Spatial Multiplexing in Cellular MIMO-CDMA Systems with Linear Receivers: Outage Probability and Capacity

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Abstract

Multiple transmit and receive antennas, and the diverse matrix channel they create, are frequently touted as the key to increasing capacity in future cellular communication systems. Spatial multiplexing in particular has been shown to increase the capacity proportionally to the number of transmit antennas in many important cases. In this paper, the effectiveness of spatial multiplexing in the forward link of cellular CDMA systems with linear (i.e. low complexity) receivers is investigated. General MIMO systems without spreading are a special case of our analysis when the spreading gain is unity. Through the development of new closed-form results on outage probability and capacity for MIMO-CDMA, we show that cellular MIMO outage capacity is severely degraded by the enhancement of other-cell interference by the linear spatial receiver, and that a large number of transmit and receive antennas is required to simply break even with a SISO system. The results indicate that a practical cellular MIMO system, which will be interference-limited and have a low-complexity receiver, requires new study on techniques to reduce the impact of the other-cell interference. In addition to the derivation of outage probability and capacity, this paper proposes a new scheme for cooperatively scheduled transmissions among base stations. It is shown that this scheme can effectively compensate for the impairment caused by the linear receivers and restore a significant capacity advantage for MIMO-CDMA over SISO-CDMA.

Index Terms

MIMO systems, CDMA, spatial multiplexing, outage probability, outage capacity.

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I. Introduction

IT is well known that wireless communication systems can dramatically improve their throughput and robustness by deploying multiple antennas at both the transmitter and receiver. A large suite of techniques, known collectively as multiple input multiple output (MIMO) communications, have been developed in the past several years to exploit the resulting multidimensional channel. Significant spectral efficiency advantages can be achieved by exploiting the characteristics of MIMO channels [1]–[4]. However, such studies are primarily for a single point-to-point link and there have been comparatively few studies on multiuser MIMO systems.

MIMO technology is heralded for its potential to significantly increase – by close to an order of magnitude – the data rate of cellular and other wide area communications, which presently are mired at data rates far too slow to attractively support wireless broadband access. Modern cellular systems, whether CDMA-based (IS-95 or 3G) or TDMA-based (GSM), are and will continue to be interference-limited, since it is in the strong interest of service providers to provide universal frequency reuse and high per-cell loading. The logical conclusion then is that in order to be effective, MIMO systems will need to function reliably in an interference-limited environment.

The new dimensions provided by the multidimensional MIMO channel can be exploited to increase the diversity of the system or for orthogonal spatial channels. Diversity is generally considered lower risk, and a well-known example is space-time codes [5], [6], which have found adoption in 3G CDMA cellular systems [7]. Diversity increases the robustness of the system by eliminating fades, and the capacity growth is logarithmic with the diversity order. On the other hand, spatial multiplexing divides the incoming data into multiple substreams and transmits each on a different antenna. If successfully decoded, it is logical that this increases the capacity linearly with the number of transmit antennas (or communication dimensions), which also has been proven in the information theory community [8]. Spatial multiplexing is thus more exciting than spatial diversity from a high-data rate point of view, but there is a fundamental tradeoff between them [9]. This implies that spatial multiplexing systems will likely need other forms of diversity, such as spread spectrum and/or sophisticated coding.

CDMA is central to the worldwide 3G cellular standards, and is likely to be a dominant multiple access technique in wide area wireless networks for the foreseeable future. CDMA systems are designed to operate in an interference-limited environment and primarily for this reason have proven efficient in modern cellular communications. Additionally, CDMA systems with low or unity spreading factors can be viewed as general interference-limited systems. For these reasons of current relevance and generality, we will adopt a MIMO-CDMA framework in this paper for analyzing the outage probability and capacity of interference-limited MIMO systems.

Initial investigations on MIMO systems with co-channel interference can be seen in [10], [11], which quantified the throughput of multicell MIMO systems with spatial multiplexing by computer simulations. They showed that co-channel interference could seriously degrade the overall capacity of a MIMO system with spatial multiplexing to the point of negligible improvement over single input multiple output (SIMO) systems. The reason for this is that the independent data streams effectively become independent interferers, and without any diversity, insufficient degrees of freedom to combat this co-channel interference are available, unless the number of receive antenna is very large. More recently, the authors of [12] developed an interference-aware MIMO receiver and showed that this multiuser receiver could in principle increase the capacity of MIMO systems with co-channel interference over the prior result of [10], [11].

Although multiuser detectors considering both spatial and co-channel interference are simpler than the optimum maximal likelihood (ML) detector, multiuser receivers have not proven viable in most wide area wireless networks due to a number of reasons including complexity, noise enhancement, and insufficient dimensionality. Therefore, one simple and hence attractive architecture is a linear spatial receiver such as zero forcing (ZF) [13] or MMSE [14], followed by a matched filter CDMA receiver. Even when the receivers do not take into account other-cell interference, conventional successive canceling receivers, known commonly as BLAST receivers [15], [16], are based on joint detection and their complexity is much higher than linear receivers. This study hence aims for a thorough investigation of the performance of spatial multiplexing with linear receivers for multiuser MIMO cellular systems, since this is a practical and straightforward option for future enhancements to current cellular systems.

Recently, an information theoretic investigation on the effectiveness of spatial multiplexing with linear receivers has been provided in [17], [18]. The authors showed that although the capacity of a single, isolated communication link could not be improved by reducing the number of substreams, it would be preferable for capacity to have all users actually use fewer substreams in an interference-limited MIMO system with linear receivers and single user detection. It also stated that reducing the number of independent streams was similar to power control since other users see effectively less interference when the number of substreams is controlled. In addition to causing less interference to the other users, using fewer streams may enable users to null interference with the extra degrees of freedom [17], [18].

Based on the intuition obtained from the recent research on the MIMO systems with co-channel interference, the effectiveness of spatial multiplexing in cellular CDMA systems with linear receivers needs to be carefully investigated. It is possible that although spatial multiplexing has a fundamental capacity advantage relative to transmit diversity, that this advantage is lost in cellular MIMO systems with linear receivers. Therefore, in this paper, we provide a general framework for analyzing the outage capacity of cellular MIMO-CDMA systems and investigate the effectiveness of spatial multiplexing in terms of outage capacity. We focus on the downlink where the demand for high speed data will be dominant, whereas most outage capacity papers have focused on the reverse link due to its analytical tractability.

Regarding the prior studies on cellular MIMO-CDMA systems, to the best of authors' knowledge, no papers addressing the effectiveness of the spatial multiplexing have appeared. Most of the papers on MIMO-CDMA systems have focused on designing a new type receiver or a space time coding scheme, and analyzing its performance, e.g. [12], [19]–[21]. Papers addressing throughput in the forward link of MIMO-CDMA systems mainly rely on computer simulations [22], [23]. A referential study on MIMO-CDMA capacity with spatial multiplexing and a modified multiuser V-BLAST receiver can be seen in [24]. They determined the spectral efficiency for this system by integrating the link level bit error probability results with the system level outage simulation results. In contrast, our contribution addresses linear receivers and provides a general closed-form analysis of outage probability and capacity. In addition, we compare the outage capacity of the proposed MIMO-CDMA system with conventional SISO-CDMA systems and investigate the effectiveness of spatial multiplexing for practical cellular systems, with the conclusion that spatial-multiplexed MIMO-CDMA systems will need to employ techniques to contend with other-cell interference. Based on this result, we propose a cooperatively scheduled transmission scheme among base stations that achieves a net capacity gain without increasing the complexity of the mobile units.

The rest of this paper is organized as follows. In section II, the system model of spatial multiplexing in cellular MIMO-CDMA systems with linear receivers is given. The outage probability and capacity of the proposed system are analyzed in section III for both zero forcing and MMSE receivers, the latter of which will not be practical in most practical scenarios. In section IV, the effectiveness of spatial multiplexing in cellular MIMO-CDMA with linear receivers is investigated and discussed, and some implementation issues are raised. Some numerical results are presented in section V. Finally, we draw conclusions and provide suggestions for future research in section VI.

II. System Model

For spatial multiplexing, incoming data is divided into multiple substreams and each substream is transmitted on a different transmit antenna. The transmitter and receiver structure of MIMO-CDMA systems for spatial multiplexing is given in Figure 1. The data of each user is spatially multiplexed into substreams to be transmitted across M_t transmit antennas and each substream is spread by a spreading code. Though the spreading codes should be different among users, either the same code or different codes can be used in spreading substreams of a given user. Even though the use of different codes can cause code scarcity, it can achieve superior performance to the use of the same code because the substreams can be differentiated by both the spatial characteristics and their codes [24]. After spreading, substreams are transmitted on their corresponding transmit antennas. The number of receive antennas is M_r and the received chip-level sampled signal for the desired user k is given in a matrix form by

$$\mathbf{Y}_k = \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{H}_{0,k}\mathbf{S}_0 + \sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{H}_{i,k}\mathbf{S}_i + \mathbf{N}_k \tag{1}$$

where $\chi_{i,k}$ and $d_{i,k}$ denote the lognormal shadow fading from the base station i to the user k and the distance from the base station i to the user k, respectively, and depend on the location of the desired user k. The superscript l is the path loss exponent. The matrix $\mathbf{H}_{i,k}$ denotes the channel matrix representing short term fading from the base station i to the user k. Each entry of the matrix $\mathbf{H}_{i,k}$ is an *i.i.d.* complex Gaussian random variable $\sim \mathcal{CN}(0,1)$ representing short term fading. This channel model is acceptable because the difference of propagation loss and long-term shadow fading among antenna elements of the antenna array in a mobile station is negligible. The matrix $\mathbf{S}_i$ denotes the transmitted signal from the base station i and the mth row of the matrix $\mathbf{S}_i$ representing substreams corresponding to the mth antenna given by

$$\mathbf{s}_i^{(m)} = \sum_{k=1}^{K} \sqrt{\varphi_{i,k}^{(m)} P_i}\; b_{i,k}^{(m)} \mathbf{c}_{i,k}^{(m)} \tag{2}$$

where K is the total number of users in a cell and $\varphi_{i,k}^{(m)}$ is the relative power portion assigned to the mth substream of user k in the base station i. The relative power portion $\varphi_{i,k}^{(m)}$ should satisfy the constraint of $\sum_{k=1}^{K}\sum_{m=1}^{M_t}\varphi_{i,k} = 1$. P_i and $b_{i,k}^{(m)}$ are the total power of the base station i and the transmitted (binary) bit of the mth substream of the user k from the base station i with value of ± 1, respectively. The vector $\mathbf{c}_{i,k}^{(m)}$ denotes the $1 \times J$ vector representing the chip-level sampled spreading code for the mth substream to the user k. Although orthogonal codes are optimal as spreading codes in downlink, the orthogonality of the orthogonal codes is damaged by the multipath channel and hence the spread signals are also scrambled by pseudo-random noise (PN) codes to improve the correlation properties. Therefore, we consider $\mathbf{c}_{i,k}^{(m)}$ as the combined orthogonal/PN code. The matrix $\mathbf{N}_k$ is the $M_r \times J$ matrix representing *i.i.d.* complex Guassian noise $\sim \mathcal{CN}(0, \sigma_n^2)$ and J is the spreading gain. Note that general MIMO systems without spreading are a special case of our model when the spreading gain is unity and thus $\mathbf{c}_{i,k}^{(m)}$ reduces to a binary random variable. Even when time division multiple access (TDMA) or frequency division multiple access (FDMA) is considered, the above model in (2) can be used by newly defining $\mathbf{c}_{i,k}^{(m)}$ as an orthogonal basis in the time or frequency domain.

At the receiver side, we consider a simple two-stage linear receiver consisting of a zero forcing (ZF) linear filter for spatial separation and a conventional linear matched filter (MF) bank for detection of the desired user's substreams as figure 1 because of the minimal complexity requirements of mobile stations. Mobile units are highly power limited and hence have limited processing power [25]. Therefore, simple linear receivers are more attractive than complicated non-linear receivers including multiuser detection schemes with joint detection if they have reasonable performance and careful studies on feasibility of the linear receivers are necessary. The MMSE linear filter can replace the ZF filter for spatial separation but it requires increased system knowledge. After matched filtering, the symbol detection for the substreams of the desired user are made and the detected symbols are de-multiplexed and reconstructed into a data stream of the desired user.

In the analysis, the following conditions are considered: (1) Multiple cell structure with 19 cells is considered. (2) Mobile stations are uniformly distributed throughout the cell. (3) Polar (r, θ) coordinates are employed and the location of the base station 0 corresponds to the origin of the coordinates. (4) MIMO structure with M_t antennas at the base station and M_r antennas at the mobile station is considered.

III. Analysis of Outage Probability and Capacity

In this section, we firstly consider the ZF filter as the pre-filter for spatial separation and then discuss the MMSE filter, which is a generalization of the ZF filter but requires increased system knowledge.

A. Analysis for ZF Filters

1) Signal-to-Interference Ratio: The composite received signal is firstly processed by the linear ZF filter in order to separate the substreams from the distinct transmit antennas. Then, the spatially separated signals are given by

$$\mathbf{Z}_{0,k} = \mathbf{H}_{0,k}^{\dagger}\mathbf{Y}_{0,k} = \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{S}_0 + \mathbf{H}_{0,k}^{\dagger}\sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{H}_{i,k}\mathbf{S}_i + \mathbf{H}_{0,k}^{\dagger}\mathbf{N}_k \tag{3}$$

where $(\cdot)^{\dagger}$ denotes the pseudoinverse of a matrix.

After spatial separation by the linear ZF filter, each substream is processed by a matched filter for detection of the desired user's substream. The mth row of the matrix $\mathbf{Z}_{0,k}$ represents the composite substream corresponding to the mth transmit antenna and is given by

$$\mathbf{z}_{0,k}^{(m)} = \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{s}_0^m + \mathbf{f}_{0,k}^{(m)}\sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{H}_{i,k}\mathbf{S}_i + \mathbf{f}_{0,k}^{(m)}\mathbf{N}_k \tag{4}$$

where $\mathbf{f}_{0,k}^{(m)}$ is the mth row of $\mathbf{H}_{0,k}^{\dagger}$. The spatially separated composite substream is similar to the general form of the chip level sampled received signal in CDMA communications. Each separated composite substream, i.e., each row of the matrix $\mathbf{Z}_{0,k}$, is matched filtered for the desired user and the MF output of the mth row of the matrix $\mathbf{Z}_{0,k}$ is obtained by

$$\begin{aligned} z_{0,k}^{(m)} &= \mathbf{z}_{0,k}^{(m)} \cdot \left(\mathbf{c}_{0,k}^{(m)}\right)^H \\ &= \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\sqrt{\varphi_{0,k}^{(m)}\ P_0}\ b_{0,k}^{(m)} + \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\sum_{j=1,j\neq k}^{K}\sqrt{\varphi_{0,j}^{(m)}\ P_0}\ b_{0,j}^{(m)} \cdot \sqrt{\alpha}\rho \\ &\quad + \sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\left(\mathbf{c}_{0,k}^{(m)}\right)^H \ + \ \mathbf{f}_{0,k}^{(m)}\mathbf{N}_k\left(\mathbf{c}_{0,k}^{(m)}\right)^H \end{aligned} \tag{5}$$

where ρ denotes the cross correlation among the random scrambling codes and α is the orthogonality factor reflecting the imperfection of the orthogonality among orthogonal codes due to multipath propagation. As the multipath fading becomes severe, the forward link orthogonality by the orthogonal spreading codes become sacrificed and α goes to 1 [26]. When the spreading gain is unity, the analysis reduces to a special case of general MIMO systems and both α and ρ become unity. The MF output of (5) consists respectively of the desired signal component, the intracell interference component, the intercell interference component, and the thermal noise component.

When the fading environment is a superposition of both fast and slow fading, i.e. log-normal shadowing and Rayleigh fading, a common performance metric is combined outage probability and average error probability where the outage occurs when the slow fading falls below some target value, i.e. a target signal-to-interference-plus-noise ratio (SINR) averaged over the fast fading [27]. Since the background noise power is negligible compared to the interference power in an interference-limited environment, the signal-to-interference ratio (SIR) of the mth substream for the desired user k averaged over the fast fading can be given by

$$\gamma_k^{(m)} = \frac{\chi_{0,k}(d_{0,k})^{-l}\varphi_{0,k}^{(m)}P_0}{\chi_{0,k}(d_{0,k})^{-l}\sum_{j=1,j\neq k}^{K}\varphi_{0,j}^{(m)}\alpha\rho^2 P_0 + \sum_{i=1}^{N_c}\chi_{i,k}(d_{i,k})^{-l}\mathbb{E}\left[\left|\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}\right|^2\right]} \tag{6}$$

$$= \frac{\varphi_{0,k}^{(m)}}{\sum_{j=1,j\neq k}^{K}\varphi_{0,j}^{(m)}\alpha\rho^2 + \sum_{i=1}^{N_c}\frac{\chi_{i,k}(d_{0,k})^l}{\chi_{0,k}(d_{i,k})^l}\frac{\mathbb{E}\left[\zeta_{i,k}^{(m)}\right]}{P_0}} \tag{7}$$

where $\zeta_{i,k}^{(m)} = \left|\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}\right|^2$. In the denominator of (6), the third term denoting other-cell interference power is obtained from the independence property among transmitted bits. The random variable $\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}$ can be approximated as an *i.i.d.* complex Gaussian random variable by the central limit theorem (CLT) because it comes from the sum of many random variables consisting of $\mathbf{f}_{0,k}^{(m)}$, $\mathbf{H}_{i,k}$, $\mathbf{S}_i$, and $\mathbf{c}_{0,k}^{(m)H}$, assuming the spreading gain J (which determines the dimensions of the matrix $\mathbf{S}_i$ and the vector $\mathbf{c}_{0,k}$) is large enough. When the spreading gain is small or unity, the Gaussian approximation for the random variable $\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}$ may still be reasonable if K is large, but typically a small spreading factor J will require a small number of users K. Although the inaccuracy of the Gaussian approximation will reduce the accuracy of the analysis in these edge cases, the exact analysis is left as further research and the Gaussian approximation is used throughout this work. When TDMA or FDMA is used (as will be the case for small or unity J), the random variable $\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}$ may appear Gaussian due to imperfect orthogonality amongst neighboring time and frequency slots from insufficient guard times or bandwidths.

Utilizing the Gaussian approximation, $\{\zeta_{i,k}^{(m)}\}$ are *i.i.d.* chi-squared distributed random variables with two degrees of freedom and mean

$$\mathbb{E}\left[\zeta_{i,k}^{(m)}\right]=\mathbb{E}\left[\mathbf{f}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i\mathbf{c}_{0,k}^{(m)H}\mathbf{c}_{0,k}^{(m)}\mathbf{S}_i^H\mathbf{H}_{i,k}^H\mathbf{f}_{0,k}^{(m)H}\right] \tag{8}$$

After some mathematical expansions, the mean can be reduced to

$$\begin{aligned}\mathbb{E}\left[\zeta_{i,k}^{(m)}\right] &= \mathbb{E}\left[\sum_{q=1}^{M_r}\sum_{n=1}^{M_r}\sum_{l=1}^{M_t}\sum_{j=1}^{M_t} f_{0,k,q}^{(m)}\ f_{0,k,n}^{(m)H} h_{i,k,n,j}\ h_{i,k,q,l}^H\ \rho^2\sum_{k'=1}^{K}\sqrt{\varphi_{i,k'}^{(l)}P_i}b_{i,k'}^{(l)}\sum_{k'=1}^{K}\sqrt{\varphi_{i,k'}^{(j)}P_i}b_{i,k'}^{(j)H}\right]\\ &= \sum_{n=1}^{M_r}\sum_{j=1}^{M_t}\sum_{k'=1}^{K}\varphi_{i,k'}^{(j)}\rho^2P_i\mathbb{E}\left[|h_{i,k,n,j}|^2\right]\mathbb{E}\left[|f_{0,k,n}^{(m)}|^2\right]\end{aligned} \tag{9}$$

where $h_{i,k,n,j}$ is the (n,j) element of matrix $\mathbf{H}_{i,k}$ and $f_{0,k,n}^{(m)}$ is the *n*-th element of the vector $\mathbf{f}_{0,k}^{(m)}$. The last equality in (9) holds from the independence among the elements of matrix $\mathbf{H}_{i,k}$ and matrix $\mathbf{H}_{0,k}$. If the allocated power to each substream of each user is equal and $\mathbb{E}\left[|h_{i,k,n,j}|^2\right]=2\sigma^2(=2)$, the mean can be given by

$$\begin{aligned}\mathbb{E}\left[\zeta_{i,k}^{(m)}\right] &= \frac{2\sigma^2\rho^2P_i}{M_t}\mathbb{E}\left[\|\ \mathbf{f}_{0,k}^{(m)}\ \|_2^2\right]\\ &= \frac{2\sigma^2\rho^2P_i}{M_t}\mathbb{E}\left[\left(\mathbf{H}_{0,k}^H\mathbf{H}_{0,k}\right)_{m,m}^{-1}\right]\end{aligned} \tag{10}$$

where $\|\cdot\|_2$ is the L^2 vector norm and $\mathbf{A}_{m,m}^{-1}$ is entry (m,m) of $\mathbf{A}^{-1}$. The derivation of the mean of $\zeta_{i,k}^{(m)}$ is given in Appendix A.

Now, the expression for the SIR of the substream of the desired user k given in (6) and (7) is obtained. Based on this SIR, the downlink outage probability and capacity is derived as follows.

2) Outage Probability and Capacity: If the allocated power to each substream of each user is equal in the home base station, the SIR of the mth substream for the desired user k in (6) and (7) can be given by

$$\gamma_k^{(m)}=\frac{\varphi_{0,k}^{(m)}}{\left(1/M_t-\varphi_{0,k}^{(m)}\right)\alpha\rho^2+\sum_{i=1}^{N_c}\frac{\chi_{i,k}(d_{0,k})^l}{\chi_{0,k}(d_{i,k})^l}\frac{\mathbb{E}\left[\zeta_{i,k}^{(m)}\right]}{P_0}} \tag{11}$$

and the relative power portion allocated to the mth substream of the desired user k at the location of (r,θ) can be obtained by

$$\begin{aligned}\varphi_{0,k}^{(m)}(r,\theta) &= \frac{\frac{1}{P_0}\mathbb{E}\left[\zeta_{i,k}^{(m)}\right]\xi_k^{(m)}\gamma_{req}^{(m)}+\frac{1}{M_t}\alpha\rho^2\gamma_{req}^{(m)}}{1+\alpha\rho^2\gamma_{req}^{(m)}}\\ &= a\xi_k^{(m)}+b\end{aligned} \tag{12}$$

where

$$\xi_k^{(m)} = \sum_{i=1}^{N_c} \left(\frac{\chi_{i,k}}{\chi_{0,k}}\right) \left(\frac{d_{0,k}}{d_{i,k}}\right)^l, \tag{13}$$

$$a = \frac{\mathbb{E}\left[\zeta_{i,k}^{(m)}\right] \gamma_{req}^{(m)} / P_0}{1 + \alpha\rho^2 \gamma_{req}^{(m)}}, \qquad b = \frac{\alpha\rho^2 \gamma_{req}^{(m)} / M_t}{1 + \alpha\rho^2 \gamma_{req}^{(m)}}, \tag{14}$$

and $\gamma_{req}^{(m)}$ is the required SIR for the mth substream of the user k, which guarantees the target average bit error probability.

Since $\left\{\frac{\chi_{i,k}}{\chi_{0,k}}\right\}$ are lognormal random variables and the sum of lognormal random variables can be well approximated as a lognormal random variable [28], the random variable $\xi_k^{(m)}$ can be approximated as a random variable with mean μ_ξ and variance σ_ξ^2 as [28]

$$\mu_\xi = \sum_{i=1}^{Nc} \left(\frac{d_{0,k}}{d_{i,k}}\right)^l \exp\left(\frac{\ln 10}{10}(m_i - m_0) + \left(\frac{\ln 10}{10}\right)^2 \frac{(\sigma_i^2 + \sigma_0^2)}{2}\right) \tag{15}$$

$$\begin{aligned} \sigma_\xi^2 = & \sum_{i=1}^{Nc} \left(\frac{d_{0,k}}{d_{i,k}}\right)^{2l} \exp\left(\frac{2\ln 10}{10}(m_i - m_0) + \left(\frac{\ln 10}{10}\right)^2 (\sigma_i^2 + \sigma_0^2)\right) \\ & \cdot \left\{ \exp\left(\left(\frac{\ln 10}{10}\right)^2 \left(\sigma_i^2 + \sigma_0^2\right)\right) - 1 \right\} \end{aligned} \tag{16}$$

where m_i and m_0 are the means of $\chi_{i,k}$ and $\chi_{0,k}$, respectively, and σ_i and σ_0 are the s.t.d.'s of $\chi_{i,k}$ and $\chi_{0,k}$, respectively.

The forward link outage occurs when the forward link power budget runs out. Therefore, the outage probability can be evaluated as

$$\begin{aligned} P_{out} &= Pr\left[\sum_{m=1}^{M_t}\sum_{k=1}^{K} \nu_k \; \varphi_{0,k}^{(m)}(r,\theta) \; > \; 1\right] \\ &= Pr\left[\int_0^R\int_0^{2\pi} \sum_{m=1}^{M_t} \nu_k \varphi_{0,k}^{(m)}(r,\theta) \; \frac{r}{\pi R^2} \; dr d\theta \; > \; 1\right] \end{aligned} \tag{17}$$

where R is the radius of a cell and ν_k is the activity factor for user k's data with $Pr[\nu_k = 1] = \nu$.

Although the closed form solution cannot be directly obtained from (17) because the distribution of the random variable $\varphi_{0,k}^{(m)}(r,\theta)$ depends on the location of each mobile station, we can obtain the closed form outage probability with the method introduced in [29]. In order to obtain the closed form outage probability, we firstly assume the worst case that all the mobile stations are at the cell boundary. Then, we apply the forward power factor η, which is a correction factor compensating for the worst case to reflect the fact that the mobile stations are uniformly distributed in the cell. Then, the outage probability can be obtained by

$$\begin{aligned} P_{out} &= Pr\left[\sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k \varphi_{0,k}^{(m)}(R,0) \;>\; \frac{1}{\eta}\right] \\ &= Pr\left[a\sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k {\xi'_k}^{(m)} + \sum_{k=1}^{K}\sum_{m=1}^{M_t} b\nu_k \;>\; \frac{1}{\eta}\right] \end{aligned} \tag{18}$$

where ${\xi'_k}^{(m)}$ denotes the random variable $\xi_k^{(m)}$ when the user k is at the cell edge.

For given $\{\nu_k\}$, $\sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k {\xi'_k}^{(m)}$ can again be approximated as a log-normal random variable. Let Y be $\sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k {\xi'_k}^{(m)}$, then the log-normal random variable Y has mean and variance as

$$\mu_y = \sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k \mu_{\xi'} \quad \text{and} \quad \sigma_y^2 = \sum_{k=1}^{K}\sum_{m=1}^{M_t} \nu_k^2 \sigma_{\xi'}^2 \tag{19}$$

Then, the conditional outage probability can be obtained by

$$\begin{aligned} P_{out|\{\nu_k\}} &= Pr\left[Y > \frac{1}{a}\left(\frac{1}{\eta} - b\sum_{k=1}^{K}\sum_{m=1}^{M_t}\nu_k\right) \mid \{\nu_k\}\right] \\ &= Pr\left[10\log Y > 10\log\left(\frac{1}{a}\left(\frac{1}{\eta} - b\sum_{k=1}^{K}\sum_{m=1}^{M_t}\nu_k\right)\right) \mid \{\nu_k\}\right] \\ &= Q\left(\frac{10\log\left(\frac{1}{a}\left(\frac{1}{\eta} - b\sum_{k=1}^{K}\sum_{m=1}^{M_t}\nu_k\right)\right) - \widetilde{\mu}_y}{\widetilde{\sigma}_y}\right) \end{aligned} \tag{20}$$

where $\widetilde{\mu}_y$ and $\sigma_{\widetilde{Y}}^2$ are the mean of dB value and the variance of dB value of Y, respectively, and given by

$$\widetilde{\mu}_y = \frac{10\ln(\mu_y)}{\ln 10} - \frac{10}{2\ln 10}\ln\left(\frac{\sigma_y^2}{\mu_y^2}+1\right) \quad \text{and} \quad \sigma_{\widetilde{Y}}^2 = \left(\frac{10}{\ln 10}\right)^2 \ln\left(\frac{\sigma_y^2}{\mu_y^2}+1\right) \tag{21}$$

Finally, the (unconditional) outage probability can be obtained by

$$\begin{aligned} P_{out} &= \mathbb{E}_{\{\nu_k\}}\left[Q\left(\frac{10\log\left(\frac{1}{a}\left(\frac{1}{\eta} - b\sum_{k=1}^{K}\sum_{m=1}^{M_t}\nu_k\right)\right) - \widetilde{\mu}_y}{\widetilde{\sigma}_y}\right)\right] \\ &= \sum_{n=0}^{K} Q\left(\frac{10\log\left(\frac{1}{a}\left(\frac{1}{\eta} - nbM_t\right)\right) - \widetilde{\mu}_y}{\widetilde{\sigma}_y}\right)\binom{K}{n}\nu^n(1-\nu)^{K-n} \end{aligned} \tag{22}$$

where $\binom{K}{n}$ denotes the "K-choose-n".

Now, the outage capacity can be evaluated based on (22) for a given target outage probability and the average throughput can be expressed as $\sum_{m=1}^{M_t} KR^{(m)}$ where $R^{(m)}$ is the data rate of the m-th substream and K is the outage capacity in terms of number of simultaneous users.

B. Analysis for MMSE Filter

In this subsection, a linear MMSE filter is considered as the pre-filter for spatial separation. If we apply an MMSE filter designed in noise-limited environment given by

$$\mathbf{G}_{0,k} = \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{R}_{0,k}\mathbf{H}_{0,k}^H \left[\sigma_n^2 \mathbf{I}_{\mathbf{M_r}} + \frac{\chi_{0,k}}{(d_{0,k})^l}\mathbf{H}_{0,k}\mathbf{R}_{0,k}\mathbf{H}_{0,k}^H\right]^{-1} \tag{23}$$

where $\mathbf{R}_{0,k}$ is the covariance matrix of the desired signals from the home base station and $\mathbf{I}_{M_r}$ is an $M_r \times M_r$ identity matrix, the performance approaches the ZF filter because the received power for the desired signal is much higher than the thermal noise power.

In an interference plus noise environment, an MMSE filter considering the interference signals is the optimum linear receiver in the sense of maximizing SIR [30] and given by

$$\mathbf{G}_{0,k} = \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{R}_{0,k}\mathbf{H}_{0,k}^H \left(\mathbf{R}_{\mathrm{I},k} + \frac{\chi_{0,k}}{(d_{0,k})^l}\mathbf{H}_{0,k}\mathbf{R}_{0,k}\mathbf{H}_{0}^H\right)^{-1} \tag{24}$$

where $\mathbf{R}_{0,k}$ and $\mathbf{R}_{\mathrm{I},k}$ are the covariance matrix of the desired signals from the home base station and the covariance matrix of interfering signals conditioned on channel fading, respectively, and given by

$$\begin{aligned}\mathbf{R}_{0,k} &= \mathbb{E}\left[\mathbf{S}_0\mathbf{S}_0^H\right]\\ &= \begin{pmatrix} \sum_{k=1}^K \varphi_{0,k}^{(1)} P_0 & 0 & 0 & 0 \\ 0 & \sum_{k=1}^K \varphi_{0,k}^{(2)} P_0 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{k=1}^K \varphi_{0,k}^{(M_t)} P_0 \end{pmatrix}\end{aligned} \tag{25}$$

$$\begin{aligned}\mathbf{R}_{\mathrm{I},k} &= \sum_{i=1}^{N_c} \frac{\chi_{i,k}}{(d_{i,k})^l}\mathbf{H}_{i,k}\ \mathbb{E}\left[\mathbf{S}_i\mathbf{S}_i^H\right]\mathbf{H}_{i,k}^H\\ &= \sum_{i=1}^{N_c} \frac{\chi_{i,k}}{(d_{i,k})^l}\mathbf{H}_{i,k} \begin{pmatrix} \sum_{k=1}^K \varphi_{i,k}^{(1)} P_i & 0 & 0 & 0 \\ 0 & \sum_{k=1}^K \varphi_{i,k}^{(2)} P_i & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{k=1}^K \varphi_{i,k}^{(M_t)} P_i \end{pmatrix} \mathbf{H}_{i,k}^H\end{aligned} \tag{26}$$

The MMSE filter given in (24) contends with not only the spatial interference but also intercell interference so that we can expect better performance. This MMSE filter is a kind of multiuser detector.

However, it requires channel state information of the interfering signals and is unlikely to satisfy minimal complexity requirements in mobile stations. Unless the filter is updated at the short-term fading rate, the optimal MMSE filter is not feasible from a practical point-of-view. Nevertheless, we consider it here to show the potential improvement of the ZF receiver due to decrease interference/noise enhancement.

For analytic tractability, we use the averaged value of shadow fading instead of instantaneous value in the MMSE filter such as

$$\mathbf{G}_{0,k} = \sqrt{\frac{\mathbb{E}\left[\chi_{0,k}\right]}{(d_{0,k})^l}}\mathbf{R}_{0,k}\mathbf{H}_{0,k}^H\left(\mathbf{R}_{\mathrm{I},k} + \frac{\mathbb{E}\left[\chi_{0,k}\right]}{(d_{0,k})^l}\mathbf{H}_{0,k}\mathbf{R}_{0,k}\mathbf{H}_0^H\right)^{-1} \tag{27}$$

$$\mathbf{R}_{\mathrm{I},k} = \sum_{i=1}^{N_c}\frac{\mathbb{E}\left[\chi_{i,k}\right]}{(d_{i,k})^l}\mathbf{H}_{i,k}\begin{pmatrix} \sum_{k=1}^K \varphi_{i,k}^{(1)}P_i & 0 & 0 & 0 \\ 0 & \sum_{k=1}^K \varphi_{i,k}^{(2)}P_i & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{k=1}^K \varphi_{i,k}^{(M_t)}P_i \end{pmatrix}\mathbf{H}_{i,k}^H \tag{28}$$

This can be regarded as a suboptimal MMSE filter and its performance can be interpreted as a lower bound of the performance of the optimal MMSE filter.

1) Signal-to-Interference Ratio: Similarly to the analysis for ZF filters, the mth substream for the desired user k after MMSE filtering and MF filtering is given by

$$\begin{aligned} z_{0,k}^{(m)} &= \sum_{q=1}^{M_t}\sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(q)}\mathbf{s}_0^{(q)}(\mathbf{c}_{0,k}^{(m)})^H \;+\; \sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{g}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i(\mathbf{c}_{0,k}^{(m)})^H \;+\; \mathbf{g}_{0,k}^{(m)}\mathbf{N}_k(\mathbf{c}_{0,k}^{(m)})^H \\ &= \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}\sqrt{\varphi_{0,k}^{(m)}\ P_0}\ b_{0,k}^{(m)} + \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\sum_{j=1,j\neq k}^{K}\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}\rho\sqrt{\alpha\varphi_{0,j}^{(m)}\ P_0}\ b_{0,j}^{(m)} \\ &\quad + \sqrt{\frac{\chi_{0,k}}{(d_{0,k})^l}}\sum_{q=1,q\neq m}^{M_t}\sum_{j=1}^{K}\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(q)}\rho\sqrt{\alpha\varphi_{0,j}^{(q)}\ P_0}\ b_{0,j}^{(q)} + \sum_{i=1}^{N_c}\sqrt{\frac{\chi_{i,k}}{(d_{i,k})^l}}\mathbf{g}_{0,k}^{(m)}\mathbf{H}_{i,k}\mathbf{S}_i(\mathbf{c}_{0,k}^{(m)})^H \\ &\quad + \mathbf{g}_{0,k}^{(m)}\mathbf{N}_k(\mathbf{c}_{0,k}^{(m)})^H \end{aligned} \tag{29}$$

where $\mathbf{g}_{0,k}^{(m)}$ is the mth row of the matrix $\mathbf{G}_{0,k}$ and $\mathbf{h}_{0,k}^{(q)}$ is the qth column of matrix $\mathbf{H}_{0,k}$. Since the thermal noise is negligible compared to interference, SIR of the mth substream for the desired user k is obtained by

$$\gamma_k^{(m)} = \frac{\varphi_{0,k}^{(m)}P_0\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right]}{I_{sc1} + I_{sc2} + I_{oc}} \tag{30}$$

where I_{sc1}, I_{sc2}, and I_{oc} denote self-cell interference power from mth substreams of other users, self-cell interference power from different substreams of other users, and other-cell interference power, respectively.

If the allocated power to each transmit antenna is equal, the interference power I_{sc1}, I_{sc2}, and I_{oc} are given by

$$I_{sc1} = \left(\frac{1}{M_t} - \varphi_{0,k}^{(m)}\right) \alpha\rho^2 P_0 \; \mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right] \tag{31}$$

$$I_{sc2} = \sum_{q=1,q\neq m}^{M_t} \frac{\alpha\rho^2 P_0}{M_t}\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(q)}|^2\right] \tag{32}$$

$$I_{oc} = \sum_{i=1}^{N_c} \frac{\chi_{i,k}\left(d_{0,k}\right)^l}{\chi_{0,k}\left(d_{i,k}\right)^l}\frac{\rho^2 P_i}{M_t}\sum_{j=1}^{M_r}\mathbb{E}\left[|g_{0,k,j}^{(m)} \; h_{i,k,j,j}|^2\right] \tag{33}$$

where $g_{0,k,j}$ is the jth element of the vector $\mathbf{g}_{0,k}$ and $h_{i,k,n,m}$ is the (n,m) element of the matrix $\mathbf{H}_{i,k}$. Compared to the case of a ZF filter, the mean values in (31)-(33) cannot be obtained in a closed form and should be evaluated by computer simulations.

2) Outage Probability and Capacity: Based on the SIR, the relative power portion assigned to the mth substream of user k at (r,θ) is obtained as

$$\begin{aligned}\varphi_{0,k}^{(m)}(r,\theta) &= \frac{\rho^2\gamma_{req}^{(m)}}{M_t}\cdot\frac{\alpha + \sum_{q=1,q\neq m}^{M_t}\frac{\alpha\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(q)}|^2\right]}{\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right]} + \sum_{i=1}^{N_c}\frac{\chi_{i,k}\left(d_{0,k}\right)^l}{\chi_{0,k}\left(d_{i,k}\right)^l}\sum_{j=1}^{M_r}\frac{\mathbb{E}\left[|g_{0,k,j}^{(m)} \; h_{i,k,j,j}|^2\right]}{\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right]}}{\alpha\rho^2\gamma_{req}^{(m)}+1}\\ &= a'\xi_k^{(m)} + b'\end{aligned} \tag{34}$$

where

$$a' = \frac{\rho^2\gamma_{req}^{(m)}}{M_t}\cdot\frac{\sum_{j=1}^{M_r}\mathbb{E}\left[|g_{0,k,j}^{(m)} \; h_{i,k,j,j}|^2\right] / \; \mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right]}{\alpha\rho^2\gamma_{req}^{(m)}+1} \tag{35}$$

$$b' = \frac{\rho^2\gamma_{req}^{(m)}}{M_t}\cdot\frac{\alpha + \sum_{q=1,q\neq m}^{M_t}\frac{\alpha\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(q)}|^2\right]}{\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)}\mathbf{h}_{0,k}^{(m)}|^2\right]}}{\alpha\rho^2\gamma_{req}^{(m)}+1} \tag{36}$$

Then, by the same procedure for ZF filters, the outage probability can be obtained by

$$P_{out} = \sum_{n=0}^{K} Q\left(\frac{10\log\left(\frac{1}{a'}\left(\frac{1}{\eta} - nb'M_t\right)\right) - \widetilde{\mu}_y}{\widetilde{\sigma}_y}\right)\binom{K}{n}\nu^n(1-\nu)^{K-n} \tag{37}$$

IV. Effectiveness of spatial multiplexing in Cellular MIMO-CDMA Systems

A. Capacity Gain of MIMO-CDMA Systems with Linear Receivers

In this section, we discuss the effectiveness of spatial multiplexing in cellular MIMO-CDMA systems with linear receivers. From the results of the previous section, we can find that the outage capacity is closely related to the power assigned to each user. Therefore, a comparison between the power allocated to a particular user of the MIMO-CDMA system with spatial multiplexing and the conventional single-input single-output (SISO)-CDMA systems can roughly predict the outage capacity gain. For the conventional SISO-CDMA systems, the relative power portion allocated to a particular user k at (r,θ) is given by

$$\varphi_{0,k}^{conv}(r,\theta)=\frac{\frac{\hat{\rho}^2 P_i}{P_0}\xi_k\hat{\gamma}_{req}+\alpha\hat{\rho}^2\hat{\gamma}_{req}}{1+\alpha\hat{\rho}^2\hat{\gamma}_{req}} \tag{38}$$

where $\hat{\gamma}_{req}$ is the required averaged SIR for conventional SISO-CDMA systems, $\hat{\rho}$ is the cross correlation value, and ξ_k is equal to $\xi_k^{(m)}$ given in (13).

For a fair comparison, the data rates of conventional SISO-CDMA systems and the MIMO-CDMA systems using spatial multiplexing are all assumed to be R. That is, the data rate of each substream of MIMO-CDMA becomes R/M_t and thus, the corresponding processing gains are J/M_t for conventional SISO-CDMA and J for each substream of MIMO-CDMA, respectively. The values of required SIR for a given average bit error rate are the same for the two systems because we assume the same data rate so that $\gamma_{req}^{(m)}=\hat{\gamma}_{req}=\gamma_{req}$. We also assume perfect orthogonality among orthogonal spreading codes in a cell ($\alpha=0$) and $P_i=P_0$. The cross correlation value is approximately $\frac{2}{3\cdot PG}$ [31], [32], that is, $\hat{\rho}^2\approx\frac{2M_t}{3J}$ and $\rho^2\approx\frac{2}{3J}$.

Now, we define a power gain as a rough metric for capacity gain, which is the ratio of the allocated power of a particular user of the MIMO-CDMA system and the conventional SISO-CDMA system, as

$$G_P=\frac{\varphi_{0,k}^{conv}}{\sum_{m=1}^{M_t}\varphi_{0,k}^{(m)}} \tag{39}$$

1) Capacity Gain for ZF filters: For ZF filters, the power gain defined in (39) can be obtained with (12) and (38) by

$$G_{P,ZF}=\frac{2P_0}{3J\mathbb{E}\left[\zeta_{i,k}^{(m)}\right]} \tag{40}$$

Based on the definition of the power gain, we can find that the MIMO-CDMA systems using spatial multiplexing always have an advantage in terms of outage capacity over the conventional SISO-CDMA systems only when the power gain is greater than 1.

For the case that $M_t=M_r=M$, the power gain $G_{P,ZF}$ becomes

$$G_{P,ZF}=\frac{M}{-2Ei\left(-1/L\right)} \tag{41}$$

Therefore, we can conclude that spatial multiplexing in cellular $M \times M$ MIMO-CDMA systems with ZF filters is always better than conventional SISO-CDMA systems in terms of outage capacity only if the number of antennas is larger than $-2Ei(-1/L)$. This fact tells us that spatial multiplexing in cellular MIMO-CDMA systems with ZF filters does not always have full advantage of capacity gain although it has been known to take full advantage of capacity in simple MIMO communications with linear receivers. This result fully agrees with the information theoretic prediction on the capacity of MIMO systems with linear receiver in the presence of co-channel interference given in [17], [18]. As explained in the introduction, the authors of [17], [18] showed that whereas the capacity of a single, isolated link could not be improved by reducing the number of substreams, it would be better to have all users use fewer than maximum number of possible substreams in order to increase the capacity of each user in interference limited MIMO systems with linear receivers.

2) Capacity Gain for MMSE Filters: For MMSE filters, the power gain defined in (39) can be obtained with (34) and (38) as

$$G_{P,MMSE} = \frac{M_t \mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)} \mathbf{h}_{0,k}^{(m)}|^2\right]}{\sum_{j=1}^{M_r} \mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)} \;\; h_{i,k,j,j}|^2\right]} \tag{42}$$

Similarly to the case for ZF filters, we can conclude that the spatial multiplexing in cellular $M_t \times M_r$ MIMO-CDMA systems with MMSE filters is always better than the conventional SISO-CDMA systems in terms of outage capacity if the number of transmit antennas M_t is larger than $\sum_{j=1}^{M_r} \mathbb{E}\left[|g_{0,k,j}^{(m)} \;\; h_{i,k,j,j}|^2\right]$ / $\mathbb{E}\left[|\mathbf{g}_{0,k}^{(m)} \mathbf{h}_{0,k}^{(m)}|^2\right]$. Compared to the ZF filters, the optimum MMSE filter achieves a capacity gain even with $M_t = M_r = 2$ because the optimum MMSE filter contends with not only spatial interference but also intercell interference. This result also agrees with the results of [12] that MUD schemes contending with other-cell interference in principle increase capacity in the presence of co-channel interference. However, the optimum MMSE filter requires channel states information of the interfering signals so is not very practical in a cellular context.

B. Spatial Multiplexing: Discussion and Cooperatively Scheduled Transmission

Alert readers have likely noticed in the previous sections that spatial multiplexing may not the best choice for cellular MIMO-CDMA in terms of outage capacity unless a technique contending with other-cell interference is introduced. The most straightforward technique is to deploy a sufficient numbers of antennas at both the transmitter and receiver. If the number of antennas is sufficiently large, sufficient degrees of freedom are attained that can be used to compensate for the interference enhancement caused by the linear receivers. Another possible scheme is to use linear filters that contend not only with spatial interference but also with the other-cell interference, such as MMSE filters as introduced in this paper, or the joint detection scheme of [12]. However, in commercial systems, the number of antennas and the complexity of mobile stations are critical restrictions and so these methods are not likely to be practical in the foreseeable future.

From the perspective of maximum user data rate, spatial multiplexing is very attractive because it can dramatically increase throughput for realistic bandwidths, modulation orders, and other system parameters. In addition, the complexity advantage of linear receivers is too attractive to be easily given up, especially in the downlink where the mobile receiver will have severe power and complexity constraints.

In order to advance the performance of MIMO-CDMA with spatial multiplexing and linear receivers, we propose a cooperatively scheduled transmission scheme among base stations. In cooperatively scheduled transmission, six adjacent base stations and an (arbitrarily chosen) home base station in a hexagonal cell structure periodically transmit their signals one at a time for all users according to a pre-determined sequence, and thus the duty cycle of each base station becomes 1/7. This scheme can be regarded as a kind of time division multiple access among base stations and has a similar effect to frequency reuse in GSM or other non-CDMA cellular systems. The principal difference is that in the proposed cooperatively scheduled transmission scheme, each cell uses the whole frequency bandwidth whereas each cell of a traditional frequency reuse system is allocated only a partial (e.g. 1/7 of the total) frequency bandwidth.

Because the penalty for linear receivers in spatial multiplexing mainly comes from other-cell interference enhancement, the cooperatively scheduled transmission should improve the performance since it dramatically reduces the perceived other-cell interference. Naturally, this comes at the expense of potentially decreasing the overall transmission rate by the time reuse factor, or duty cycle. Specifically, the instantaneous throughput of each cell is multiplied by 1/7 for fair comparison with the universal frequency scheme that must tolerate higher levels of other-cell interference. Our numerical results in the next section will demonstrate that a substantial net capacity increase is achieved with this scheme. Importantly, this improvement is attained without increasing the complexity of mobile units.

Although in this paper we only consider a simple manifestation of cooperatively scheduled transmission, a more sophisticated transmission scheduling algorithm might more effectively address the impairment by other-cell interference while minimizing the average throughput reduction by the duty cycle. For example, each base stations could adopt a cooperatively scheduled transmission only for the users outside a certain range of each base station. This scheme requires the location information of each user and more complicated scheduling but could more effectively reduce the other-cell interference since users near the cell boundary are the principal sources of other-cell interference. The analysis and investigation of such schemes is left for future research.

V. Numerical Results

In the previous section, we investigated the effectiveness of the spatial multiplexing by considering a newly defined power gain, which quantified the ratio between the required transmit power for MIMO-CDMA and for SISO-CDMA for a fixed SIR. The concept of power gain is useful in understanding the effectiveness of the spatial multiplexing but cannot give us exact outage capacity comparisons. The exact outage capacity can be obtained from the outage probability formula (25). In this section, we provide sample outage capacities for spatial multiplexing in cellular MIMO-CDMA systems with various numbers of antennas and compare with conventional SISO-CDMA systems. In these numerical results, we assume the constraint imposed on the inverted chi-squared random variable in (10), L, to be 1000. Generally, the value for L depends on available transmit power and varies according to systems and this value can be regarded as an example. We also assume that the path loss exponent l is 4, the number of interfering base stations, N_c is 18, and the spreading bandwidth to be 3.84MHz, which is the bandwidth of UMTS WCDMA systems. The standard deviations of the lognormal fading from adjacent base stations and a home base station are assumed as 8dB and 2dB, respectively, the forward power factor η is 0.177, and the data activity ν is set to 1 to indicate full-rate transmission.

Figures 2 and 3 show the outage probability according to the number of simultaneous users when the orthogonality factor α is 0 and 0.5, respectively. We assume that the data rate of the conventional SISO-CDMA system is 60Kbps and thus, the processing gain is 64. For a fair comparison, the data rate of MIMO-CDMA systems with spatial multiplexing and linear receivers should also be 60Kbps.

That is, the data rate of each substream of MIMO-CDMA becomes $60\mathrm{Kbps}/M_t$ and thus, the data rate of each substream of a 2×2, 4×4, 8×8, and 12×12 MIMO-CDMA system becomes 30Kbps, 15Kbps, and 7.5Kbps, and 5Kbps, respectively. The processing gains corresponding to the data rates become 128, 256, 512, and 768. We also assume the required (uncoded) SIR for a given averaged bit error rate to be 1.5dB. From Figure 3, if the target outage probability is 0.1, the average throughput of the conventional SISO-CDMA system, 2×2, 4×4, 8×8, and 12×12 MIMO-CDMA systems with ZF filters becomes approximately 1560Kbps, 264Kbps, 540Kbps, 1368Kbps, and 2340Kbps, respectively. The 2×2 and 4×4 MIMO-CDMA systems with ZF filters do not outperform the conventional SISO-CDMA systems because of the lack of degrees of freedom to combat the enhanced interference by the ZF filter whereas even 2×2 MIMO-CDMA system with optimum MMSE filters. The 8×8 MIMO-CDMA system has smaller outage probability than conventional SISO-CDMA systems only when the number of simultaneous user is large and the 12×12 MIMO-CDMA system always outperform the conventional SISO-CDMA system since there are so many degrees of freedom.

Figure 4 shows the power gains of the ZF filter and the optimum MMSE filter according to the number of antennas. As the definition in section IV.A, the orthogonality factor is 0 and other conditions are the same as those of Figure 3. Note that since the other-cell interference dominates the performance, there is only a negligible performance difference between $\alpha = 0$ and $\alpha = 0.5$. Although the power gain does not provide an exact capacity gain because it just quantifies the ratio between the required transmit power to a particular user at (r, θ) only, it is useful in roughly predicting the outage capacity gain according to the number of antennas. If the power gain is greater than unity, spatial multiplexing with multiple antennas is always better than single transmission in terms of outage capacity. From Figure 4, we can confirm that the power gain of a ZF filter becomes greater than unity only if the number of antennas is greater than twelve whereas that of the optimum MMSE filter is always greater than unity regardless of the number of antennas because the optimum MMSE filter contends with both spatial and other-cell interference.

Figure 5 shows the outage probability according to the number of simultaneous users per a cell when the cooperatively scheduled transmission among adjacent base stations is used. All conditions are the same as those of Figure 3 except that adjacent six base stations and a home base station in the hexagonal cell structure cooperatively transmit their signals. This figures shows a raw outage capacity with the cooperatively scheduled transmission before applying the reduction in capacity by the periodic transmission. The cooperatively scheduled transmission can achieve high capacity gain over conventional CDMA system without cooperatively scheduled transmission because it can significantly reduce other-cell interference and mitigate the impairment by interference/noise enhancement.

A throughput comparison taking into account the throughput reduction by the periodic transmission is given in Figure 6. This figure shows throughput of various systems according to various target outage probabilities. For fair comparison, throughput with the cooperatively scheduled transmission is multiplied by 1/7 to reflect average throughput reduction by duty cycle. For a target outage probability is 0.1, the average throughput of the 2×2 and 4×4 MIMO-CDMA systems becomes 703Kbps (=4920Kbps/ 7) and 1646Kbps (=11520Kbps/ 7), respectively, whereas the throughput of the conventional SISO-CDMA system is 1560Kbps. This means that while achieving a superior data rate, a 4×4 MIMO-CDMA system with cooperatively scheduled transmission could also markedly reduce the outage probability.

VI. CONCLUSIONS

In this paper, the outage probability and capacity of cellular MIMO-CDMA systems with spatial multiplexing and linear receivers has been derived. Using this new analytical framework, we have investigated the effectiveness of spatial multiplexing in cellular MIMO-CDMA systems, and compared with conventional SISO-CDMA. Although spatial multiplexing is known to have a large capacity advantage for single user or single cell systems, we have shown that, unfortunately, this capacity gain is lost when MIMO is adopted into an interference-limited setting with low-complexity receivers. The basic explanation for this loss is that linear MIMO receivers are forced to enhance some of the interference, which hurts the capacity more than multiple substreams help it. With this in mind, we have shown that cooperatively scheduled transmission among base stations can mitigate the impairment by interference enhancement and a net capacity improvement can be achieved without increasing the complexity of mobile units. Based on these results, an important conclusion is that cellular MIMO systems with linear receivers must properly consider the tradeoff between maximizing user data rates and maximizing outage capacity. Future research should further consider practical techniques for increasing the attainable throughput of interference-limited MIMO systems.

APPENDIX A
DERIVATION OF THE MEAN OF $\zeta_{i,k}^{(m)}$

In (10), the distribution of $1/\left(\mathbf{H}_{0,k}^{H}\mathbf{H}_{0,k}\right)_{m,m}^{-1}$ is known to be a chi-squared random variable with $2(M_r - M_t + 1)$ degrees of freedom [33]–[35]. Therefore, the mean can be obtained with the first moment of the inverted chi-squared random variable with $(2M_r - N_t + 1)$ degrees of freedom. Unfortunately, inverted chi-squared random variables do not have finite first or second moments for all degrees of freedom. But if there is an upper limit on the inverted chi-squared random variable, the first and the second moments can be found [36]. The random variables $\{\zeta_{i,k}^{(m)}\}$ contribute to interference level, but if the total interference is above a certain threshold, additional interference caused by the $\{\zeta_{i,k}^{(m)}\}$ will not effect the practical system performance. This observation can be justified from the fact that all transmitters have a constraint on the available transmit power and thus strong interference over a threshold cannot be arbitrarily compensated for with power control. Therefore, without loss of generality, we can impose the following constraint on the inverted chi-squared random variable with $2(M_r - M_t + 1)$ degrees of freedom.

$$\left(\mathbf{H}_{0,k}^{H}\mathbf{H}_{0,k}\right)_{m,m}^{-1} = \begin{cases} \left(\mathbf{H}_{0,k}^{H}\mathbf{H}_{0,k}\right)_{m,m}^{-1} & \text{if } \left(\mathbf{H}_{0,k}^{H}\mathbf{H}_{0,k}\right)_{m,m}^{-1} < L \\ 0 & \text{else} \end{cases} \tag{A.1}$$

The technique of imposing a constraint on the inverted chi-squared random variable to find moments was introduced in [37] and it is known that the particular choice of value of L is not important if the constraint is sufficiently large. Then, the mean can be obtained as in [36]

$$\mathbb{E}\left[\left(\mathbf{H}_{0,k}^{H}\mathbf{H}_{0,k}\right)_{m,m}^{-1}\right] = \begin{cases} -Ei\left(\frac{-1}{L}\right) & \text{for } M_r = M_t \\ \frac{e^{-1/L}}{M_t - 1}\sum_{n=0}^{M_r - M_t - 1}\frac{1}{n!\ L^n} & \text{for } M_r - M_t \geq 1 \end{cases} \tag{A.2}$$

where $Ei(x)$ is the exponential integral function ($Ei(x) = -\int_{-x}^{\infty} e^{-t}/t\ dt$) and can be easily calculated with popular numerical tools such as MATLAB, MATHEMATICA, and MAPLE. Then, the mean of $\zeta_{i,k}^{(m)}$ can be obtained with (10) and (A.2) as

$$\mathbb{E}\left[\zeta_{i,k}^{(m)}\right] = \begin{cases} -\frac{2\sigma^2\rho^2 P_i}{M_t} Ei\left(\frac{-1}{L}\right) & \text{for } M_r = M_t \\ \frac{2\sigma^2\rho^2 P_i e^{-1/L}}{M_t(M_r - M_t)}\sum_{n=0}^{M_r - M_t - 1}\frac{1}{n!\ L^n} & \text{for } M_r - M_t \geq 1 \end{cases} \tag{A.3}$$

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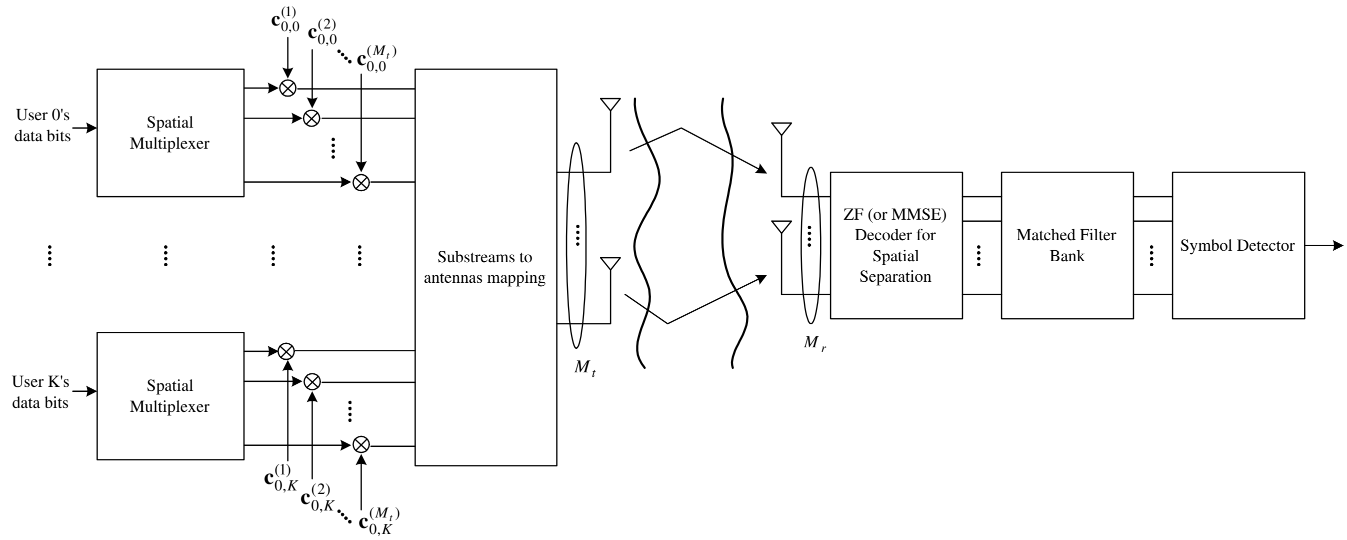

Fig. 1. Transmitter and receiver structure of downlink MIMO-CDMA systems using spatial multiplexing

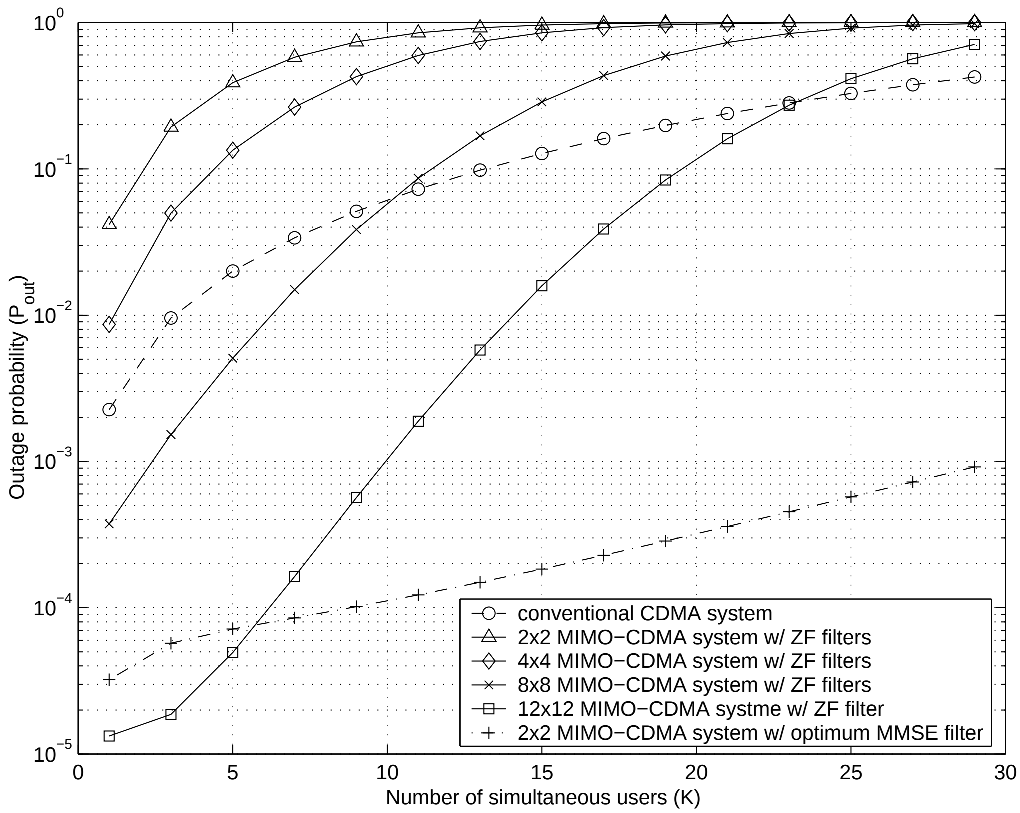

Fig. 2. Outage probability according to the number of simultaneous users ($\alpha = 0$)

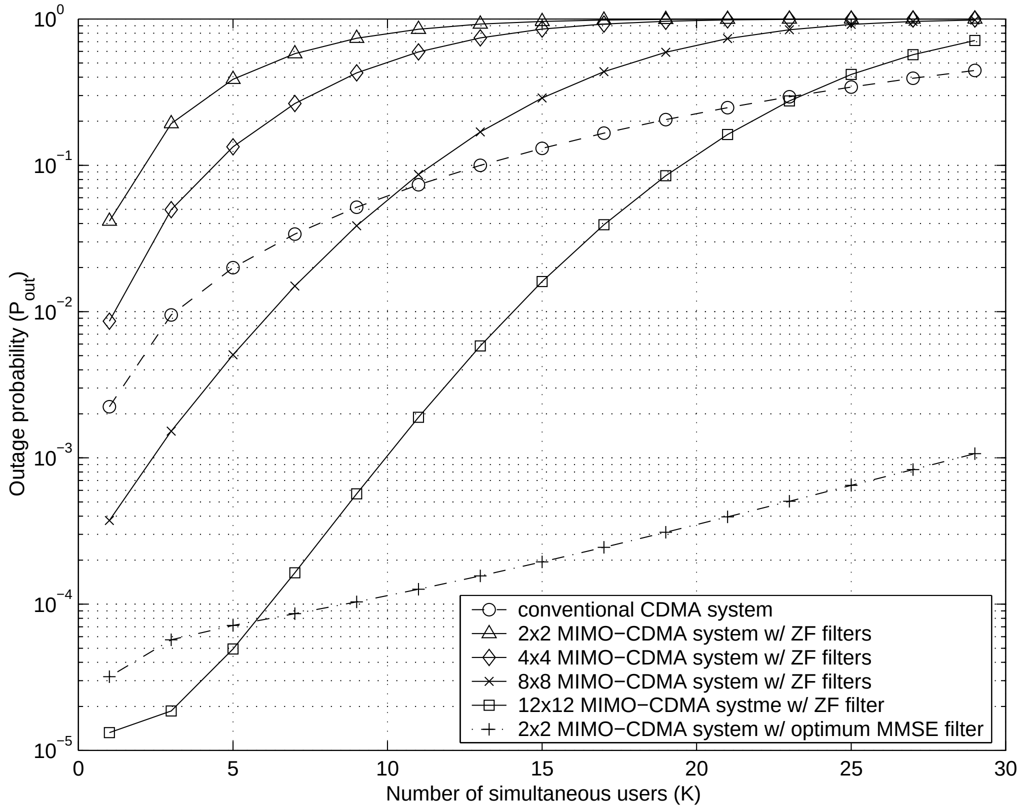

Fig. 3. Outage probability according to the number of simultaneous users ($\alpha = 0.5$)

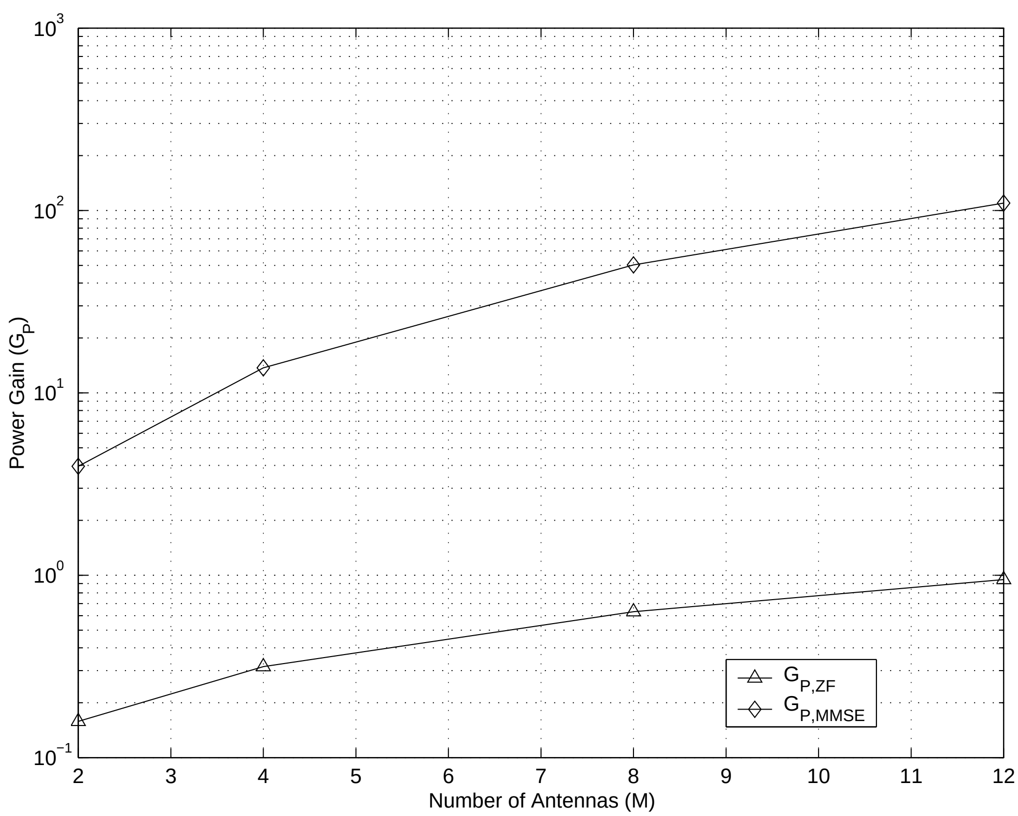

Fig. 4. Power Gains for a ZF filter and an optimum MMSE filter

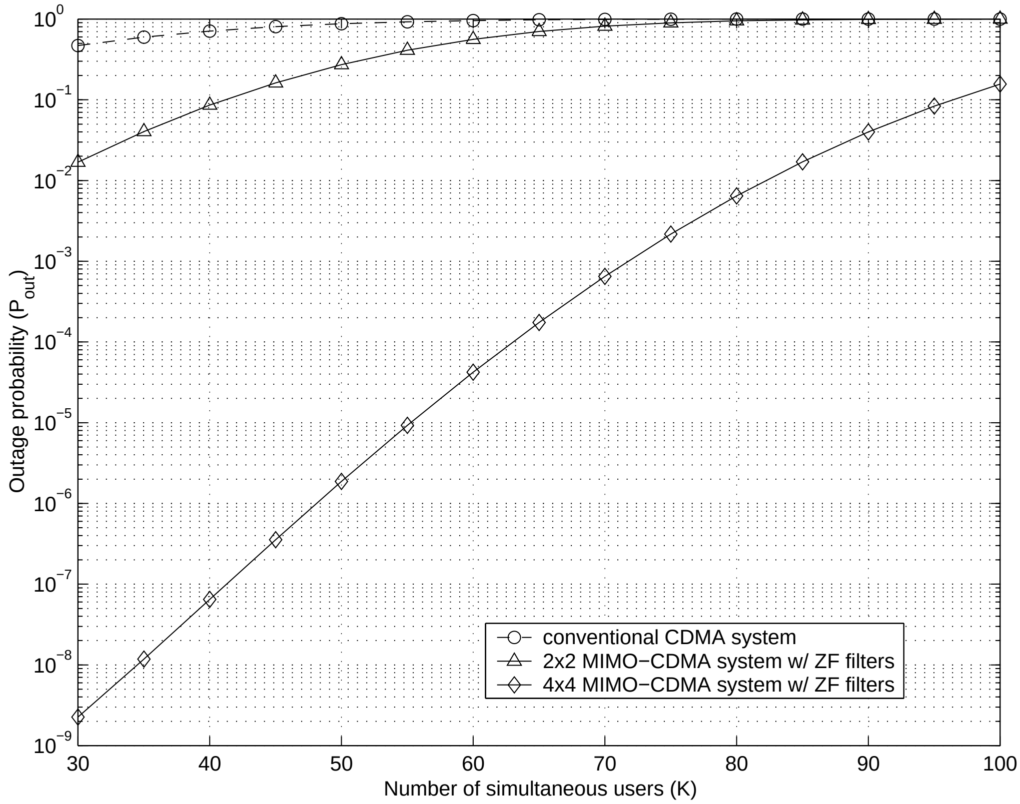

Fig. 5. Outage probability of the cooperative transmit system ($\alpha = 0.5$)

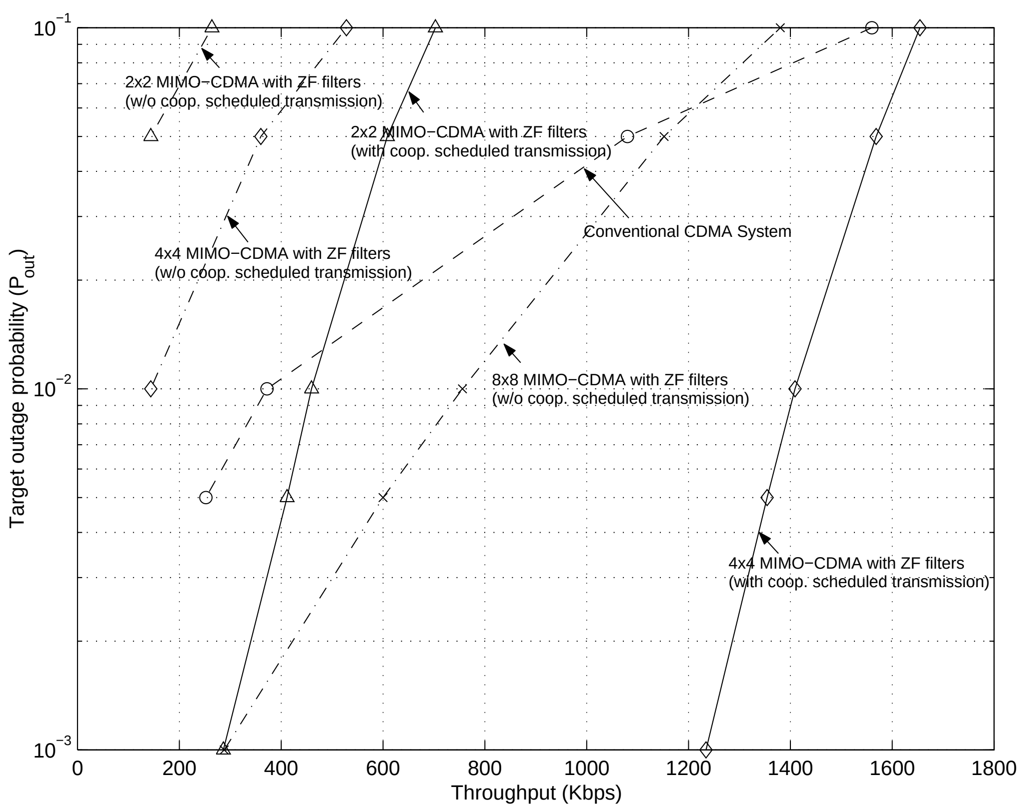

Fig. 6. Throughput according to target outage probability ($\alpha = 0.5$)