel and bit allocation information is sent to each user by a separate channel.

In this paper, rate adaptive optimization is considered. The rate adaptive problem can be formulated as:

$$\max_{P_{k,n},s_k} \min_{k} \sum_{n\subset s_k} \frac{B}{N} \log_2\left(1+\frac{P_{k,n}\alpha_{k,n}}{N_0 B/N}\right) \tag{1}$$

$$\text{subject to : } \sum_{n=1}^{N} \sum_{k=1}^{K} P_{k,n} \le P_{\max}$$

$$P_{k,n} \ge 0 \quad \text{for all } k,n$$

$S_1, S_2, \ldots S_K$ are disjoint

$S_1 \cup S_2 \cup \ldots \cup S_K = \{1,2,\ldots,N\}$

Where N is the total number of subchannels, K is the total number of users, B is the total bandwidth, $P_{k,n}$ is the power allocated to user k's subchannel n, $\alpha_{k,n}$ is the channel gain in user k's subchannel n, S_k is the set of subchannels that are assigned to user k, N_0 is the power spectrum density of additive white Gaussian Noise, P_{max} is the total power constraint. Notice that S_k are mutually exclusive since we do not allow a subchannel to be shared by different users.

This optimization problem is non-linear and very hard to solve. By introducing a sharing factor [2], this optimization problem can be converted to a convex optimization problem, which can be solved by some optimization tools, such as AMPL [2]. However, even with the relaxation that sharing a subchannel is allowed, the solvers in the optimization tool need to perform matrix decomposition and inversion to find the optimal solution efficiently. These operations are not practically in real-time implementation of dynamic resource allocation algorithms. Thus suboptimal algorithms that balance the computation requirement and performance are always of interest. Both [2] and [3] propose their suboptimal solution to the optimal solution. In this paper, the optimization problem is solved by allocating subchannel allocation and power allocation sequentially.

III. Subchannel Allocation

On the reality that optimal solution cannot be easily achieved in really time implementation, suboptimal algorithm is developed in this paper. While it is very difficult to optimize the subchannel and power allocation jointly, these two tasks are performed separately. In this section, a new subchannel allocation algorithm is developed and optimal power distribution is discussed in the next section.

The flow chart of the subchannel algorithm is shown in figure 2. Since rate adaptation is the objective in this paper, equalizing all the users' capacity while trying to keep their capacity as high as possible can provide a suboptimal solution to the rate adaptive optimization problem. On the observation that the poorest user suffers from the lowest averaged channel gain, the algorithm first allocates the power in a way to make the averaged equalized channel gain equal to all the users. Then a subchannel is initially assigned to the user who has the highest equalized channel gain in that subchannel. After all the subchannels are assigned, the capacity for each user is calculated, assuming equal power distribution. Then the user with the highest capacity gives up one of his subchannels (the one which has the smallest equalized channel gain to him) to another user. The whole process continues until the capacity difference between all the users drops below a certain threshold.

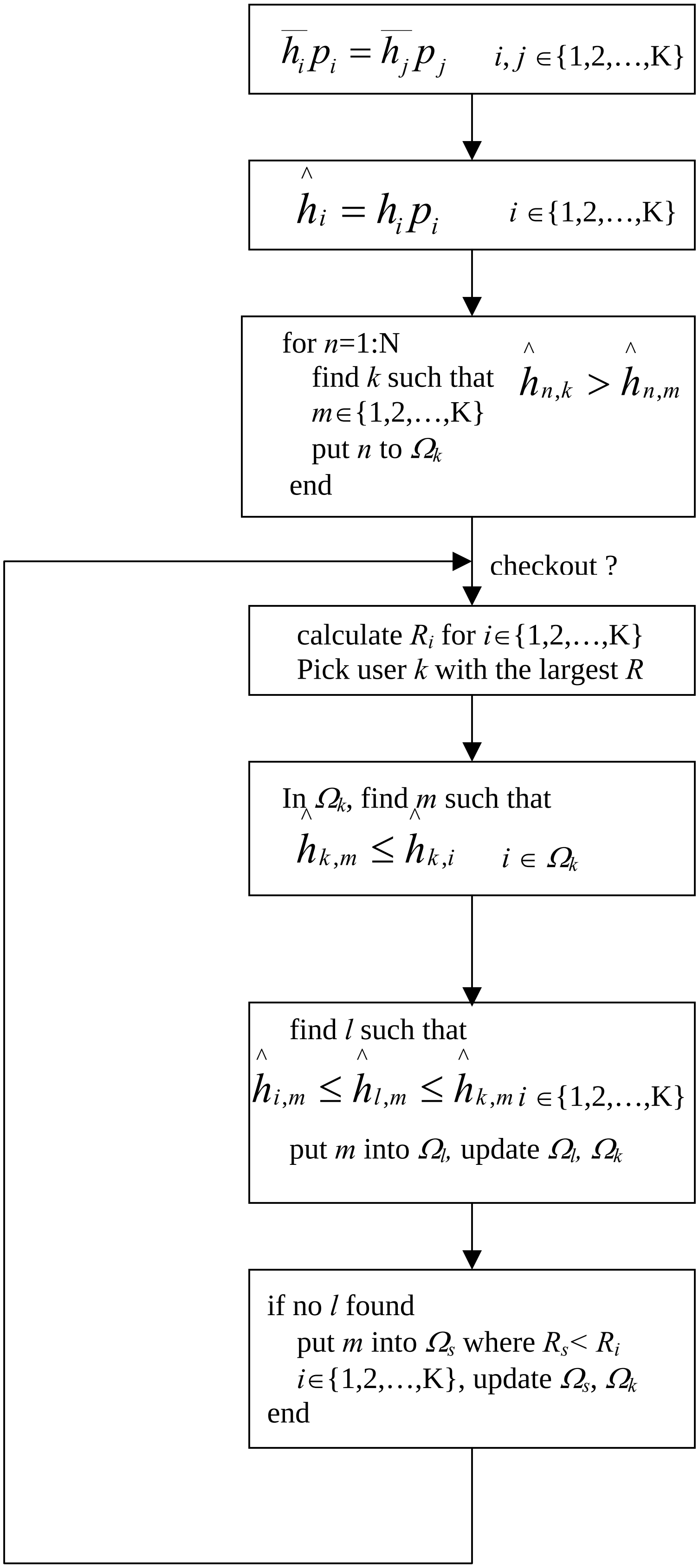

Figure 2: Subchannel allocation algorithm

Where h_i, p_i, h_i, Ω_i are respectively the averaged channel gain, the total power, the equalized channel gain and the set of subchannels that are allocated for user i.

IV. Power Allocation

After subchannel allocation is performed, optimal power allocation in a single user can be achieved by water-filling algorithm. Here we derive the water-filling algorithm as follows:

Suppose M subchannels are assigned to a user. The single user water-filling algorithm will maximize the following cost function:

$$C = \sum_{i=1}^{M} \frac{1}{N} \log_2(1 + p_i \alpha_i) \qquad (2)$$

$$\text{subject to : } \quad \sum_{i=1}^{M} p_i \le P_{tot}$$

Where p_i, α_i are respectively the power and the channel gain for the i^{th} subchannel. P_{tot} is the total power allocated to the user. We also assume that α_i's are in the descending order: $\alpha_M \ge \alpha_{M-1} \ge \ldots \ge \alpha_1$

This optimization problem can be readily solved by forming the following Lagrange cost function:

$$L = \sum_{i=1}^{M} \frac{1}{N} \log_2(1 + p_i \alpha_i) + \lambda\left(\sum_{i=1}^{M} p_i - P_{tot}\right) \qquad (3)$$

Where λ is the Lagrangian multiplier.

The optimal solution to the Lagrange cost function can be expressed as:

$$p_j = p_i + \frac{\alpha_j - \alpha_i}{\alpha_j \alpha_i} \qquad \text{i, j} \in \{1,2,\ldots M\} \qquad (4)$$

Let i=1 and substitute all the p_j into the overall power constraint, we can get:

$$M p_1 + \sum_{j=2}^{M} \frac{\alpha_j - \alpha_1}{\alpha_j \alpha_1} \le P_{tot} \qquad (5)$$

The above equation shows us a way to calculate the optimal power for 1st subchannel. The power for other subchannel can be calculated by equation (4). There is one condition that needs some discussion: if the summation in the right side of equation (5) is larger than Ptot, p1 will be negative value, which is not possible since all the power value should be non-negative. The negative value for p1 indicates that the gain in the 1st subchannel is so poor that it should not be used. A recursive method can be used to find which subchannels should be used.

V. Partial Channel Information

In the multiuser OFDM literature, one important assumption is that the channel information is perfectly known the base station. However, in reality, various kinds of delay make the current allocation scheme not optimal to the current channel condition. These delays includes:

- Processing delay, that is the time needed to solve the optimization problem.
- Round trip delay. In FDMA systems, the downlink and uplink work in orthogonal frequency band. In order to get the downlink channel condition, the base station will send a probe to the mobile. The mobile then estimates the downlink channel and feeds the information back to the base station. This delay might not be a problem in TDMA system since both the downlink and uplink work in the same frequency band. However, there will still be some delay since guard time is used in TDMA system.

In a word, the channel information that the base station can collect is either partial or delayed, or even both. The performance of dynamic allocation scheme with partial information is reported in the next section.

VI. Simulation Results

In this section, various sets of simulation results are reported. In all the simulations, the wireless channel is modeled as a frequency-selective multipath channel with six independent Rayleight multipaths (with an exponentially decaying profile). The maximum delay spread is 0.25 usec. Channel information is sampled every 0.5 msec to update the subchannel and power allocation.

Figure 3 shows the performance of the proposed suboptimal algorithm. Four users are in the system and the maximum path loss difference is 40 dB. The x-axis is the worst signal to noise ratio (WSNR) defined as the worst user's averaged SNR. The plot shows the capacity difference between all the users vs. WSNR. We can see that around 0 dB of WSNR, the standard deviation of all the users' capacities is less 7% of the mean capacity.

Figure 4 shows the capacity gain over path loss difference. Two users are assumed in the system and the WSNR is 10 dB. From the plot, we can see that the capacity gain approaches a constant when the path loss difference is larger than 20 dB.

Figure 5 and 6 show the capacity vs. user number. The maximum path loss in the system is 40 dB and the WSNR is fixed at 10 dB. Figure 5 shows that as user number increases, the capacity for a single user drops, since the overall capacity is almost fixed. However, from figure 6, it is obvious that as user number increases, the capacity gain (percentage) of dynamic resource allocation scheme over fixed TDMA increases. The reason is with large number of user in system, more diversity can be used by the base station and hence more capacity gain.

Figure 7 and 8 show the capacity gain vs. Doppler spread if the overall system delay is fixed at 0.01 second. Four users are assumed to be in the system. The WSNR is 10dB and the maximum path loss difference is 40 dB. Figure 7 tells us that the capacity gain of dynamic scheme over fixed TDMA drops drastically as Doppler spread goes large, while the capacity gain remains almost constant if perfect channel information is available at the base station. Figure 8 shows the percentage of the capacity gain with partial channel information to perfect channel information. It can be read from figure 8 that at the Doppler spread around 50 Hz, the percentage goes below 20% and hence it brings little gain by adopting dynamic resource allocation scheme over fixed TDMA, while the computation requirement is much higher in the former case.

VII. Conclusions

In this paper, rate adaptive optimization in multiuser OFDM systems is studied. While it is computationally expensive to optimize subchannel allocation and power distribution jointly, subchannel allocation and power distribution are performed sequentially in the paper. A suboptimal subchannel allocation algorithm is developed and proved to be good by simulation. Another contribution of this paper is that a more realistic assumption -- partially channel information, is taken and used in the simulations. With this assumption, simulation results show that with a fixed system delay of 0.01 second, the capacity gain of dynamic multiuser OFDM system over fixed TDMA system is insignificant when the maximum Doppler frequency is larger than 200 Hz.

VIII. References

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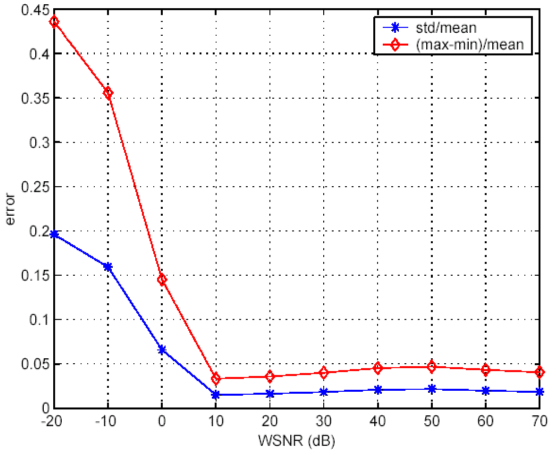

Fugire 3: error vs. WSNR

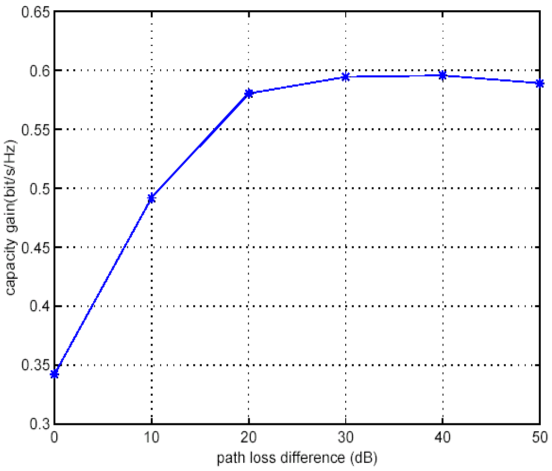

Figure 4: capacity gain vs. path loss difference

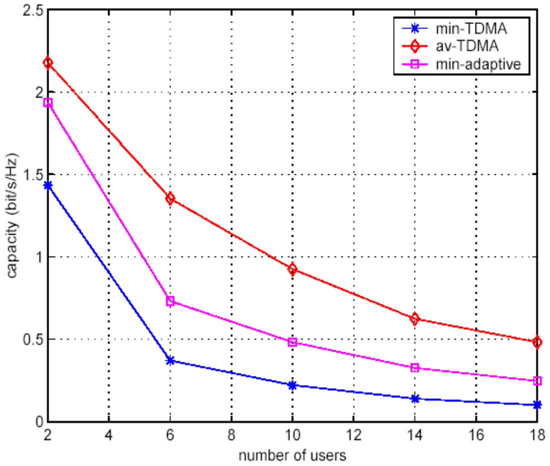

Figure 5: capacity vs. user number

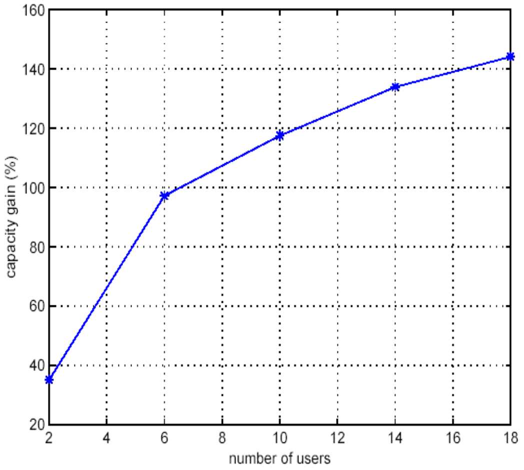

Figure 6: capacity gain vs. user number

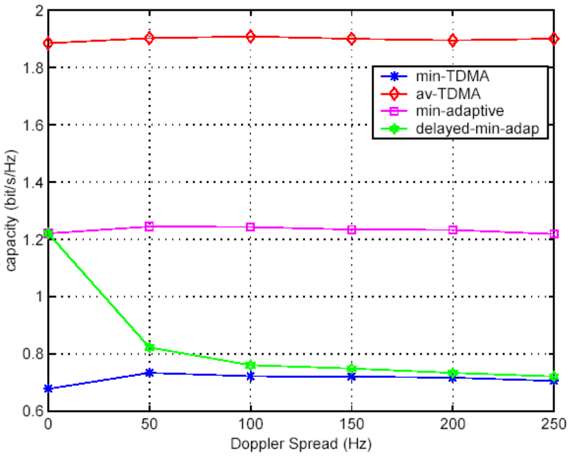

Figure 7: capacity vs. Doppler Spread

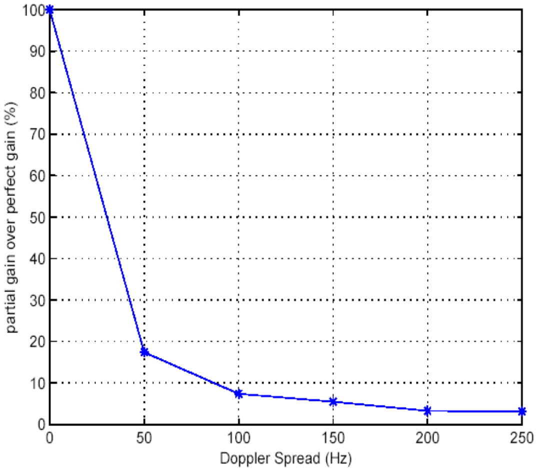

Figure 8: gain vs. Doppler Spread